Analysis of continuous beams – including settlement of supports; Single bay portal frames with and without Side sway.

Introduction:- * GASPER KANI, A german Engineer, developed another distribution procedure based on slope deflection equations.

* This method is very useful for the analysis of multistorey frames.

* The greatest advantage of this method is, even if a mistake is committed in distribution in one of the cycles, it converges finally to the correct answer.

* Even today, many practicing engineers who are not familiar with computer methods, use Kani's method for the analysis of 3 to 4 storey building frames.

Sign convention: 1. clockwise end moments are positive
2. clockwise rotations are positive.

Let member AB shown below is an intermediate member of a beam/frame, which has no relative displacements at the ends.

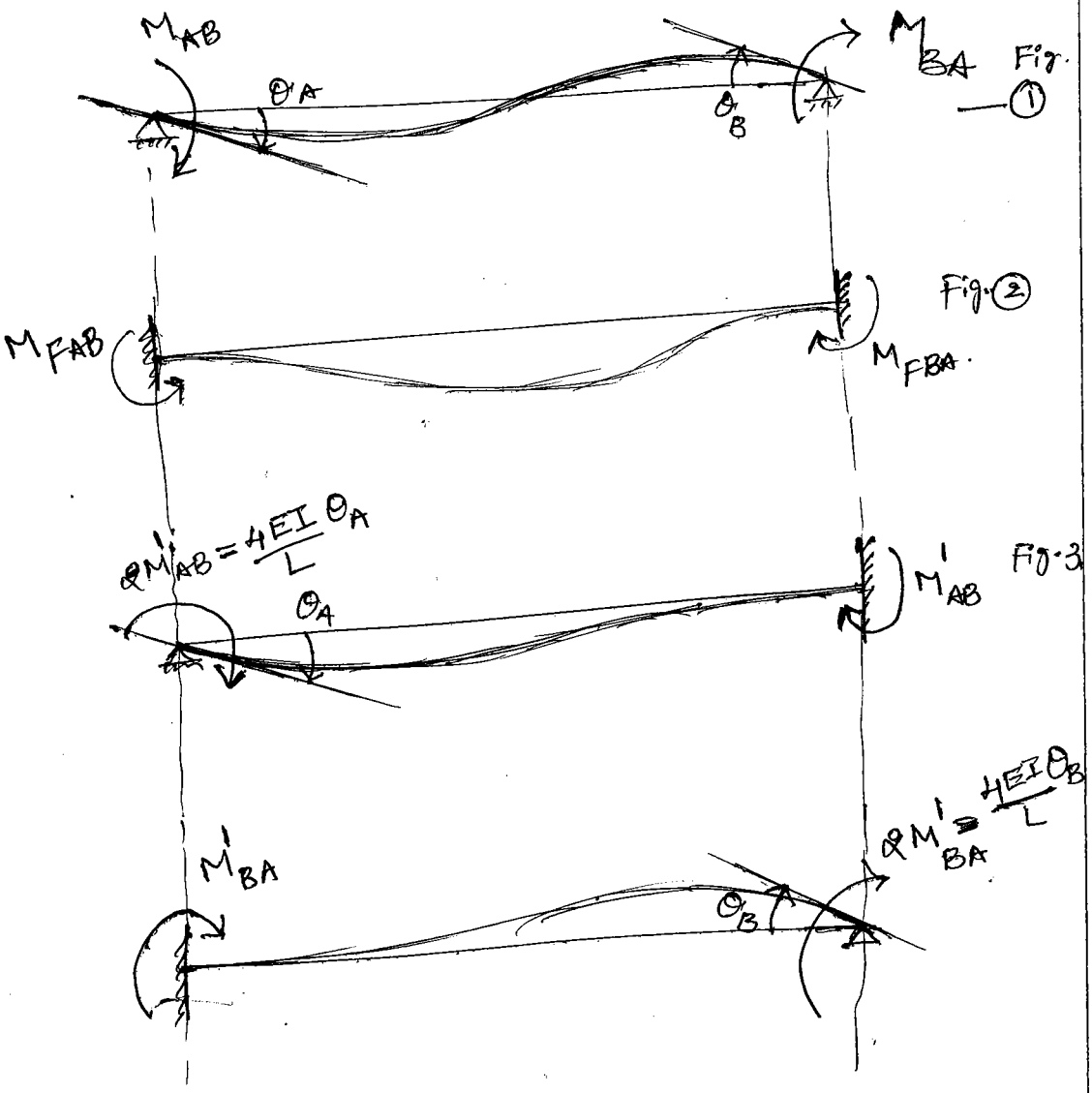
→ Let M_{AB} and M_{BA} are ~~final~~ end moments

M_{AB} may consists of:

(i) Fixed end moments ($\theta_A = \theta_B = 0$)

(ii) moment due to rotation of end A only.

(iii) " " " B "



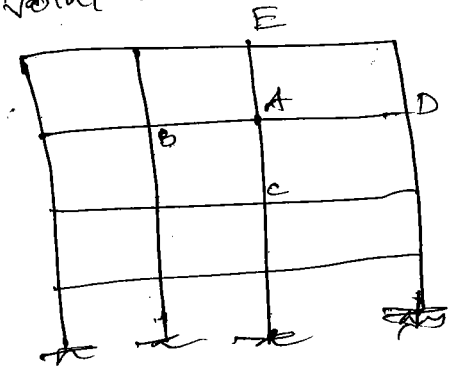
$$\therefore \quad M_{AB} = M_{FAB} + \tfrac{1}{2}M'_{AB} + M'_{BA}$$

$$M_{BA} = M_{FBA} + M'_{AB} + \tfrac{1}{2}M'_{BA}$$

M'_{AB}, M'_{BA} are called rotation contributions.

$\therefore$ Final moment = Fixed end moment + $\frac{1}{2}$(Rotation contribution of near end) + Rotation Contribution of far end

Now, Consider Moments at Joint A shown in the frame.



$$M_{AB} = M_{FAB} + \tfrac{1}{2}M'_{AB} + M'_{BA}$$
$$M_{AC} = M_{FAC} + \tfrac{1}{2}M'_{AC} + M'_{CA}$$
$$M_{AD} = M_{FAD} + \tfrac{1}{2}M'_{AD} + M'_{DA}$$
$$M_{AE} = M_{FAE} + \tfrac{1}{2}M'_{AE} + M'_{EA}.$$

at A; $\Sigma M_{AB} = \Sigma M_{FAB} + 2\Sigma M'_{AB} + \Sigma M'_{BA}$

from Joint equilibrium $M_A = 0$

$$\Sigma M_{FAB} + 2\Sigma M'_{AB} + \Sigma M'_{BA} = 0$$

$$\Sigma M'_{AB} = -\frac{1}{2}\left(\Sigma M_{FAB} + \Sigma M'_{BA}\right) \quad \text{---(1)}$$

from, $2M'_{AB} = \frac{4EI\theta_A}{L} = (K_{AB})\theta_A$

$$M'_{AB} = \left(\frac{K_{AB}}{2}\right)\theta_A$$

$$\therefore \Sigma M'_{AB} = \left(\frac{\Sigma K_{AB}}{2}\right)\theta_A = \frac{1}{2}\theta_A \Sigma K_{AB}$$

for all the members meeting at A θ_A is same.

$$M'_{AB} = \left(\frac{K_{AB}}{2}\right)\theta_A$$

Taking the ratio, $\frac{M'_{AB}}{\Sigma M'_{AB}} = \frac{K_{AB}}{\Sigma K_{AB}}$

$$\therefore M'_{AB} = \frac{(K_{AB})}{(\Sigma K_{AB})}\Sigma M'_{AB} \quad \text{---(2)}$$

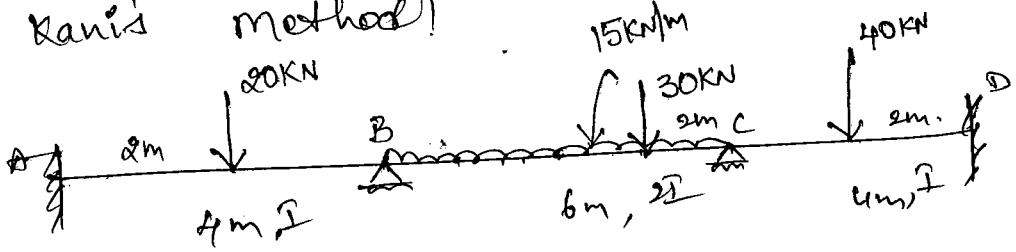
from (1); $M'_{AB} = -\frac{1}{2}\left(\frac{K_{AB}}{\Sigma K_{AB}}\right)\left(\Sigma M_{FAB} + \Sigma M'_{BA}\right)$ ---(3)

$\therefore -\frac{1}{2}\left(\frac{K_{AB}}{\Sigma K_{AB}}\right)$ is "Rotation factor"

procedure :-

1. find fixed end moments at all joints
2. calculate near end rotation contributions by assuming far end contributions as 0.
3. Similarly, calculate Rotation Contributions at all joints which completes first cycle
4. Now rotation contributions are available at far ends. Using the eqn (3) calculate near end contributions at all joints to complete one cycle
5. Repeat the procedure till a change in the rotation contributions of two successive iterations are negligible
6. calculate final end moments.

Prob: Analyse the continuous beam using Kani's method.



FEMs:- $M_{FAB} = \frac{-PL}{8} = -10\,KNm$; $M_{FBA} = +10\,KNm$.

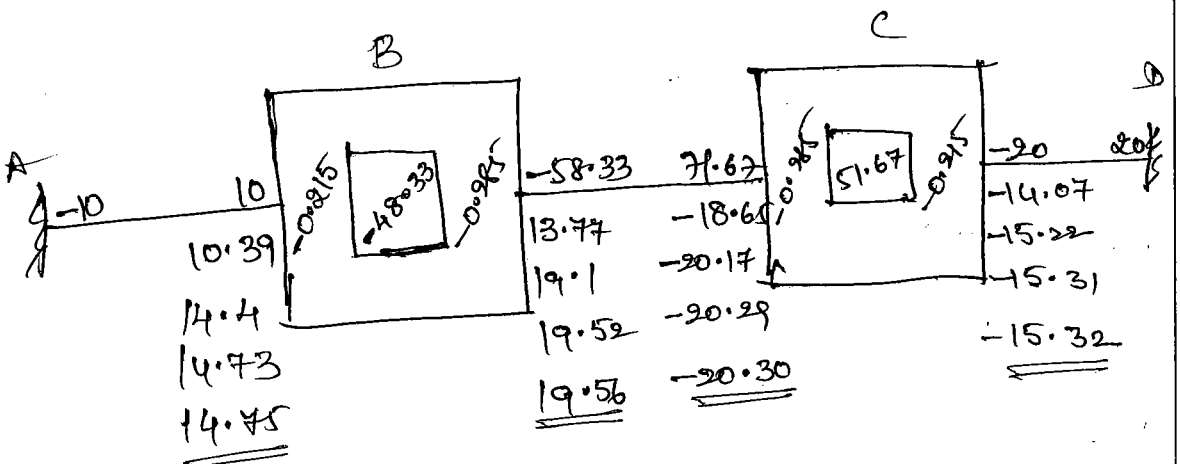
$M_{FBC} = -45 - 13.33 = -58.33\,KNM$

$M_{FCB} = 45 + 26.67 = 71.67\,KNm$.

$M_{FCD} = -80\,KNm$; $M_{FDC} = 80\,KNm$.

Rotation factor:- $R.F = -\frac{1}{2}\left(\frac{K}{\Sigma K}\right)$

Jt	Memb	K	ΣK	R.F
B	BA	$\frac{4EI}{4} = EI$	2.33EI	-0.215
	BC	$\frac{4E(2I)}{6} = 1.33EI$		-0.285
C	CB	$\frac{4E(2I)}{6} = 1.33EI$	2.33EI	-0.285
	CD	EI		-0.215



$M'_{AB} = R.F\left(\Sigma M_{FAB} + \Sigma M'_{BA}\right)$

cycle-I :- $M'_{BA} = -0.215(-48.33+0) = 10.4$

$M'_{BC} = -0.285(-48.33+0) = 13.77$

$M'_{CB} = -0.285(51.67+13.77) = -18.65$

$M'_{CD} = -0.215(51.67+13.77) = -14.07$

cycle II :- $M'_{BA} = -0.215(-48.33+0-18.65) = 14.4$

$M'_{BC} = -0.285(-48.33+0-18.65) = 19.1$

$M'_{CB} = -0.285(51.67+19.1+0) = -20.17$

$M'_{CD} = -0.215(51.67+19.1+0) = -15.22$

cycle III :- $M'_{BA} = -0.215(-48.33+0-20.17) = 14.73$

$M'_{BC} = -0.285(-48.33+0-20.17) = 19.52$

$M'_{CB} = -0.285(51.67+19.52+0) = -20.29$

$M'_{CD} = -0.215(51.67+19.52+0) = -15.31$

Final End moments :- $M_{BB} = M_{FAB} + 2M'_{BB} + M'_{BA}$

$M_{BB} = -10+2(0)+14.75 = 4.75$ KNm.

$M_{BA} = 10+2\times14.73+0 = 39.5$ KNm.

$M_{BC} = -58.33+2(19.52)-20.3 = -39.51$ KNm

$M_{CB} = 71.67+2(-20.3)+19.52 = 50.63$ KNm

$M_{CD} = -20+2(-15.32)+0 = -50.64$ KNm

$M_{DC} = 20+2(0)-15.32 = 4.68$ KNm.

Continuous beams with Hinge ends:-

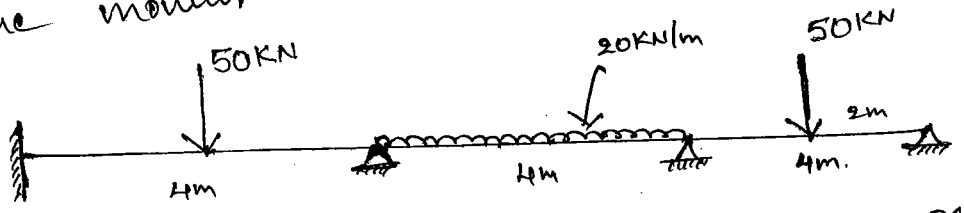
- Stiffness of member with far end Hinge is $\frac{3EI}{L}$.
- far end condition from Hinge is zero.
- Modify the moments of the member AB if 'B' is Hinge.

eg:- $M_{FAB} = M_{FAB} + COM. \text{ from } 'B'.$

$M_{FBA} = M_{FBA} - M_{FBA} = 0$

i.e., Balance the hinge end and carry the moment to the far end

Prob:-



Sol: $M_{FAB} = -25$; $M_{FBA} = 25 KNm.$; $M_{FBC} = -26.67$, $M_{FCB} = 26.67$

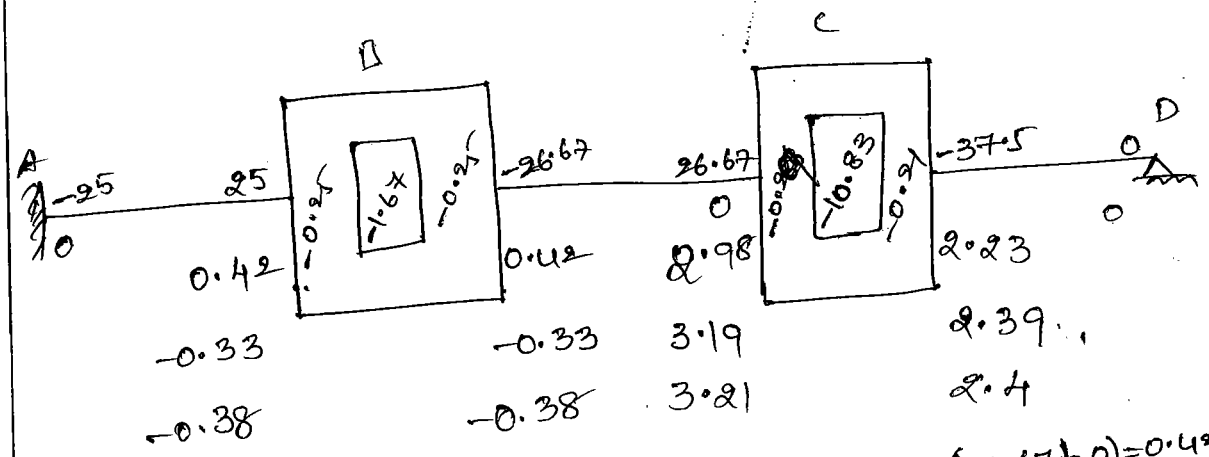
$M_{FCD} = -25$; $M_{FDC} = 25$

modified moments :- $M_{FDC} = 25 - 25 = 0$

$M_{FCD} = -25 + \left(\frac{25}{2}\right) = -37.5 \text{ KNm.}$

R.f Table:-

Jt	Memb	K	Σk	R.f
	BA	$\frac{4EI}{4} = EI$		-0.25
B			2EI	
	BC	$\frac{4EI}{4} = EI$		-0.25
	CB	$\frac{4EI}{4} = EI$		-0.29
C			1.75EI	
	CD	$\frac{3EI}{4} = 0.75EI$		-0.21



Cycle I :- $M'_{BA} = R.F\,(\Sigma M_{FBA} + \Sigma M'_{AB}) = -0.25(-1.67 + 0) = 0.42$

$M'_{BC} = R.F\,(\Sigma M_{FBC} + \Sigma M'_{BA}) = -0.25(-1.67) = 0.42$

$M'_{CB} = R.F\,(\Sigma M_{FCB} + \Sigma M'_{BC}) = -0.29(-10.83 + 0.42 + 0) = 2.98$

$M'_{CD} = -0.21(-10.83 + 0.42 + 0) = 2.23.$

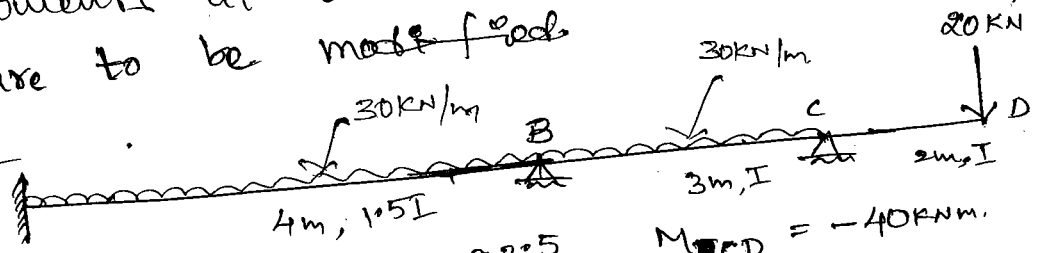
Only, Continue till any two successive iterations gives almost same Rotation contributions

Memory	A	B		C	D
F.EMs	-25	25	-26.67	26.67	-37.5	0
Near end (M'_{AB})	0	-0.76	-0.76	6.42	4.8	0
far end (M'_{BA})	-0.38	0	3.21	-0.38	0	0
Final end Moments	-25.38	24.24	-24.24	32.71	-32.71	0

Continuous beams with Overhangs :- $K_{BC} = \frac{3EI}{L}$

- moments at the support of overhang portion are to be modified.

eg:-

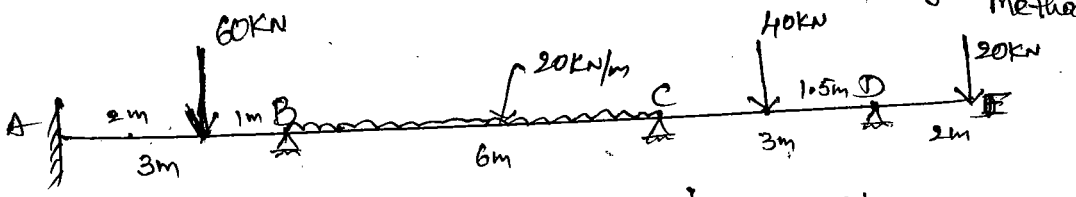


$M_{FBC} = -22.5$; $M_{FCB} = 22.5$ $M_{FCD} = -40$ KNM.

for, $M_{FCB} + M_{CD} = 0$ $\Rightarrow$ coe need to add 17.5 KNM to M_{FCB}, so ther it will become 40KNM. Further $(17.5/2) = 8.75$ KNM will be carried to M_{FBC}, hence M_{FBC} will be modified as -13.75 KNM.

www.Jntufastupdates.com

Prob: Analyse the continuous beam shown by Kani's Method.



$M_{FAB} = -13.33$

$M_{FBA} = 26.67$ kNm

$M_{FBC} = -60$

$M_{FCB} = 60$

$M_{FCD} = -15$

$M_{FDC} = (15)$

$M_{DE} = -40$.

Modified moments

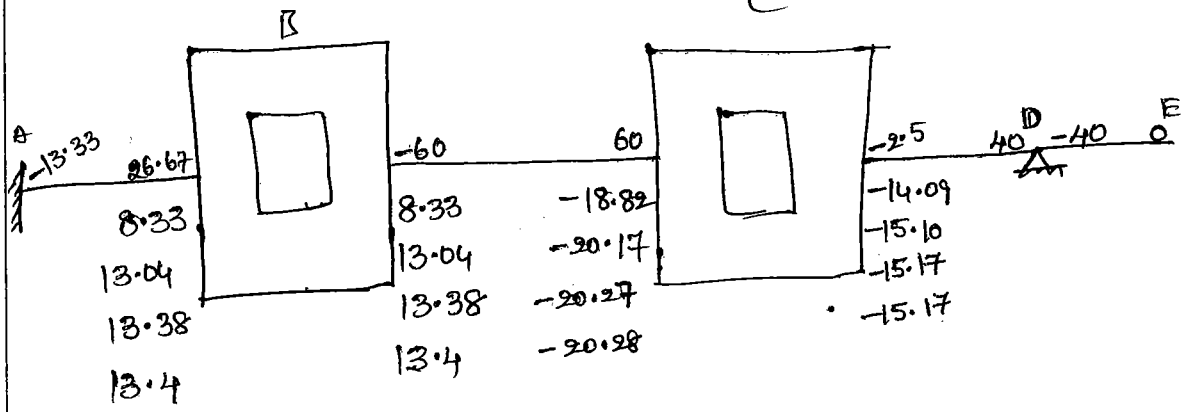
$M_{FDC} = 15 + 25 = 40$

$M_{FCD} = -15 + \left(\frac{25}{2}\right)$

$= -2.5$ kNm.

R.F BA = −0.25 ; CB = −0.29

BC = −0.25 ; CD = −0.21



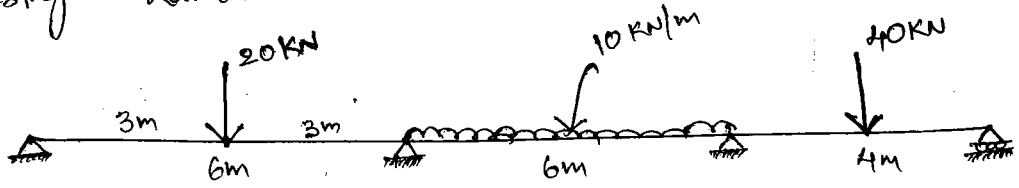
Final end moments:-

Remarks	A	B		C	D		E
FEM	-13.33	26.67	-60	60	-2.5	40	-40	0
2KNEC	2x0	2x13.4	2x13.4	2x-20.28	2x-15.17	0	0	0
FEC	13.4	0	-20.28	13.4	0	0	0	0
Final End moments	0	53.47	-53.47	32.84	-32.84	40	-40	0

Continuous beam with Sinking of Supports:-

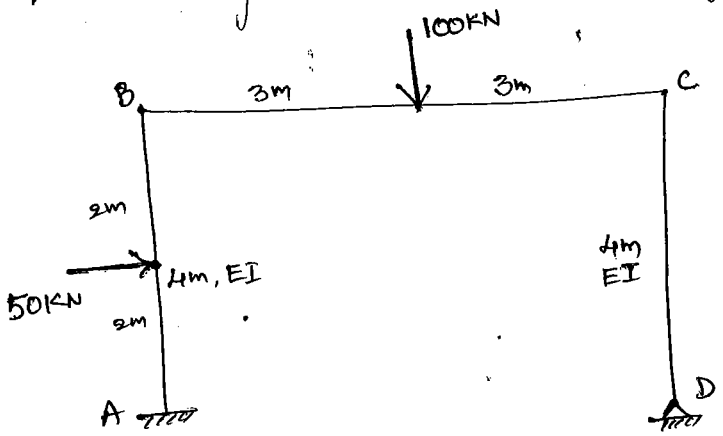
JNTUK Prob: A continuous beam ABCD SS over each support consists of spans AB, BC & CD of length 3m, 4m & 3m resp. Beam carries UDL of 12KN/m over the whole length. Support B & C Sink by 2mm and 4mm resp. Det the moments over each support by using Kani's Methods. Take $E = 2 \times 10^5$ N/mm²; $I = 4 \times 10^7$ mm⁴.

using Kani's method :

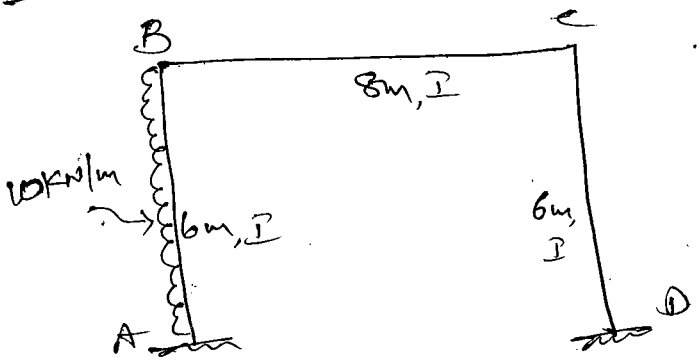


(Supply) JNTUKT A rectangular portal frame ABCD consists of legs AB and DC of ht 4m and 6m resp. Beam BC of length 8m. Carries UDL of 12 KN/m and a point load of 80KN at centre. Legs are fixed at their bottom. $I_{ab} : I_{bc} : I_{cd} = 1:2:1$. Analyse the frame using Rotation Contribution Method?

(Supply) JNTUKT Analyse the portal frame by using RCM?



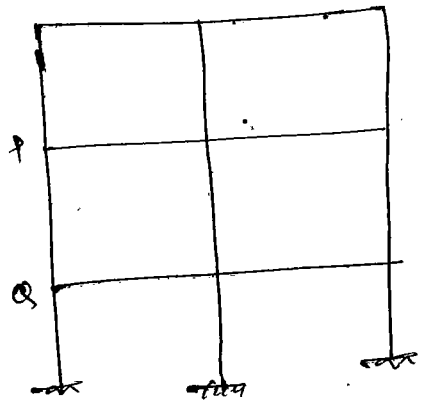
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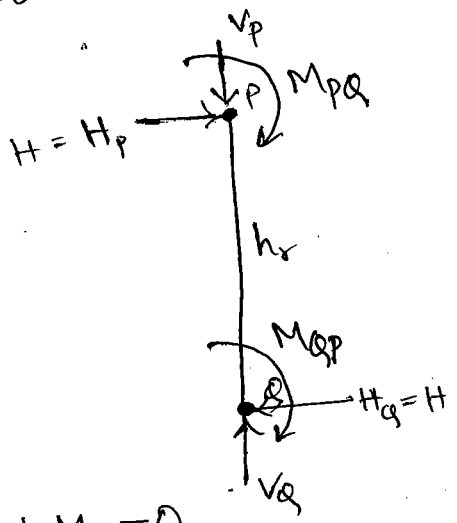


frames with sway: (i) Having at same
(ii) " " Different.

(i) Frame with sway having same column ht:-

Consider a frame shown below, r^{th} storey.





$\Sigma M_p = 0 \quad H \times h_r + M_{PQ} + M_{QP} = 0$

$$H = -\frac{(M_{PQ} + M_{QP})}{h_r}$$

Summing horizontal forces in r^{th} storey.

$$\Sigma H = -\frac{(\Sigma M_{PQ} + \Sigma M_{QP})}{h_r} = S_r \text{ (Storey shear)}$$

$$\therefore S_r h_r = -(\Sigma M_{PQ} + \Sigma M_{QP}) \quad \text{— (1)}$$

Let, ΣM^*_{PQ} be the ~~additional~~ Moment caused due to sway,

Final moment,

$$M_{PQ} = M_{FPQ} + 2M'_{PQ} + M'_{QP} + M^*_{PQ} \quad \text{— (2)}$$

$$M_{QP} = M_{FQP} + 2M'_{QP} + M'_{PQ} + M^*_{QP} \quad \text{— (3)}$$

If Loads are zero, $M_{FPQ} + M_{FQP} = 0$

~~Adding~~ (2) & (3) and taking no load condition,

$$M_{PQ} + M_{QP} = 3M'_{PQ} + 3M'_{QP} + M^*_{PQ} + M^*_{QP}.$$

For all columns

$$\Sigma M_{pq} + \Sigma M_{qp} = 3\left(\Sigma M'_{pq} + \Sigma M'_{qp}\right) + 2\Sigma M^*_{pq}$$

$\because M^*_{pq} = M^*_{qp}$ → moments due to sway are same.

$$\therefore \Sigma M^*_{pq} = \frac{1}{2}\left(\Sigma M_{pq} + \Sigma M_{qp}\right) - \frac{3}{2}\left(\Sigma M'_{pq} + \Sigma M'_{qp}\right) \longrightarrow (4)$$

from (1) to (2):

$$\Sigma M^*_{pq} = -\frac{S_r h_r}{2} - \frac{3}{2}\left(\Sigma M'_{pq} + \Sigma M'_{qp}\right)$$

$$\Sigma M^*_{pq} = -\frac{3}{2}\left[\frac{S_r h_r}{3} + \left(\Sigma M'_{pq} + \Sigma M'_{qp}\right)\right]$$

But $M^*_{pq} = \frac{-6EI\Delta}{L^2} = \frac{-6^3 \times 4EI\Delta}{4L \times L}$

$$= -1.5\,K\left(\frac{\Delta}{L}\right) \longrightarrow (5)$$

$$\Sigma M^*_{pq} = -1.5\left(\frac{\Delta}{L}\right)\Sigma K.$$

$$\frac{M^*_{pq}}{\Sigma M^*_{pq}} = \frac{K}{\Sigma K} \Rightarrow M^*_{pq} = \left(\frac{K}{\Sigma K}\right)\Sigma M^*_{pq}$$

$$\therefore M^*_{pq} = \frac{-3}{2}\left(\frac{K}{\Sigma K}\right)\left[\frac{S_r h_r}{3} + \left(\Sigma M'_{pq} + \Sigma M'_{qp}\right)\right]$$

" $\frac{-3}{2}\left(\frac{K}{\Sigma K}\right)$ " is called Displacement factor.

" $\frac{S_r \cdot h_r}{3}$ " is called Storey moment.

"S_r" Storey shear can be found by taking sum of all horizontal forces above the section through the storey in consideration

$$M^*_{PQ} = D.F\left[\frac{\Sigma hr}{3} + \left(\Sigma M_{PQ} + \Sigma M'_{QPJ}\right)\right]$$

Disp. Contribution = D.F [Storey moment + Σ (Rotation contributions at top and bottom ends of column in ~~that~~ Storey)]

$$M'_{AB} = R.F\left(\Sigma M_{FAB} + \Sigma M'_{BA} + \Sigma M^*_{PQ}\right)$$

Columns with different height:-

Choose a convenient height as Storey height. (h_r)

$$\Sigma M^*_{PQ}\left(\frac{h_r}{h_{PQ}}\right) = -\frac{3}{2}\left[\because \frac{\Sigma hr}{3} + \left(\Sigma M'_{PQ} + \Sigma M'_{QP}\right)\right]$$

Denoting: $\frac{h_r}{h_{PQ}} = C_{PQ}$

$$\Sigma C_{PQ} M^*_{PQ} = -\frac{3}{2}\left[\frac{\Sigma hr}{3} + \Sigma C_{PQ}\left(M'_{PQ} + M'_{QP}\right)\right]$$

$$M^*_{PQ} = \frac{-6EI\Delta}{h_{PQ}^2} = \frac{-1.5\,K\Delta}{h_{PQ}} \Rightarrow M^*_{PQ} \propto \frac{K}{h_{PQ}} \quad (\delta)$$

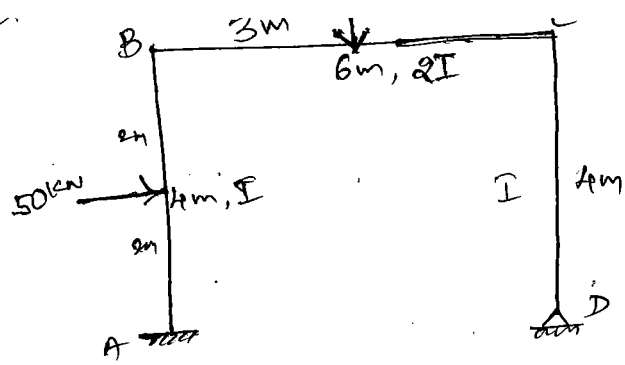
$$M^*_{PQ} \propto K\left(\frac{h_r}{h_{PQ}}\right) \propto K C_{PQ} \quad (\because h_r \text{ is const})$$

$$\frac{M^*_{PQ}}{\Sigma M^*_{PQ}} = \frac{K C_{PQ}}{\Sigma C_{PQ}^2 K}$$

$$\therefore M^*_{PQ} = -\frac{3}{2}\left[\frac{K C_{PQ}}{\Sigma C_{PQ}^2 K}\left(\frac{\Sigma hr}{3} + \left(\Sigma M'_{PQ} + \Sigma M'_{QP}\right) C_{PQ}\right)\right]$$

$$D.f = -\frac{3}{2}\left(\frac{K C_{PQ}}{\Sigma K C_{PQ}^2}\right)$$

$$D.C = D.f\left[\frac{\Sigma hr}{3} + \Sigma\left(M'_{PQ} + M'_{QP}\right) C_{PQ}\right].$$



$M_{FAB} = -25$ KNm

$M_{FBA} = 25$ KNm

$M_{FBC} = -75$ KNm

$M_{FCB} = 75$ KNm

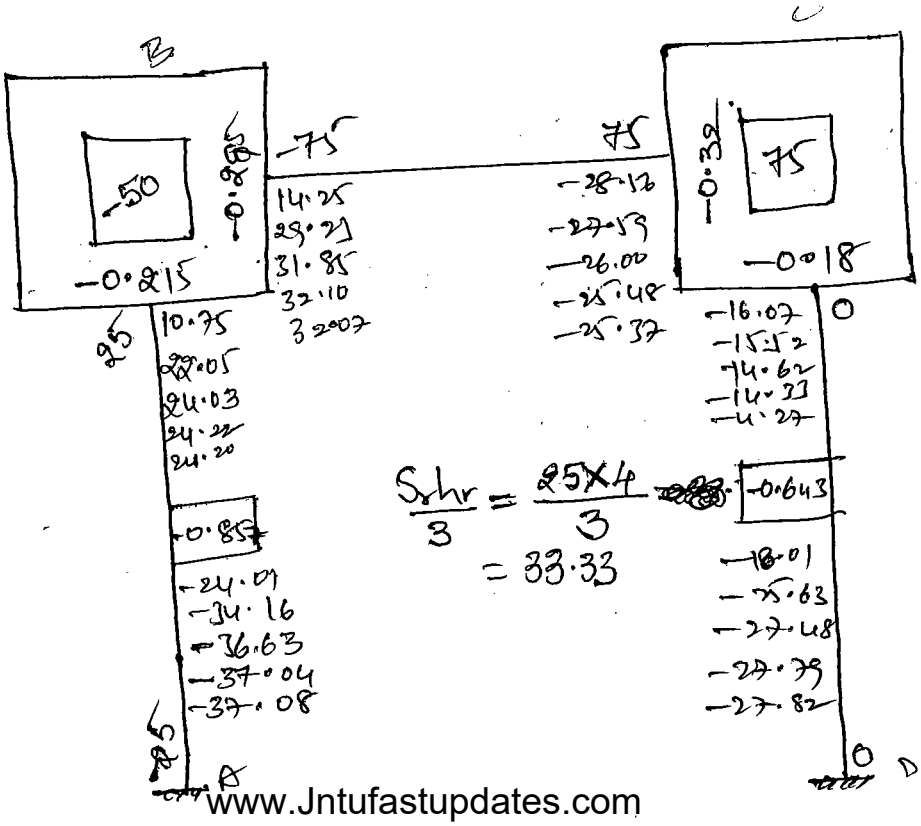
$M_{FCD} = M_{FDC} = 0$

R.F Table

Jt	Memb	K	ΣK	R.F
B	BA	$\frac{4EI}{4} = EI$	2.33EI	-0.215
	BC	$\frac{4 E \times 2I}{6} = 1.33$ EI		-0.285
C	CB	1.33EI	2.083EI	-0.32
	CD	$\frac{3EI}{4} = 0.75$EI		-0.18

Disp. factor Table

Memb	K	ΣK	D.F = $\frac{-3}{2}\left(\frac{K}{\Sigma K}\right)$
AB	EI	1.75EI	-0.857
CD	0.75EI		-0.643



$\frac{S.hr}{3} = \frac{25 \times 4}{3} = 33.33$

Rot. contr. $M'_{AB} = R.F(\Sigma M_{FAB} + \Sigma M'_{OB} + \Sigma M^*_{PQ})$

Disp. contr. $M^*_{PQ} = D.F\left(\frac{Sr.hr}{3} + (\Sigma M'_{PQ} + \Sigma M'_{QP})\right)$

cycle I :- $M'_{BA} = -0.215(-50+0+0+0) = 10.75$

$M'_{BC} = -0.285(-50+0+0+0) = 14.25$

$M'_{CB} = -0.32(75+14.25+0+0) = -28.56$

$M'_{CD} = -0.18(\quad " \quad) = -16.07$

$M^*_{AB} = -0.857(33.33+10.75+0-16.07+0) = -24.01$

$M^*_{CD} = -0.643(\quad " \quad) = -18.01$

Illy for all cycles.

find End moments:-

$M_{AB} = -25 + 2\times0 + 24.2 - 37.08 = -37.88$

$M_{BA} = 25 + 2\times24.2 + 0 - 37.08 = 36.32$

$M_{BC} = -75 + 2\times32.07 - 25.37 = -36.23$

Illy

If columns have different lengths

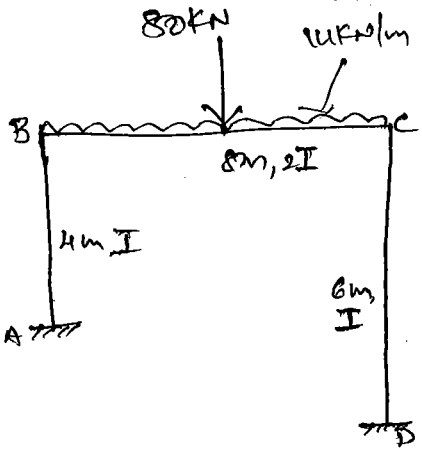
D.F Table

Storey	Memb	K	Cpq	$\Sigma K C_{pq}^2$	D.F
1	AB	$\frac{4EI}{L}$	$\frac{3}{3}=1.0$	1.85	-1.055
	CD	$\frac{4EI}{L}$	$\frac{3}{4}=0.75$		-0.5

$C_{pq} = \frac{h_r}{h_{pq}}$; $D.F = \frac{K C_{pq}}{\Sigma K C_{pq}^2}$?

Different column [illegible]

prob: A rectangular portal frame ABCD consists of legs AB and DC of ht 4m and 6m resp. Beam BC of length 8m. Beam carries UDL of 14 KN/m and a point load at centre of 80KN. legs are fixed at their bottom. $I_{ab} : I_{bc} : I_{cd} = 1 : 2 : 1$. Analyse the frame using rotation Contribution method.



Sol

$M_{FAB} = 0 = M_{FBA}$

$M_{FBC} = -74.67 - 80 = -154.67$

$M_{FCB} = 154.67$

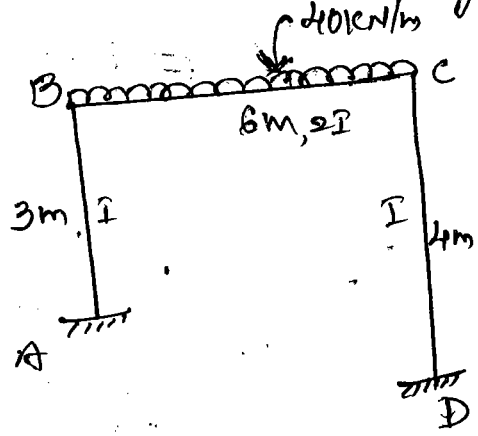
$M_{FCD} = 0 = M_{FDC}$

R.F Table

Jt	Memb	K	ΣK	R.F
B	BA	$\frac{4EI}{4} = EI$	2EI	
	BC	$\frac{4E \times 2I}{8} = EI$		

Analysis of frames with Sway when columns in a Storey have different Heights.

Ex. Analyse the portal frame shown below by Kani's method?



Sol. FEMs :- $M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$

$M_{FBC} = -120$ KNm

$M_{FCB} = +120$ KNm.

R.F :-

Jt	Memb	K	ΣK	R.F
B	BA	$\frac{4EI}{3}$	$\frac{8EI}{3}$	-0.25
	BC	$\frac{4E(2I)}{6} = \frac{4EI}{3}$		-0.25
C	CB	$\frac{4E2I}{6} = \frac{4EI}{3}$	$\frac{7EI}{3}$	-0.286
	CD	$\frac{4EI}{4} = EI$		-0.214

D.F :- $-\frac{3}{2}\left(\frac{K C_{pq}}{\Sigma K C_{pq}^2}\right)$ Let $h_r = 3m$ (ref. h_r)

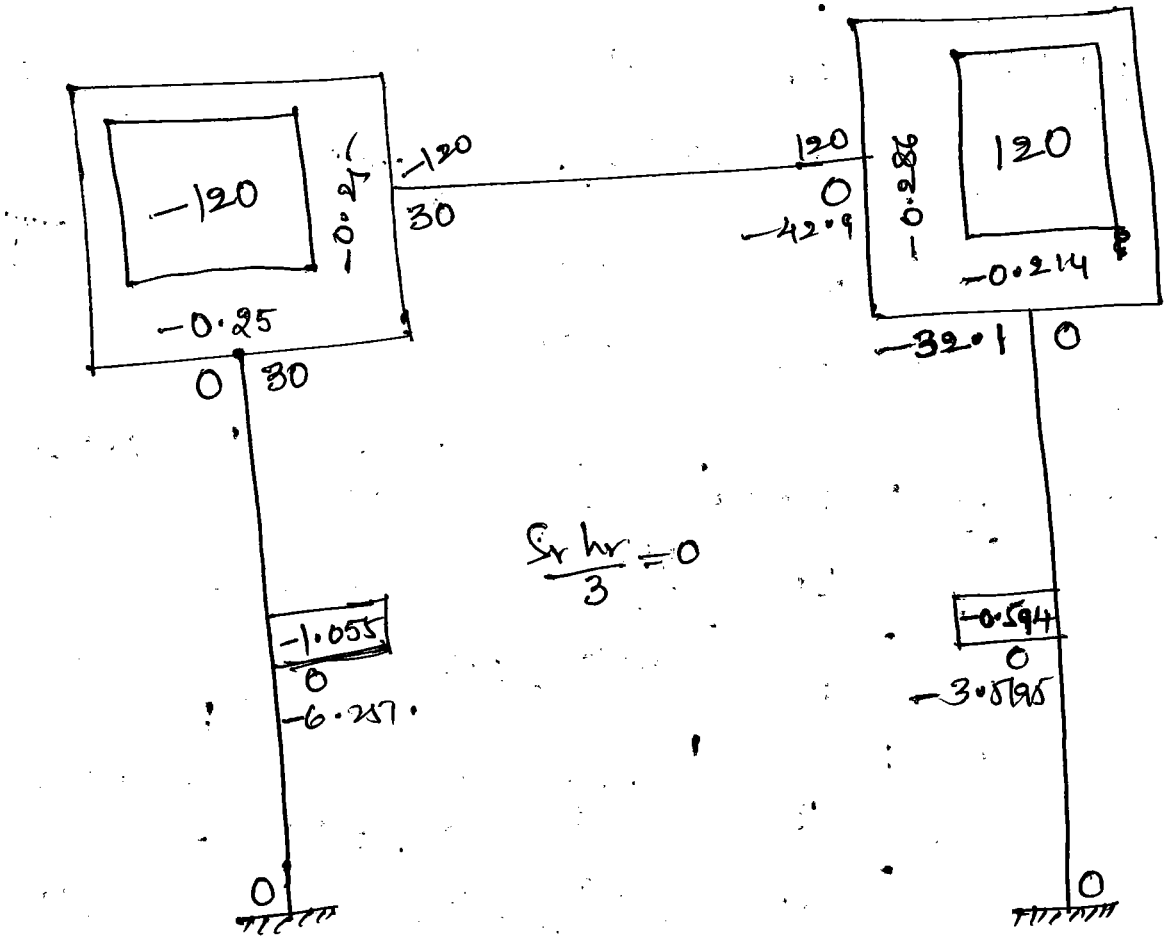
$C_{AB} = \frac{h_r}{h_{AB}} = \frac{3}{3} = 1.0$; $C_{CD} = \frac{3}{4} = 0.75$

Storey	Memb	K	C_{pq}	KCpq	ΣKC_{pq}^2	ΣKC_{pq}^2	D.F
I	AB	$\frac{4EI}{3} = 1.33EI$	1.0	1.33EI	1.33EI	2.33EI	-1.057
	DC	$\frac{4EI}{4} = EI$	0.75	0.75EI	0.5625 EI		-0.594

Total D.F need not be equal to 1.0.

Displacement Contribution must be modified as,

$$M^{*}_{pq} = D.F\left[\frac{S r h r}{3} + \Sigma\left(M'_{pq} + M'_{qp}\right) C_{pq}\right]$$



C-I :- $M'_{BA} = -0.25(-120+0+0+0) = 30$

$M'_{BC} = \quad " \quad = 30$

$M'_{CB} = -0.286(120+30+0+0) = -42.9$

$M'_{CD} = -0.214\ (\quad " \quad) = -32.1$

$M^{*}_{AB} = -1.055\ (0+(0+30)1+(0-32.1)0.75 = -6.257$

$M^{*}_{CD} = -0.594\ (0+\quad " \quad) = -3.5195$

C-II :- $M'_{BA} = -0.25(-120+0-42.9-6.257) =$

---- Continue.