Unit-IV (MDM)

Stiffness and carry over factor - Distribution factor - Analysis of continuous beams with and without Sinking of Supports. portal frames - including Sway. Substitute frame Analysis by two cycle

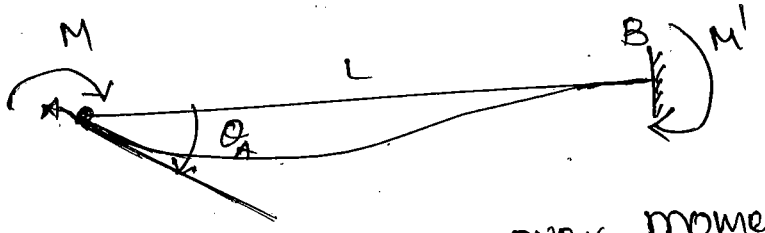
Intro:- * This method is Suggested by prof Hardy cross in 1930's

* Solution of Simultaneous equations in SDM is replaced by an iterative distribution procedure

SDE are, $M_{AB} = M_{FAB} + \frac{2EI}{L}\left[(2\theta_A + \theta_B) - \frac{3\Delta}{L}\right]$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left[(2\theta_B + \theta_A) - \frac{3\Delta}{L}\right]$$

COM :- carry over moment ; when a ~~member~~ moment is applied at one end of a member allowing rotation of that end and fixing the far end. Some moment develops at far end also, this is called COM.

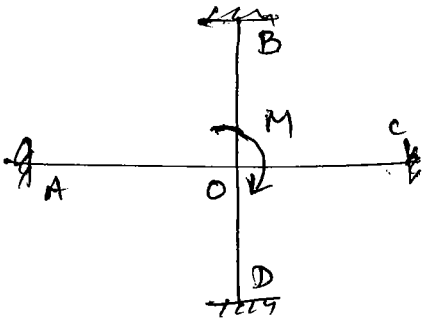


COF :- $= \frac{M'}{M} = \frac{\text{Carry over moment}}{\text{Applied moment.}}$

Stiffness :- moment required to rotate an end by unit angle.

$$K = \frac{M}{\theta_A}$$

D.F :- Distribution factor :- The ratio of moment shared by a member to the applied moment at the joint is called D.F.

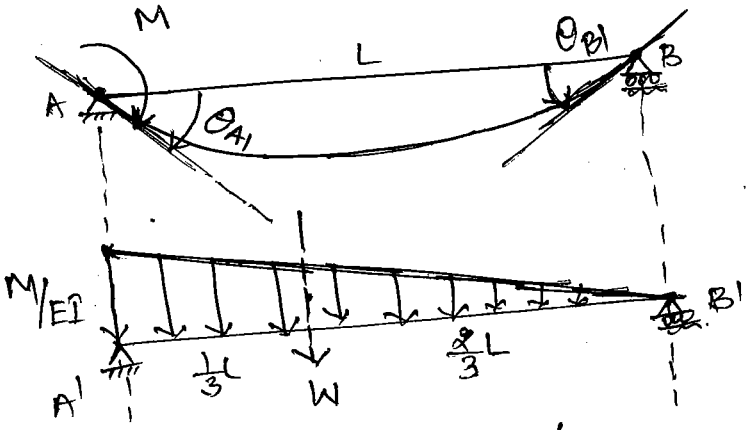


$$D.F \text{ of } OA = \frac{M_{OA}}{M}$$

Sign convention :- 1. C.W moments are +ve
2. ACW " " -ve

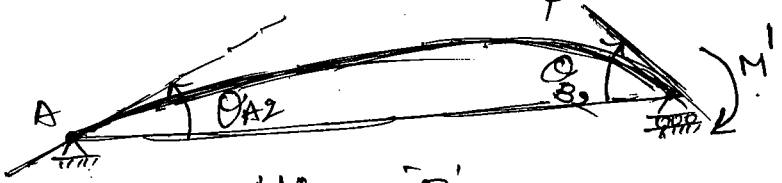
Expressions for COF, D.F, Stiffness

CBM :- consider a SSB of span L applied by moment M' at A



$$\theta_{A1} = R_{A'} = \left(\frac{1}{2} \times L \times \frac{M}{EI}\right) \times \frac{\left(\frac{2}{3}L\right)}{L} = \frac{ML}{3EI}$$

$$\theta_{B1} = R_{B'} = \left(\frac{1}{2} \times L \times \frac{M}{EI}\right) \times \frac{(L/3)}{L} = \frac{ML}{6EI}$$



Applying Reduction "1" M' at B'

$$\theta_{A2} = \frac{M'L}{6EI} \qquad \theta_{B2} = \frac{M'L}{3EI}.$$

But, end B of the beam is fixed

hence, $\theta_B = 0 \quad \theta_{B1} - \theta_{B2} = 0$

$$\theta_{B1} = \theta_{B2} \Rightarrow \frac{ML}{6EI} = \frac{M'L}{3EI}$$

$$\boxed{M' = M/2} \Rightarrow \boxed{\frac{M'}{M} = \frac{1}{2} = COF}$$

$$\theta_A = \theta_{A1} - \theta_{A2} = \frac{ML}{3EI} - \frac{M'L}{6EI}$$

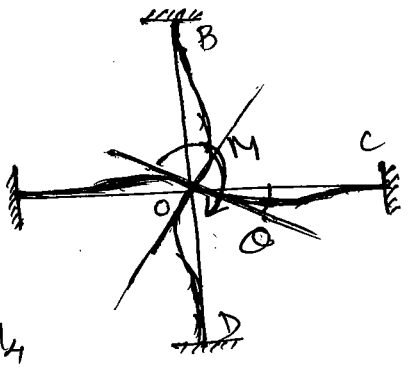
$$= \frac{ML}{3EI} - \frac{\left(\frac{M}{2}\right)L}{6EI}$$

$$= \frac{ML}{3EI} - \frac{ML}{12EI} = \frac{ML}{EI}\left(\frac{12-3}{12\times 3}\right)$$

$$= \frac{ML}{EI} \times \frac{9}{12\times 3} = \frac{ML}{4EI}$$

$$\therefore K_\theta = \frac{M}{\theta_A} = \frac{M}{\theta} = \boxed{\frac{4EI}{L} = K_\theta}$$

Consider a Joint, consisting of four members.-



$$M = M_1 + M_2 + M_3 + M_4$$

$$\theta = \frac{M_1}{K_1} = \frac{M_2}{K_2} = \frac{M_3}{K_3} = \frac{M_4}{K_4}$$

$$\theta = \frac{M}{K} = \frac{M_1 + M_2 + M_3 + M_4}{K_1 + K_2 + K_3 + K_4}$$

$$= \frac{M}{\sum_{i=1}^{4} K_i}$$

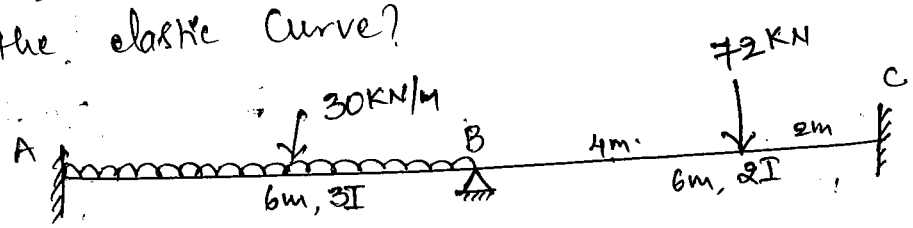
$$M_i = K_i\theta = K_i\left(\frac{M}{\Sigma K_i}\right) \Rightarrow \boxed{DF = \frac{M_i}{M} = \frac{K_i}{\Sigma K_i}}$$

procedure:-

1. Assume all ends are fixed and find fixed end moments.
2. Find D.f for all members at a Joint.
3. Balance a Jt by distributing balancing moment to various members meeting at the Joint proportional to their D.F. Apply same procedure to all Joints.
4. carry over half the distributed moment to far ends. This represents balance of Joints.
5. Repeat Step3 & 4 till Distributing moments are negligible.
6. Sum all moments at a particular end of a member to get final moment.

Note:- Fixed end is a self balancing Joint.

prob:- Analyse the continuous beam shown by MDM and draw BMD & SFD. Also draw the elastic curve?



Sol:- FEM's :- $M_{FAB} = -\frac{wl^2}{12} = -\frac{30\times 6^2}{12} = -90\,KNm$

$M_{FBA} = 90\,KNm.$

$M_{FBC} = -\frac{Pab^2}{l^2} = -\frac{72\times 4\times 2^2}{6^2} = -32\,KNm$

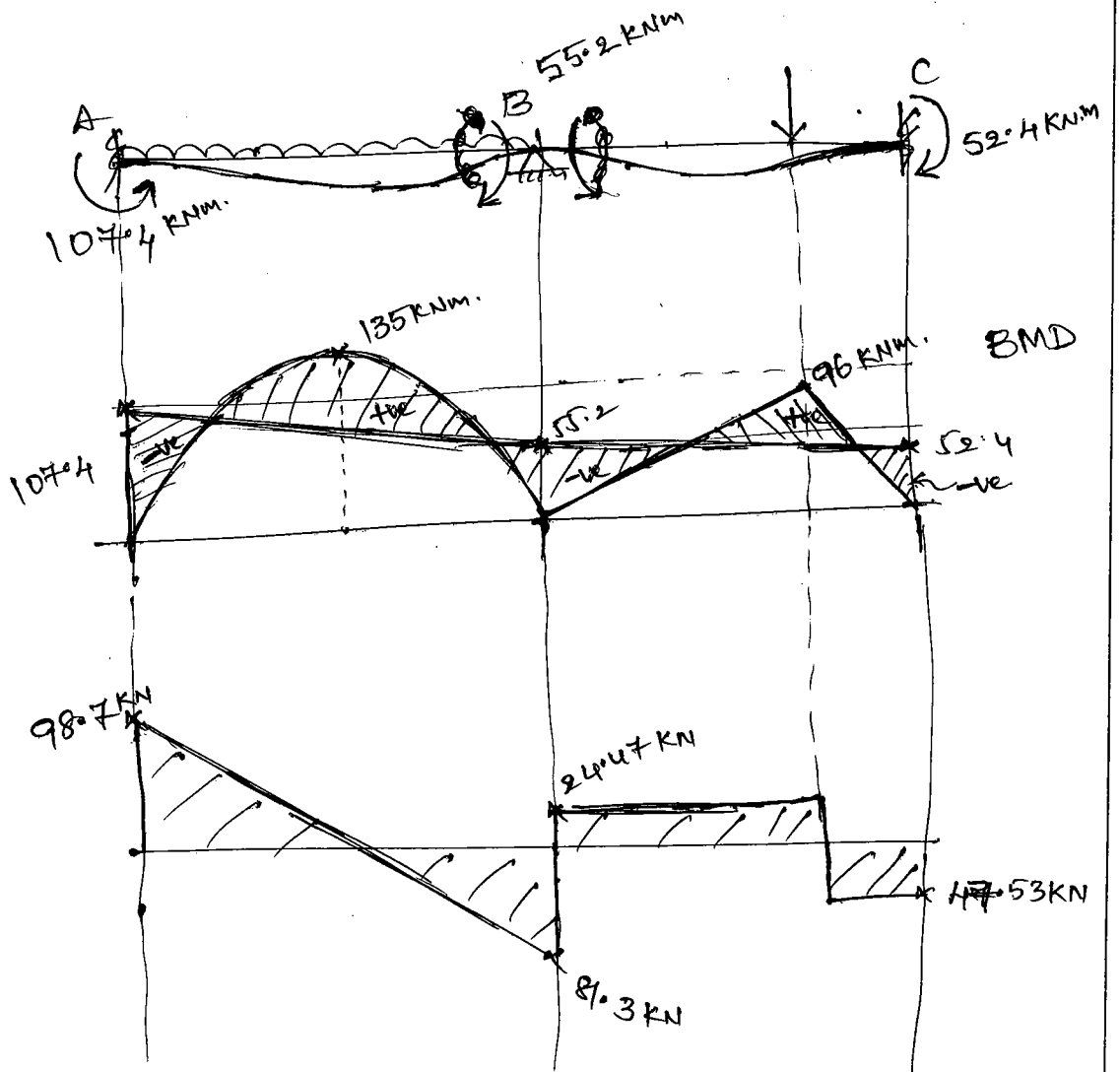
$M_{FCB} = \frac{Pba^2}{l^2} = \frac{72\times 2\times 4^2}{6^2} = 64\,KNm.$

D.F:-

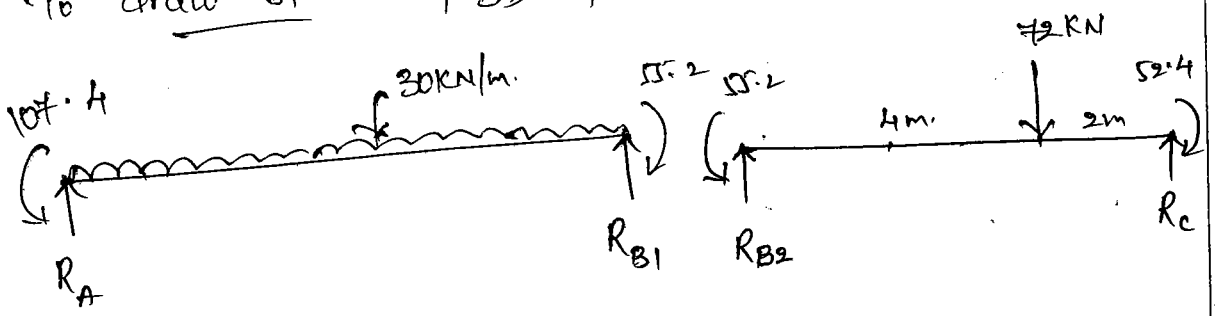
Jt	Members	K	ΣK	DF
B	BA	$\frac{4EI}{6} = 2EI$	3.33EI	0.6
	BC	$\frac{4E(2I)}{6} = 1.33EI$		0.4

Distribution Table

	A	B		C
D.F		0.6	0.4	
FEMs	-90	90	-32	64
Bal.		-34.8	-23.2	
COM.	-17.4			-11.6
Final End Moments	-107.4	55.2	-55.2	52.4



To draw SFD:- FBD of each beam.



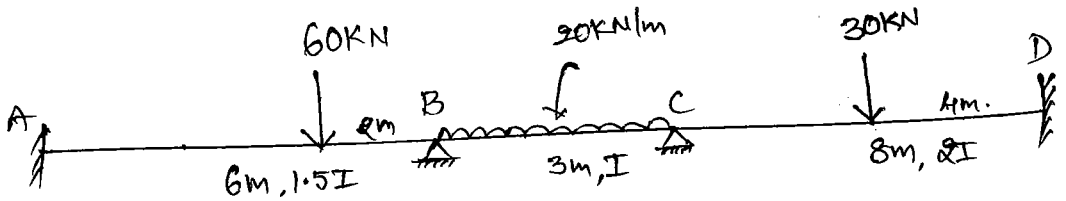
$$R_{B1} = 81.3\ KN,\quad R_A = 98.7\ KN$$

$$R_{B2} = 24.47\ KN,\quad R_C = 47.53\ KN.$$

C.H. prob

① Using MDM analyze the two span continuous beam. The moment of Inertia of AB = I while that of BC = 2I. The ends A & C are fixed. Sketch the BMD & S.F.D. Span AB carries a concentric load of 36 KN with a span of 6m and span BC carries a UDL of 20KN/m over a span of 8m?

Prob:- Analyse the continuous beam shown below and draw BMD?



Sol:- $M_{FAB} = -26.67$ $M_{FBC} = -15$ $M_{FCD} = -30 = \frac{PL}{8}$

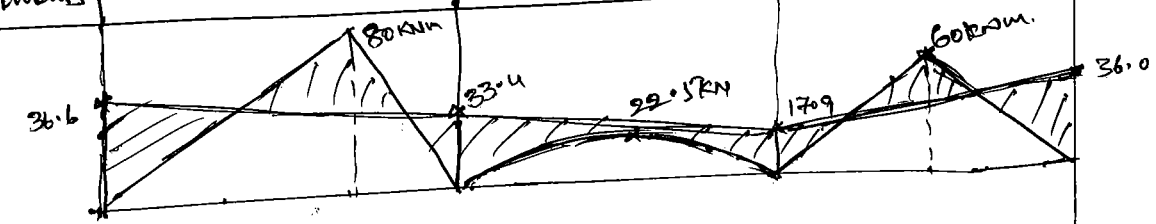
$M_{FBA} = 53.33$ $M_{FCB} = 15$ $M_{FDC} = 30 = \frac{PL}{8}$

D.F:- $DF_{BA} = 0.43$ $DF_{CB} = 0.57$

$DF_{BC} = 0.57$ $DF_{CD} = 0.43$

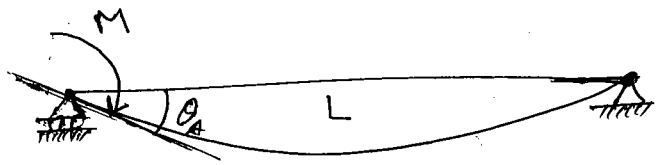
M.D:-

	A	B		C	D
D.F		0.43	0.57	0.57	0.43	
FEM	-26.67	53.33	-15	15	-30	30
Bal		-16.48	-21.85	8.55	6.45	
COM	-8.24		4.28	-10.93		3.23
Bal		-1.84	-2.44	6.23	4.7	
COM	-0.92		3.12	-1.22		2.35
Bal		-1.34	-1.78	0.7	0.53	
COM	-0.67		0.35	-0.89		0.27
Bal		-0.15	-0.2	0.51	0.38	
COM	-0.075		0.25	-0.1		0.16
Bal		-0.11	-0.14	0.057	0.043	
final end Moments	-36.6	33.4	-33.4	17.9	-17.9	36



BMD

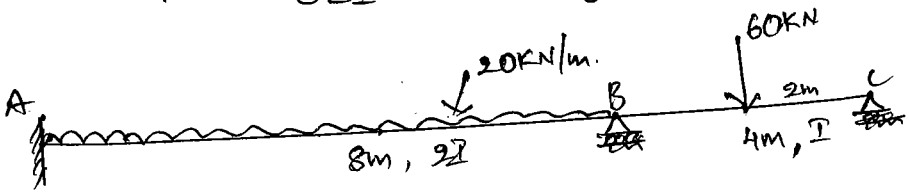
Continuous beam with simple end:-



$M' = 0 \Rightarrow COM = 0 \quad COF = 0$

$\theta_A = \frac{ML}{3EI} \Rightarrow \frac{M}{\theta} = \frac{3EI}{L} = K_\theta$

Prob:



$M_{FAB} = -106.67 \qquad M_{FBC} = -30$

$M_{FBA} = 106.67 \qquad M_{FCB} = 30$

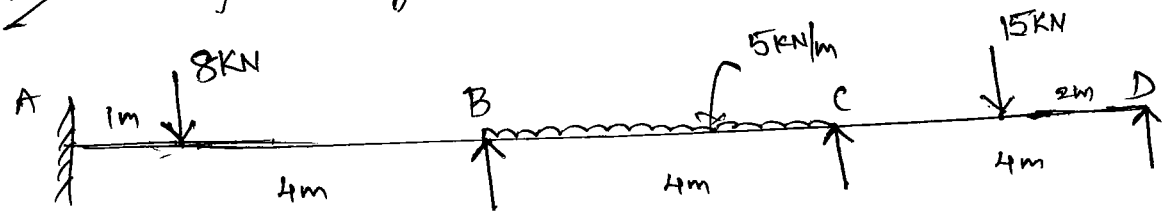
D.F.:-

Jt	mem	K	ΣK	D.F
B	BA	$\frac{4EI}{L} = EI$	$1.75EI$	0.57
	BC	$\frac{3EI}{L} = 0.75EI$		0.43

M.D:-

	A	B		C
DF		0.57	0.43	
FEM	-106.67	106.67	-30	30
Bal				-30
COM			-15	
Bal		-35.15	-26.12	
COM	-17.6			0
Final	-124.27	71.52	-71.52	0

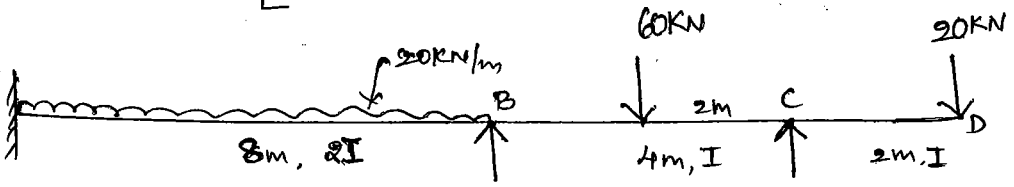
C.T. Draw BMD for continuous beam shown in fig by using MDM? May-2016,



Continuous beams with over hang:-

* Stiffness of overhanging portion is taken as zero (D.F = 0). Since moment in this portion do not depend on loading in other portions. Support C is considered similar to simply supported end. Hence Stiffness of BC is $\frac{3EI}{L}$ and no carry over from B to C.

Prob:-



Sol:- $M_{FAB} = -106.67$ $M_{FBC} = -30$ $M_{FCD} = -40$

$M_{FBA} = 106.67$ $M_{FCB} = 30$

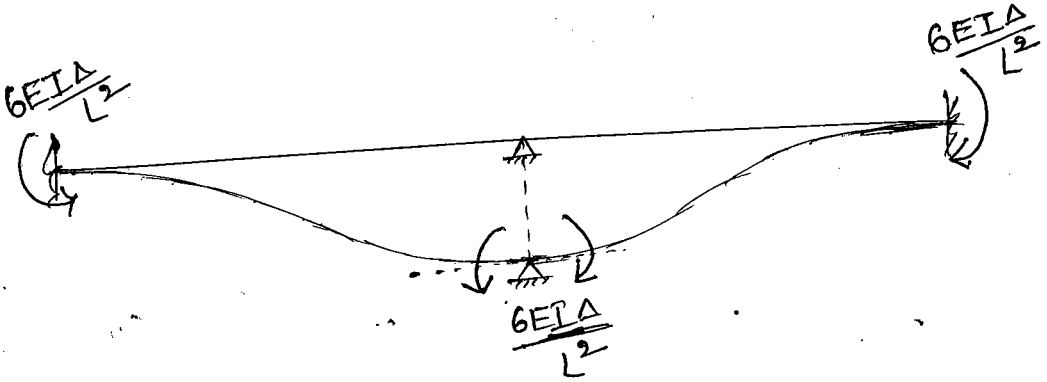
DF:-

JT	memb	K	ΣK	DF
B	BA	$\frac{4EI}{L} = EI$	$1.75EI$	0.57
	BC	$\frac{3EI}{L} = 0.75EI$		0.43
C	CB	$\frac{4EI}{L} = EI$	EI	1.00
	CD	0		0

	A		B		C	overhang D
D.F		0.57	0.43		1.0	0
FEM	-106.67	106.67	-30		30	-40
Bal.					10	0
COM			5			
Bal		-40.92	-30.75			
COM	-20.46				0	
final EM.	-127.13	65.75	-65.75		40	-40

Beams with Sinking of Supports:-

* fixed end moments due to Settlement of supports are added to fixed end moments due to loads. Rest of analysis is same.



RHSS → -ve moments

LHSS → +ve "

prob:- A fixed beam of span 6m carries a UDL of 18 KN/m. If the right support sinks by 6.5mm, find fixing end moment of the supports. Draw SFD & BMD. Take $E = 200 KN/mm^2$, $I = 5 \times 10^7 mm^4$. Analyse by MDM?

May/June 2015 :- A continuous beam ABCD is simply supported at A, B and C and is fixed at D. The spans AB, BC and CD are 3m, 4m and 6m long. The beam carries point load of 15KN on AB at 2m from A, a pt load of 25KN at middle of BC and a pt load of 8KN at middle of CD. If $I_{AB} : I_{BC} : I_{CD} = 1 : 1 : 2$ find support moments and reactions using MDM?

SA-II
IV-(6)

Frames with Sway:-

(i) Assume the sway in the frame is prevented by giving external support at beam level. Carryout analysis which is called non-sway analysis.

(ii) Considering FBD of columns, find horz forces developed at supports. then considering horizontal equilibrium of entire system find force 'S' developed at additional support assumed at beam level.

(iii) Assuming arbitrary end moments due to sway carryout analysis to find sway force 'S'.

(iv) find sway correction factor $K = \frac{S}{S'}$

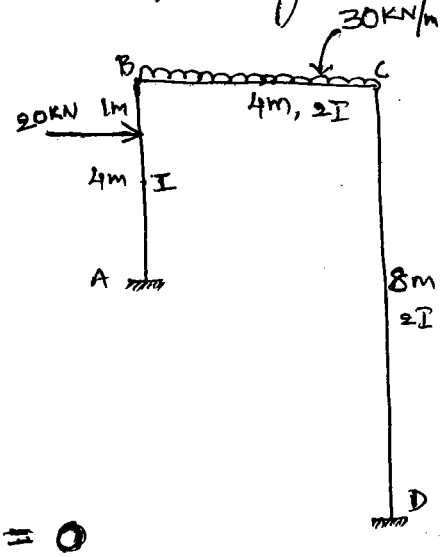
(v) Cal. final moments by the eqn.

Final End moment = non-sway moment + K * sway moments.

S → sway force

S' → Arbitrary sway force

Prob: Analyse the given frame by using ...



Sol: Non-sway analysis

Step1:- Assuming a support at C.

$$M_{FAB} = -\frac{20\times3\times1^2}{4^2} = -3.75 \text{ KNm}$$

$$M_{FBA} = \frac{20\times3^2\times1}{4^2} = 11.25 \text{ KNm}$$

$$M_{FBC} = \frac{-30\times4^2}{12} = -40 \text{ KNm.}$$

$M_{FCB} = +40$ KNm. ; $M_{FCD} = M_{FDC} = 0$

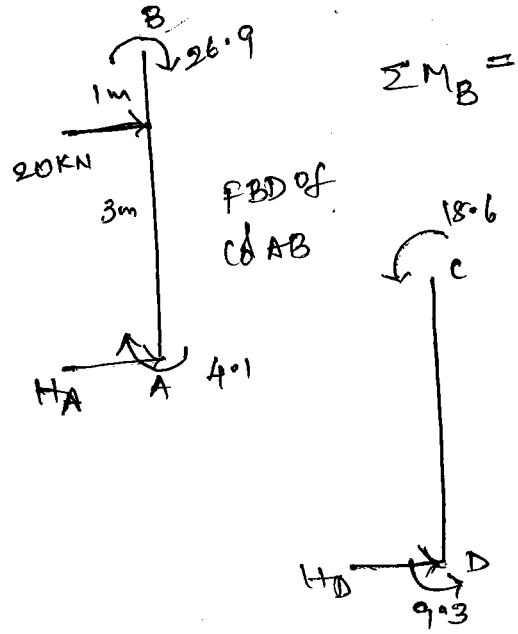
Step2:- D.F

Jt	Member	K	ΣK	D.F
B	BA	$\frac{4EI}{4} = EI$	3EI	0.33
	BC	$\frac{4E(2I)}{4} = 2EI$		0.67
C	CB	$\frac{4E(2I)}{4} = 2EI$	3EI	0.67
	CD	$\frac{4E(I)}{8} = EI$		0.33

Step3,4,5:-

	A		B		C	D
D.F		0.33	0.67	0.67	0.33	
FEM	-3.75	11.25	-40	40	0	0
Bal		9.5	19.26	-26.8	-13.2	
COM	4.75		-13.4	9.63		-6.6
Bal		4.42	9.0	-6.45	-3.2	
COM	2.21		-3.225	4.5		-1.6
Bal		1.1	2.9	-3.02	-1.5	
COM	0.55		-1.51	1.1		-0.75
Bal		0.5	1.0	-0.74	-0.36	
COM	0.25		-0.37	0.5		-0.18
Bal		0.12	0.25	-0.165	-0.335	
COM	0.06		-0.0825	0.125		-0.1675
Final End Moments	4.01	26.09	-26.09	18.6	-18.6	-9.3

Sway Force (Reaction at restrained support)



$\Sigma M_B = 0 \quad 4.1 + 26.9 - 20\times1 - H_A\times4 = 0$

$H_A = \frac{11}{4} = 2.75$ KN.

$\Sigma M_C = 0 \quad 9.3 + 18.6 + H_D\times8 = 0$

$H_D = -3.5$ KN

$\Sigma F_x = 0 \quad H_A + H_D + 20 - S = 0$

$2.75 - 3.5 + 20 - S = 0$

$\boxed{S = 19.25 \text{ KN}}$

Sway Analysis :- Let M_{F1} & M_{F2} be fixed end moments in AB & CD column for Δ sway.

$$\therefore \frac{M_{F1}}{M_{F2}} = \frac{\left(\frac{6EI_1\Delta}{L_1^2}\right)}{\left(\frac{6EI_2\Delta}{L_2^2}\right)} = \frac{I}{4^2}\times\frac{8^2}{2I} = \frac{8\times8}{4\times4\times2} = 2$$

Let $M_{F1} = -100 \quad M_{F2} = -50$

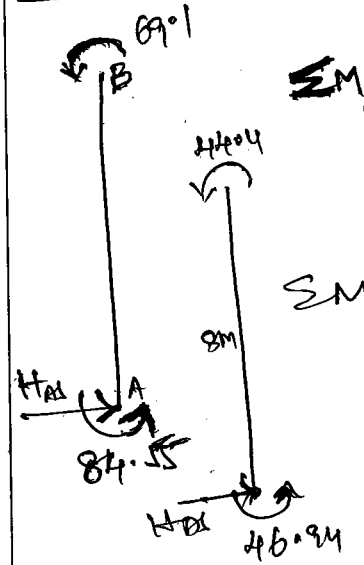
$\therefore M_{FAB} = M_{FBA} = -100 \quad ; \quad M_{FCD} = M_{FDC} = -50$

M.D for Arbitrary sway

D.F	A	0.33 B	0.67	0.67 C	0.33 D
FEMs	-100	-100	0	0	-50
Bal		33	67	33.5	16.5
Com	16.5		16.75	33.5	8.25
Bal		-5.53	-11.22	-22.45	-11.06
Com	-2.765		-11.225	-5.61	-5.53
Bal		3.7	7.52	3.76	1.85
Com	1.85		1.88	3.76	0.925
Bal		-0.62	-1.26	-2.52	-1.24
Com	-0.31		-1.26	-0.63	-0.62

	A		B		C	D
COM	-0.31		-1.26	-0.63		-0.62
Bal		0.42	0.84	0.42	0.21	
COM	0.21		0.21	0.42		0.11
Bal		-0.07	-0.14	-0.28	-0.14	
COM	-0.035		-0.14	-0.07		-0.07
Find End moments	-84.55	-69.1	69.1	44.4	-44.4	-46.94

To find S':-

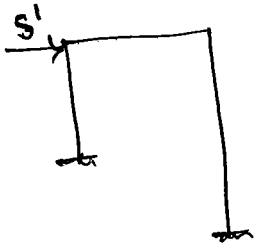


$\Sigma M_B = 0 \rightarrow 84.55 + 69.1 + H_{AS} \times 4 = 0$

$H_{AS} = -38.4125$ KN

$\Sigma M_C = 0 \quad 46.94 + 44.4 + H_{DS} \times 8 = 0$

$H_{DS} = -11.4175$ KN



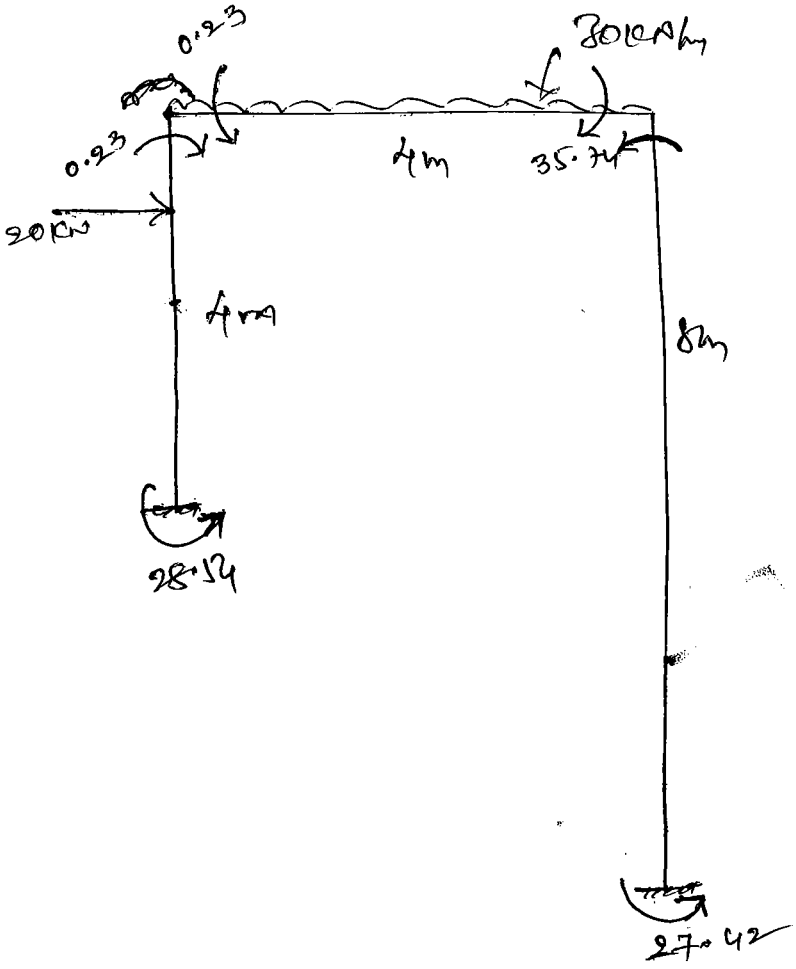
$\Sigma F_x = 0 \quad S' + H_{AS} + H_{DS} = 0$

$S' = 49.83$ KN.

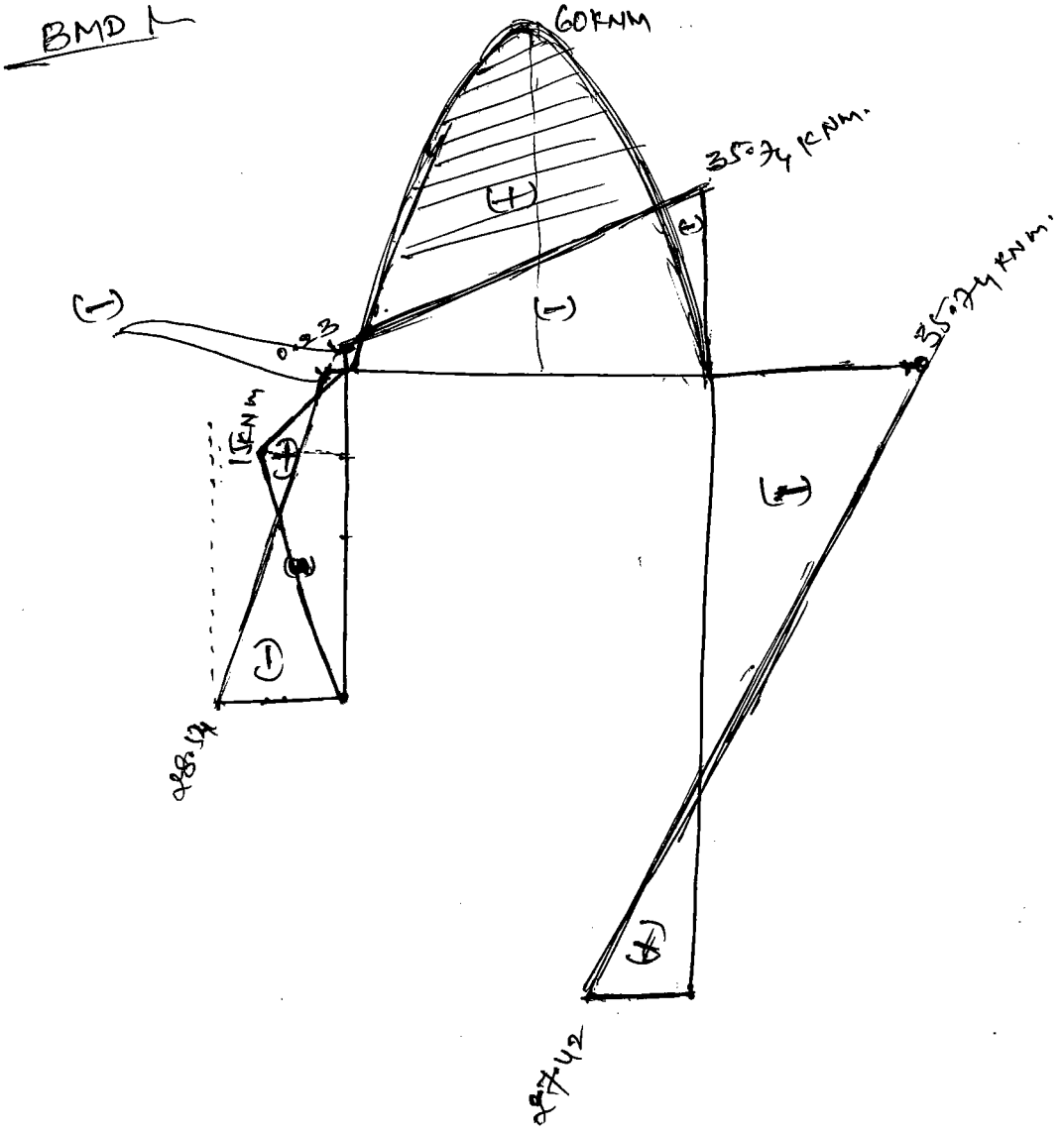
$\therefore K = \frac{S}{S'} = \frac{19.25}{49.83} = 0.386$

Find End moments

	A		B		C	D
Non-Sway	40.1	26.9	-26.9	18.6	-18.6	-9.3
Arbitrary Sway	-84.55	-69.1	69.1	44.4	-44.4	-46.94
Corrected Sway moments	-32.64	-26.67	26.67	17.14	-17.14	-18.12
Final End moments	-28.54	0.23	-0.23	35.74	-35.74	-27.42


0.23
0.23
30KN/m
4m
35.74
20KN
4m
8m
28.14
27.42

BMD :-


60KNM
35.74 KNM.
35.74 KNM.
0.23
15KNm
28.14
27.42