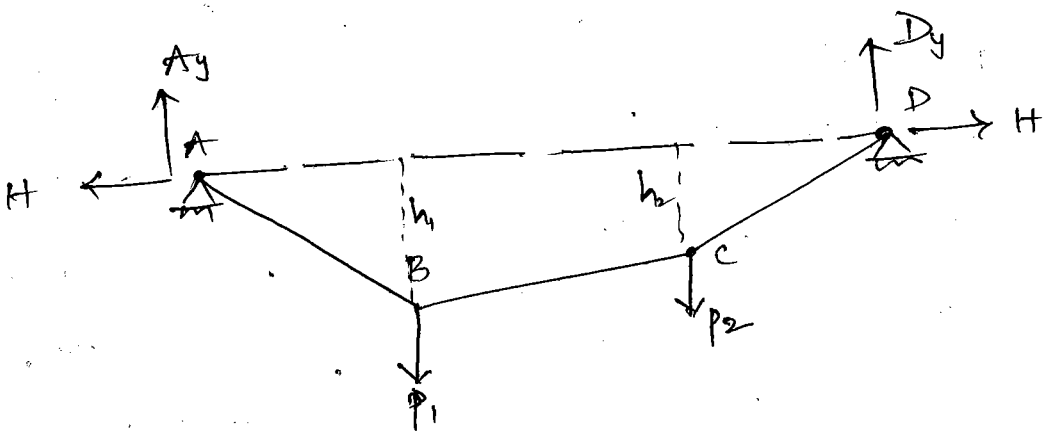
UNIT-III

cable structures and Suspension Bridges:-

Introduction:- cables are flexible wire-like systems having no flexural stiffness (bending) and they can carry only axial tension and no other type of force.

→ Being fully flexible against bending the shape of a cable is determined by the external forces that are acting on the cable (fig. below)



→ the shape of above cable b/w two supports depends on location and magnitude of P_1 & P_2.

→ A cable is unable to carry BM, SF, torsion & axial comp^n. Nevertheless, cables can be very effectively used in achieving long-span light-weight systems, such as bridges & roofs for large Areas.

→ Two kinds of bridge structural systems where cables are used are the Suspension-cable systems and Cable-Stayed systems.

eg:- Golden gate bridge, S.F, USA is Suspension-cable bridge

ANZAC Bridge, Sydney, AUS. is cable-stayed

→ cables are usually made of multiple strands of cold-drawn high-strength steel wires twisted together.

→ Generally, they have strength four to five times that of structural steel and practically inextensible under operating loading conditions.

→ Since cables carry only axial tension, full potential of the cable C/S can be utilized in transferring forces. Therefore, cables are able to carry the same amount of force with a much smaller C/S compared to other structural systems.

→ The High-strength-to-weight ratio makes cables very useful where light-weight systems are needed. On the other hand, a beam over a very long span could require a very large (and deep) C/S, and most of its potential could be used in carrying internal forces due to its own weight.

→ If we use cables replacing the beam or in combination with a beam instead, a lighter structure could be required, whose self-wt could not add significantly to load effects

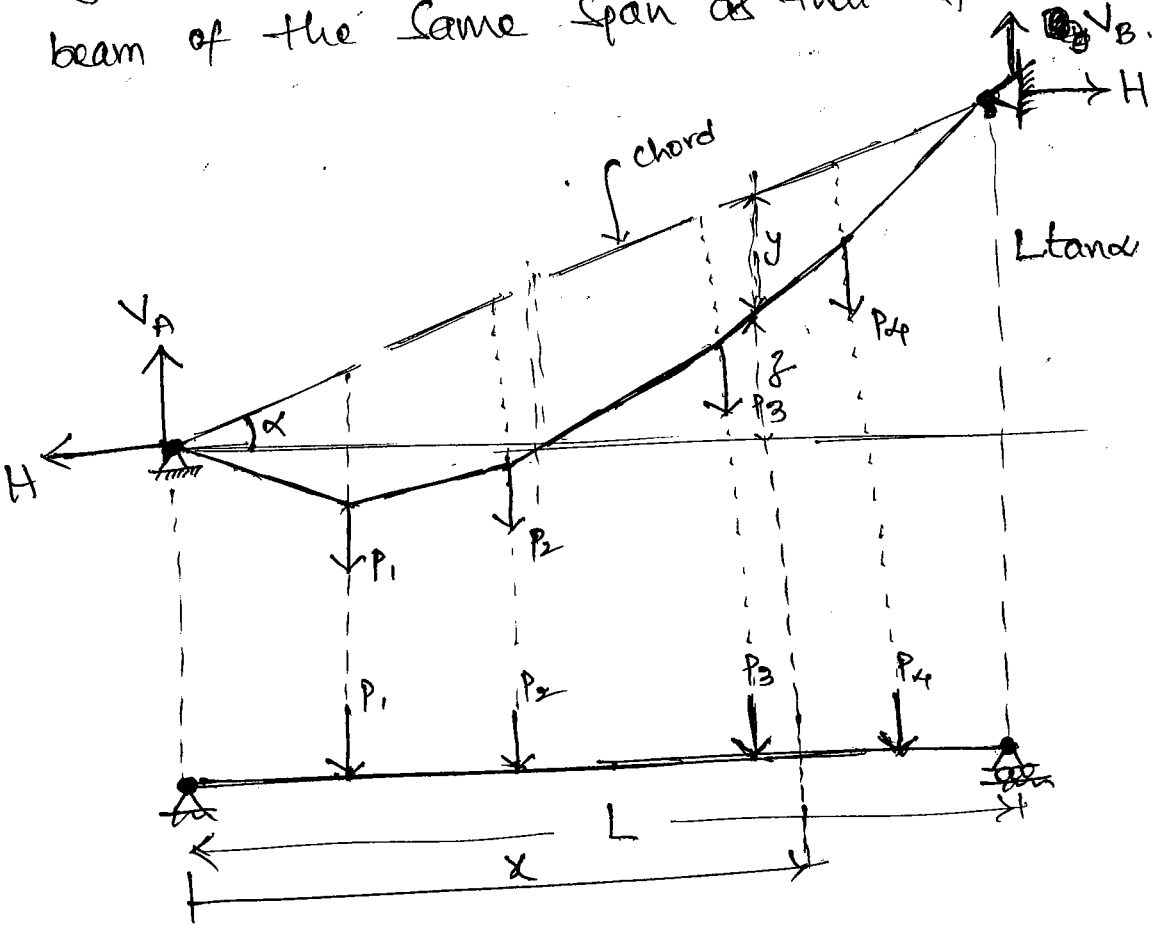
→ the main disadvantage with cables is due to their flexible geometry. As the loading on a cable system changes (moving load) there can also be large change in the cable geometry and subsequently on forces acting in the cable. Unexpected forces may destabilize a cable system, causing excessive deformations. A designer should be very careful on this regard while designing a cable system, along with other issues such as, Large forces at the anchors, large oscillations, etc

eg: TACOMA NARROW Bridge failure

The general cable theorem:-

It helps us determine the shape of a cable supported at two ends when it is acted upon by vertical forces.

Statement :- At any point on a cable acted upon by vertical loads, the product of the horizontal component of cable tension and the vertical distance from that point to the cable chord equals the moment which would occur at that section if the loads carried by the cable were acting on an simply-supported beam of the same span as that of the cable.



→ Consider cable AB, the line AB joining the two supports is known as the chord and the horizontal distance b/w the supports is known as the span.

→ vertical distance b/w chord and the cable at any ck is known as the dip.

* moment equilibrium about 'B':-

$$V_A L + H L \tan\alpha = \Sigma M_{BP} \quad \text{——} (1)$$

Since the cable is totally flexible against bending, BM at any ck is zero. By equating Bm at x from A to zero,

$$V_A \times x + H y = \Sigma M_{xp} \quad \text{——} (2)$$

$$V_A x + H(x\tan\alpha - y) = \Sigma M_{xp} \leftarrow \text{(moments due to loads to the left of 'x'}$$

Substituting V_A from (1)

$$V_A = \frac{\Sigma M_{BP}}{L} - H\tan\alpha$$

$$\therefore \left(\frac{\Sigma M_{BP}}{L} - H\tan\alpha\right) x + Hx\tan\alpha - Hy = \Sigma M_{xp}$$

$$\frac{\Sigma M_{BP}\, x}{L} - Hx\tan\alpha + Hx\tan\alpha - Hy = \Sigma M_{xp}$$

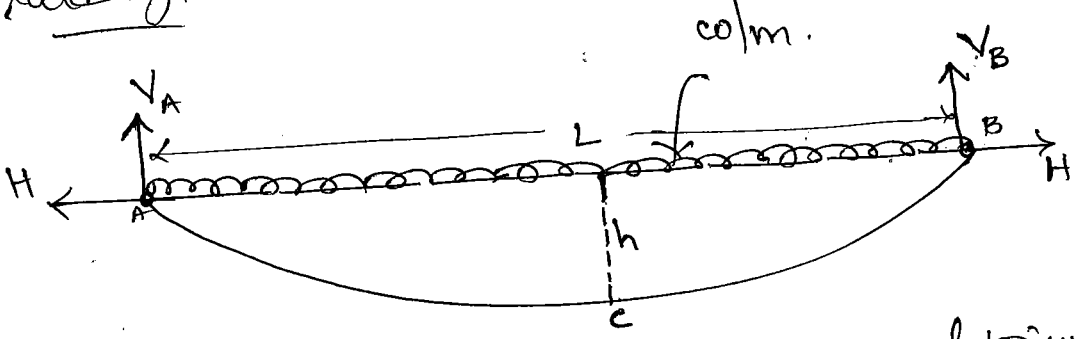
$$\boxed{Hy = \Sigma M_{BP}\left(\frac{x}{L}\right) - \Sigma M_{xp}}$$

from SSB, $R_A = \frac{\Sigma M_{BP}}{L}$

BM at a distance x for the beam $= \frac{\Sigma M_{BP}}{L} x - \Sigma M_{xp}$.

$$\therefore \boxed{Hy = \text{Moment at } x \text{ for SSB}}$$

Application of cable theorem for distributed loading:-



$\Rightarrow V_A = V_B = \frac{coL}{2}$ from vertical equilibrium.

from general cable theorem,

$$Hy = \left(\frac{x}{L}\right)\Sigma M_{BP} - \Sigma M_{xp}$$

$$= \frac{x}{L}\left(\frac{coL^2}{2}\right) - \frac{cox^2}{2} = \frac{cox}{2}(L-x)$$

$$\therefore Hy = \frac{cox}{2}(L-x) \quad \text{——— (1)}$$

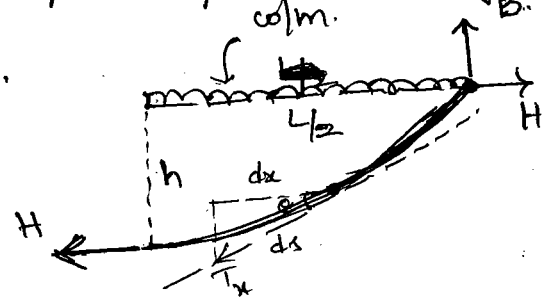
applying general cable theorem at point 'C'.

$x = \frac{L}{2}$; $y = h$. $\Rightarrow H \cdot h = \frac{coL}{4}\left(L - \frac{L}{2}\right)$

$$H \cdot h = \frac{coL}{4} \times \left(\frac{L}{2}\right) = \frac{coL^2}{8}$$

$$\therefore H \cdot h = \frac{coL^2}{8} \quad \text{——— (2)}$$

at point 'C', due to symmetry the cable tension is horizontal.



from eqn (2), $H = \frac{coL^2}{8h}$.

from eqn (1), $y = \frac{cox}{2H}(L-x)$

To define general shape of cable in terms of mid-span dep 'h',

$$y = \frac{\omega x}{2H}(L-x) \quad (\because \text{Substituting for } H)$$

$$y = \frac{\omega x \times 8h}{2 \times \omega L^2}(L-x)$$

$$\therefore y = 4hx\left(\frac{L-x}{L^2}\right)$$

$$\therefore y = \frac{4hx}{L^2}(L-x) \quad \text{---(eq1)}$$

Let T be the axial tension in cable at a distance 'x'. This axial tension acts along the tangent of the cable geometry. Let us measure the length of cable 's'.

$$(\cos\theta)\, T = H \Rightarrow T = \frac{H}{\cos\theta} = \frac{H}{(dx/ds)} = H\left(\frac{ds}{dx}\right)$$

$$= H \cdot \frac{\sqrt{dx^2+dy^2}}{dx} = H\sqrt{1+\left(\frac{dy}{dx}\right)^2} \rightarrow (1)$$

but, $\frac{dy}{dx} = \frac{d\left[\frac{4hx}{L^2}(L-x)\right]}{dx}$

$$\boxed{\frac{dy}{dx} = \frac{4h(L-2x)}{L^2}} \leftarrow \text{substituting this in eq}^{n}(1)$$

$$T = H\sqrt{1+\frac{16h^2}{L^4}(L-2x)^2}$$

* when $x=0$ and $x=L$. maximum Tension occurs also slope is maximum.

* At mid span minimum Tension occurs.

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To obtain total length of cable 'S'

$$S = \int_0^l ds = \int_0^l \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx \quad \left[y = \frac{4hx^2}{l^2}\right]$$

$$= 2\int_0^{l/2} \left[1+\left(\frac{8hx}{l^2}\right)^2\right]^{1/2} dx = 2\int_0^{l/2} \left[1+\frac{64h^2x^2}{l^4}\right]^{1/2} dx.$$

Binomial theorem.

$$= 2\int_0^{l/2} \left[1+\frac{1}{2}\left(\frac{64h^2x^2}{l^4}\right)+\ldots.\right] dx$$

$$= 2\left[x+\frac{32h^2}{L^4}\left(\frac{x^3}{3}\right)\right]_0^{l/2}$$

$$= 2\left[\frac{L}{2}+\frac{32h^2}{L^4}\times\frac{L^3}{3\times 8}\right]$$

$$\boxed{S = L+\frac{8h^2}{3L}}$$

The shape of a flexible cable supported at two ends and hanging only under its self-weight is known as a CATENARY.

* The self-wt of the cable is uniform along its curved length and not along its span. The wt/m that is considered for obtaining equation.1 is uniform along the span. and (Galileo) not along the curved shape.

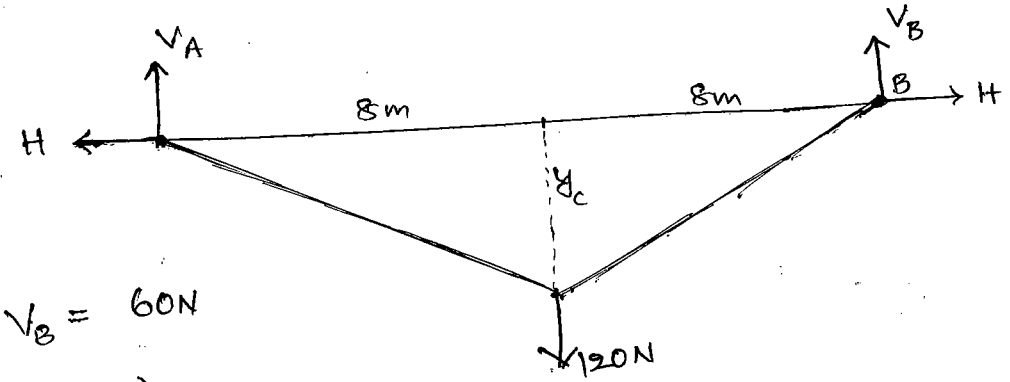
The equation of Caterary is $y = a\cosh\left(\frac{x}{a}\right)$

$$= \frac{a}{2}\left(e^{x/a}+e^{-x/a}\right)$$

(Leibnitz)

(1) A Light cable 18m long is supported at two ends at the same level. The supports are 16m apart. The cable supports 120N load dividing the distance in to two equal parts. Find the shape of the cable and tension in cable?

Sol:-



$V_A = V_B = 60N$

$\left(\sqrt{8^2 + y_c^2}\right) + 9 = 18$

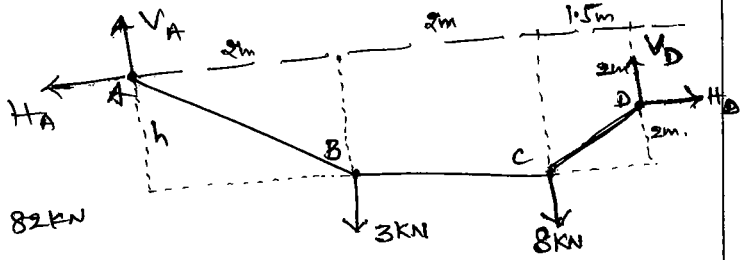
$y_c = 4.123m.$

$\sum M_c = 0$; $H \times 4.123 = 60 \times 8$

$H = 116.42 KN.$

$T = \sqrt{H^2 + V_A^2} = \sqrt{(60)^2 + (116.42)^2} = 130.97N$

(2) Determine the tension in each segment of the cable shown below. Also calculate the dimension 'h'?



Sol:- $T_{CD} = 6.79 KN$

$\theta_{BC} = 32.3^\circ$; $T_{BC} = 4.82KN$

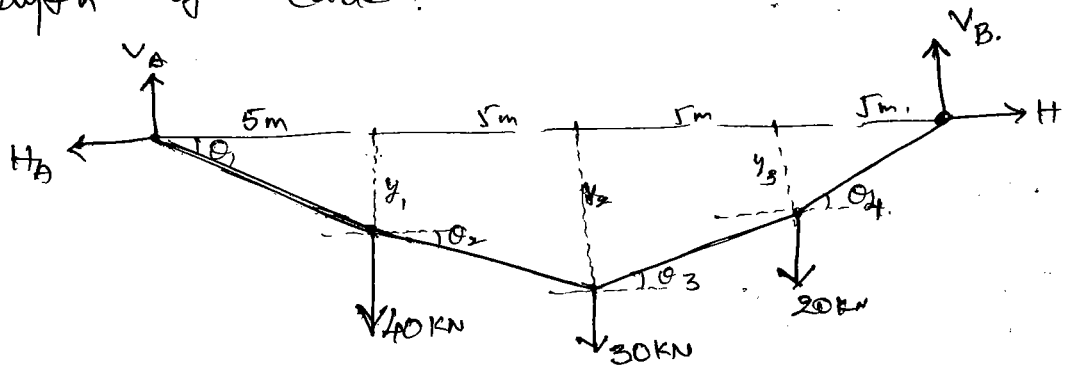
$\theta_{AB} = 53.8^\circ$; $T_{AB} = 6.9KN$

$h = 2.74 m.$

By taking $\sum M_A = 0$, we get T_{CD}. (from cable FBD)

By taking FBD of points C, B we get T_{BC}, T_{AB}.

Prob. A light cable is supported at two points 20m apart which are at same level. Cable supports three concentrated loads as shown below. The deflection at first point is found to be 0.8m. Det the tension in the different segments and total length of cable?



Sol: $V_A = 50$ KN ; $V_B = 40$ KN.

$y_1 = 0.8$m ; $y_2 = 0.96$m ; $y_3 = 0.64$m.

$\theta_1 = 9.09°$; $\theta_2 = 1.833°$; $\theta_3 = 3.662°$; $\theta_4 = 7.29°$

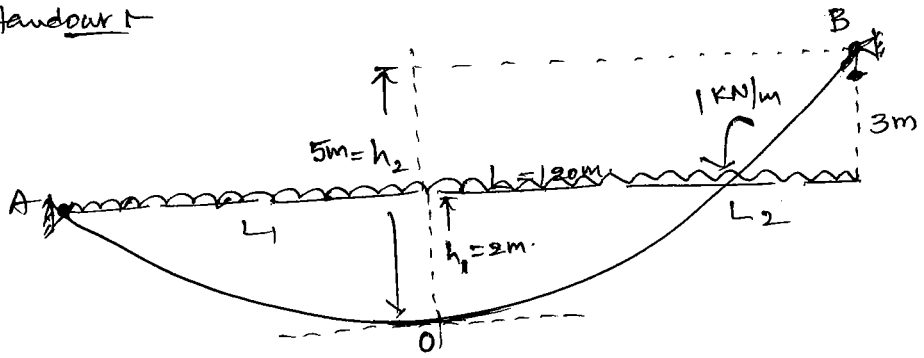
$T_1 = 316.49$ KN ; $T_2 = 312.678$ KN ; $T_3 = 318.162$ KN

$T_4 = 315.072$ KN.

$S = 20.117$m.

Cables with ends at different level:-

prob(6) from Handouts



Considering Origin at 'O'. $y = \frac{4hx^2}{l^2}$

$$L_1 = \frac{L\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} \quad ; \quad L_2 = \frac{L\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}$$

$$L_1 = 46.5\text{ m} \quad ; \quad L_2 = 73.5\text{ m}.$$

$\Sigma M_A = 0 \quad V_B \times 120 - H \times 3 - 120 \times 60 = 0$

$BM_O = 0$ (Taking OB Segment)

$$V_B \times 73.5 - H \times 5 - 1 \times \frac{73.5^2}{2} = 0$$

$V_B = 73.5$ KN $\qquad H = 540.4$ KN.

$V_A = 46.5$ KN

$$\therefore T_A = \sqrt{V_A^2 + H^2} = 542.4\text{ KN}$$

$$T_B = \sqrt{V_B^2 + H^2} = 545.4\text{ KN}.$$

$\therefore T_{max} = 545.4$ KN

Temp. Effect on cables:-

$$S = L + \frac{8h^2}{3L}$$

$$\frac{dS}{dh} = \frac{16h\,dh}{3L}$$

$$\delta S = \frac{16h}{3L}\,\delta h$$

$$\delta S = S\alpha t = \alpha t\left(L + \frac{8h^2}{3L}\right) = L\alpha t + (\,)$$

$$\delta S = L\alpha t$$

$$\therefore \delta h \times \frac{16h}{3L} = L\alpha t$$

$$\boxed{\delta h = \frac{3L^2\alpha t}{16h} = \frac{3L}{16h}(L\alpha t)}$$

Taking moment about Lower pt:-

$$H \cdot h = M_c \Rightarrow H = \frac{M_c}{h}$$

$$\frac{dH}{dh} = -\frac{M_c}{h^2} = -\left(\frac{M_c}{h}\right) \times \frac{1}{h}$$

$$\frac{dH}{dh} = -\frac{H}{h}$$

$$\frac{\delta H}{H} = -\frac{\delta h}{h}$$

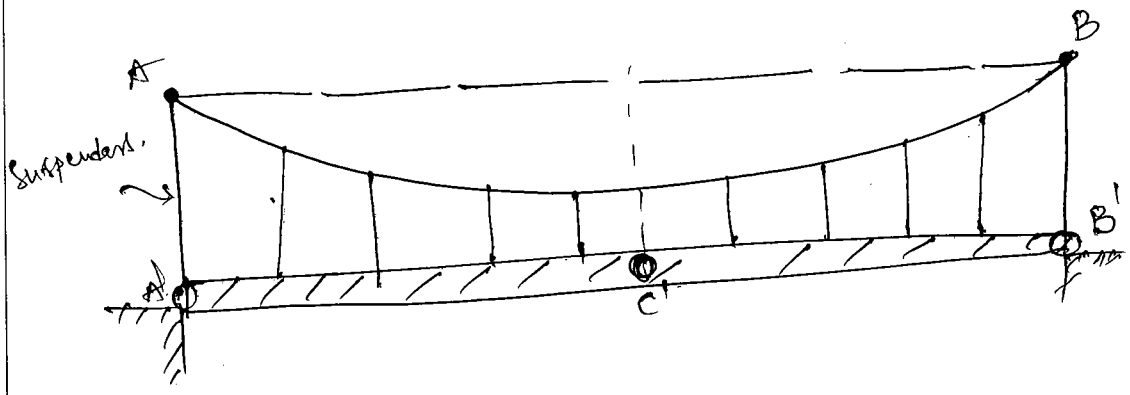
$$\frac{\delta H}{H} = -\frac{3L(L\alpha t)}{16h^2}$$

$$* \quad \delta H = -\frac{3L(L\alpha t)}{16h^2}\,[H]$$

(∴ H decreases with increase in Temp)

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Suspension Bridge with 3H stiffening girder



Prob: A 3H stiffening girder of a suspension bridge of span 100m is subjected to two point loads of 200 KN and 300 KN at a distance of 25m & 50m from left. Find SF and BM for the girder at a distance 30m from left. The supporting cable has a central dip of 10m. Find also max. tension and its slope in the cable?

Sol: Taking FBD of cable & S.G. $\Rightarrow$

$$\Sigma M_{A'} = 0 \Rightarrow R_{B'} \times 100 - 200 \times 25 - 300 \times 50 = 0$$

$$R_{B'} = 200 \text{ KN}.$$

FBD of S.G :- $(B'C')$

$$M_{C'} = 0 \Rightarrow R_{B'} \times \frac{l}{2} - \frac{wl^2}{8} = 0$$

$$R_{B'} \times \frac{l}{2} = \frac{wl^2}{8} \Rightarrow 200 = w \times \frac{100}{4}$$

$$\Rightarrow w = 8 \text{ KN/m}.$$

$$R_{A'} = 300 \text{ KN}.$$

$$SF_{\text{at } 30m} :- = 300 - 200 - w_e\left(\frac{l}{2} - x\right) = -60 \text{ KN}$$

$$BM :- = 300 \times 30 - 200 \times 5 - \frac{we}{2}(l - x) = -400 \text{ KNM}.$$

for cable:- $V_A = \frac{w_e l}{2} = \frac{8 \times 100}{2} = 400\ KN$

$$H \cdot h = \frac{w_e l^2}{8} = \frac{8 \times 100^2}{8} = 10{,}000$$

$$H = \frac{10{,}000}{10} = 1000\ KN.$$

$$T_{max} = \sqrt{1000^2 + 400^2} = 1077.03\ KN$$

$$T_{max} \cos\theta = H \Rightarrow \cos\theta = \frac{1000}{1077.03} \Rightarrow \boxed{\theta = 21.8^\circ}$$

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prob:- A 3H Suspension girder bridge has a span of 200m over the supports at same level. It has a central dip of 20m. The girder carries three point loads of 15KN, 25KN and 20KN acting at 35m, 80m & 150m resp. from left end. Draw BMD?

prob:- A Suspension cable 100m span and 15m dip is stiffened with a three hinged girder. If a concentrated load of intensity 100KN acts at 30m from left. Determine max. tension in the cable. Also det. max BM & SF?

Suspension cable with two-Hinged stiffening girder:-

$$w_e = \frac{W}{L}$$

This is statically Indeterminate structure However the analysis is made simple by assuming that the girder is very stiff and hence whatever is the load on the girder the stiffeners are equally strened

Prob:- A suspension bridge of span 80m and width 6m is having two cables stiffened with two Hinged girders. The central dip of cables is 8m. Dead load on the bridge is $5 KN/m^2$ and LL is $10 KN/m^2$ which covers the left half of span. Determine S.F and BM at 20m from left end. Find also the max tension in the cable?

Sol:- $l = 80m, \quad h = 8m, \quad DL = \frac{5 \times 6}{2} = 15 KN/m$

$$LL = \frac{10 \times 6}{2} = 30 KN/m \text{ (left half)}$$

$$W = 15 \times 80 + 30 \times 40 = 2400 KN$$

$$w_e = \frac{W}{l} = \frac{2400}{80} = 30 KN/m.$$

$$H = \frac{w_e l^2}{8h} = \frac{30 \times 80^2}{8 \times 8} = 3000 KN$$

$$V = \frac{w_e l}{2} = \frac{30 \times 80}{2} = 1200 KN.$$

$$T_{max} = \sqrt{V^2 + H^2} = \sqrt{3000^2 + 1200^2} = 3231.1 KN.$$

Girder:- $R_A = \frac{15 \times 80 \times 40 + 30 \times 40 \times (80 - 20)}{80} = 1500 KN.$

Beam $SF_D = 1500 - 15 \times 20 - 30 \times 20 = 600 KN$

SF D^1 = $600 - \left(\frac{\text{Load}}{2} - 30\right)$

$= 600 - \left(\frac{30 \times 30}{2} - 30 \times 20\right)$

$= 0$

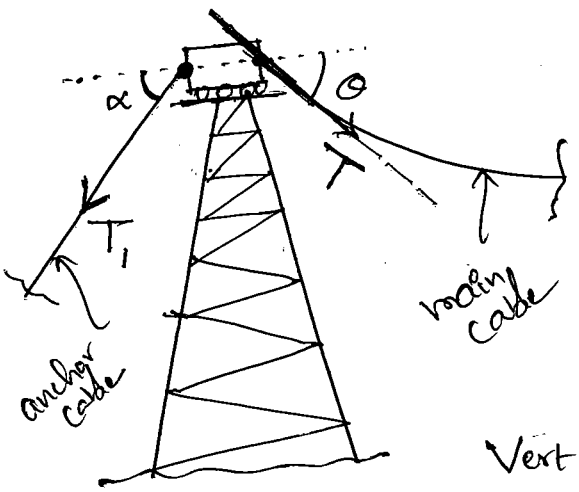
$BM_B = 1500 \times 20 - 15 \times 20 \times 10 - 30 \times 20 \times 10 - \left(\frac{30 \times 20 \times 80}{2} - 30 \times \frac{20 \times 20}{2}\right) = 3000 \text{ KNm}$

Prob:- A suspension cable 100m span and 12m dip is stiffened with a 2H girder. The girder carries DL of 10KN/m over entire span and a load of 800 KN at 40m, from left. Det. max tension in cable and SF & BM at 30m from left?

Anchor cable:- main cable is supported by anchor cable in two ways.

(i) Saddle (ii) pulley.

(i) **Saddle** :- Horizontal roller on tower.



$\Sigma F_x = 0 \Rightarrow$

$T\cos\theta = T_1\cos\alpha$ — (1)

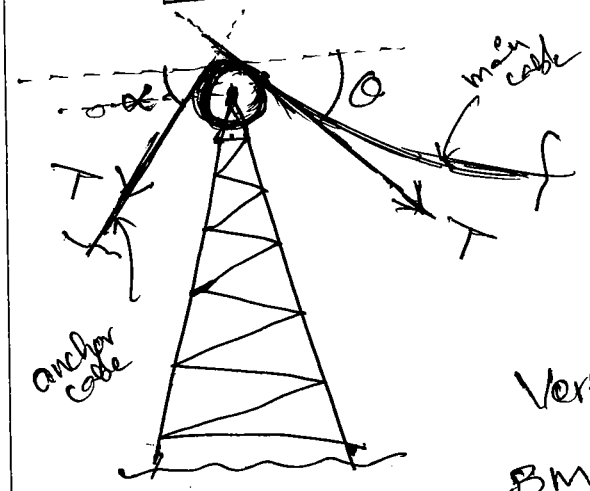
$\theta, \alpha, T \rightarrow$ solved from cable.

T_1 may be calculated from (1)

Vert force $= T\sin\theta + T_1\sin\alpha$

moment at base of tower = can be calculated

(ii) **Pulley:-** mounted on tower.



T, θ, α solved from cable

Horz force $= T\cos\theta - T\cos\alpha$

$= T(\cos\theta - \cos\alpha)$

Vert force $= T(\sin\theta + \sin\alpha)$

BM at base of tower = Can be calculated

prob: A Bridge cable is suspended from towers 80m apart and carries a load of 30KN/m on the entire span. If the max Sag is 8m. Calculate maximum tension in cable If the cable is supported by saddles which are stayed (anchored) by wires inclined at 30° to horizontal. Determine forces on towers. If the same inclination of back stay passes over pulley determine forces on towers?

Sol: $V_A = 1200 KN$ (Taking moment about 'C')

$H = 3000 KN.$

$T_{max} = \sqrt{1200^2 + 3000^2} = 3231.1 KN$

$H = T_{max} \cos\theta \Rightarrow \theta = 21.8°$

Saddle:

$T_1 \cos\alpha = T \cos\theta$

$T_1 = 3464.1 KN$

Vert force $= 2939.0 KN$

pulley:

Hor force $= 201.82 KN$

Vert force $= 2815.48 KN$