SA-II : Unit-I

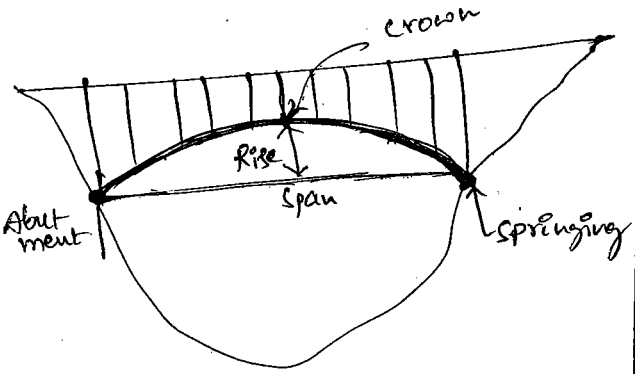
Three Hinged Arches:- Elastic theory of Arches - Eddy's theorem Determination of horizontal thrust, Bending moment, normal thrust and radial shear - effect of Temperature

Two Hinged Arches:- Determination of horizontal thrust, BM, normal thrust, radial shear - Rib shortening and temp Stresses, tied arches - fixed arches (NO analytical Qs)

Arches :- Intro:- For large spans, beams are very uneconomical and mostly the self wt of beams contributes to the stress in such large proportions that it is difficult to design beams for large spans.

eg: Bridges

Def: Arches are nothing but curved beams (Vert plane) that transfer loads to their plane.

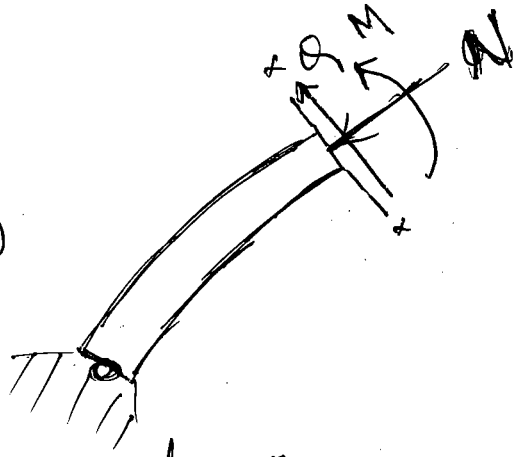


- Arches transfer loads to abutments at Springing points.
- Hinges may be provided at these points.
- The top most point is crown some times has a hinge.
- The ht of crown above the support level is called rise.

→ (Because of curved nature of arches, they develop horizontal forces)*

→ Any section in the arch will be subjected to normal thrust, radial shear, and Bending moment. However BM is less compared to a beam of same span. Thus loads get transferred partly by axial Comph and partly by flexure.

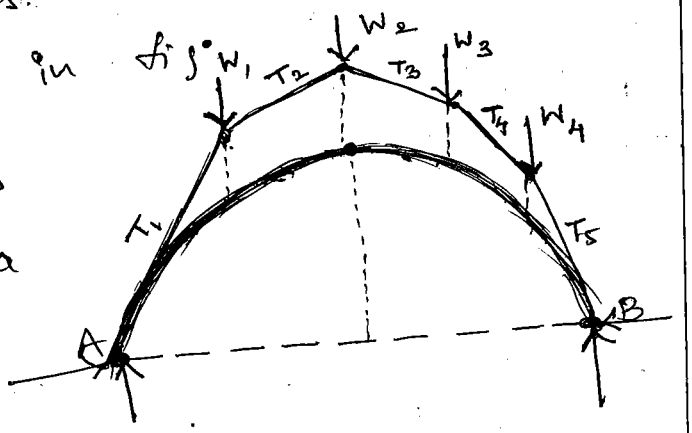
Internal forces in the Arch:



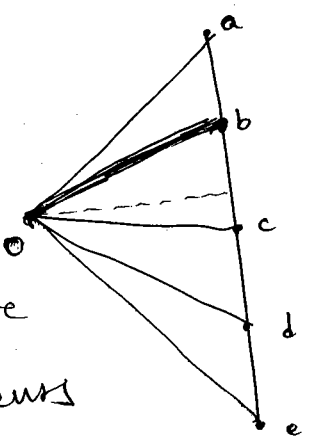
Types of Arches (No. of Hinges)

(i) Three-Hinged
(ii) Two-Hinged
(iii) Hingeless (fixed) (iv) Single Hinge

Elastic theory of Arches:- Consider an arch with loads as shown in fig.



- Represent vertical loads to some scale on a vertical line.
- Select a point outside vertical line at some distance
- Join Oa, Ob, Oc, Od etc
- Draw lines parallel to Oa, Ob etc on the arch, which represents T_1, T_2, etc.



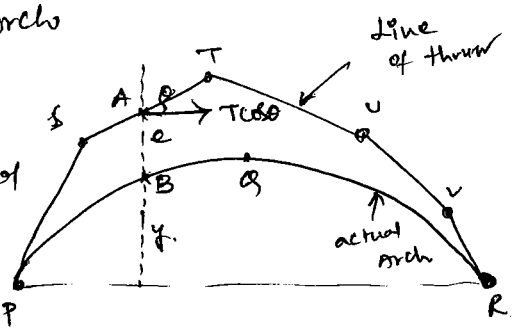
- No. of polygons are possible as O can be chosen at any distance and anywhere.

→ Funicular polygon which lies within the actual arch is called as line of thrust.

→ polygonal arch following the path of true compn. is called theoretical Arch (or) Linear arch.

Eddy's theorem:- BM at any point on the arch axis is proportional to the vertical intercept b/w theoretical and actual arch.

proof:- • Let PQR is actual arch



• Let PSTUVR is funicular polygon representing theoretical arch.

• Select a point 'A' on Line of thrust of Comph.

• draw a vertical line which will intersect actual arch at 'B'.

• NCosθ is the hor? Compoent of thrust 'N'

• BM = NCosθ × AB.

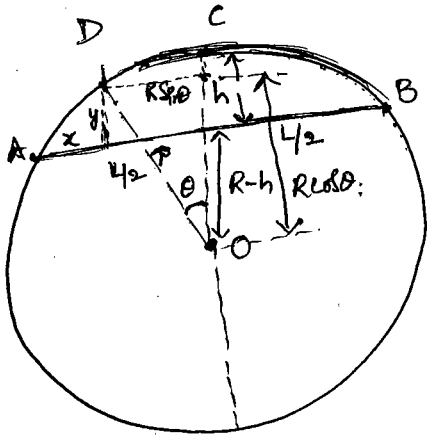
• As NCosθ being constant throughout the arch,

BM ∝ AB

∴ BM is proportional to vertical intercept.

Types of arches:- (Geometry) (i) circular
(ii) parabolic
(iii) elliptical.

Circular Arch:-



From the property of circle,

$$\frac{L}{2} \times \frac{L}{2} = h(2R-h)$$

$$\frac{L^2}{4h} = 2R-h$$

$$2R = h + \frac{L^2}{4h}$$

$$\boxed{R = \frac{h}{2} + \frac{L^2}{8h}}$$ ← radius of circular Arch

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Taking Origin at A, $x = \frac{L}{2} - R\sin\theta$

$y = R\cos\theta - (R-h)$

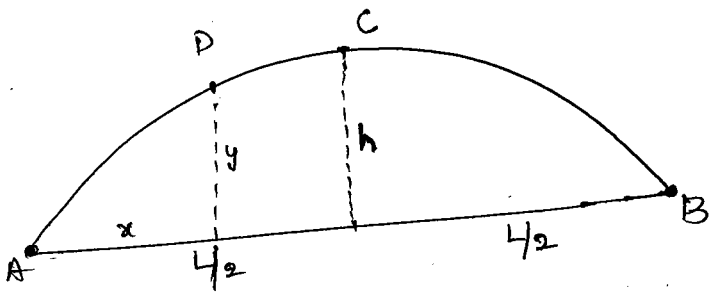
$= R\cos\theta - R + h$

$= R(\cos\theta - 1) + h$

$y = h - R(1-\cos\theta)$

For parabolic Arch:- Taking Origin at A:-

$y = \frac{4hx}{L^2}(L-x)$



If Origin is at 'C' :- $\frac{x^2}{y} = a$

if $x = L/2$ $y = h$ $\Rightarrow$ $a = \frac{L^2}{4h}$

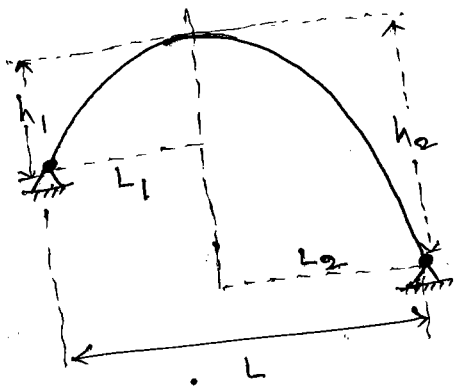
$\frac{x^2}{y} = \frac{L^2}{4h} \Rightarrow y = \frac{4hx^2}{L^2}$

If Springing points are not at same level:-

$\frac{x^2}{y} =$ constant

$\frac{x}{\sqrt{y}} =$ Constant

$\frac{L_1}{\sqrt{h_1}} =$ Const ; $\frac{L_2}{\sqrt{h_2}} =$ Const.



$a = \frac{L_1}{\sqrt{h_1}}$; $b = \frac{L_2}{\sqrt{h_2}}$ $\Rightarrow$ $a+b = \frac{L_1+L_2}{\sqrt{h_1}+\sqrt{h_2}}$

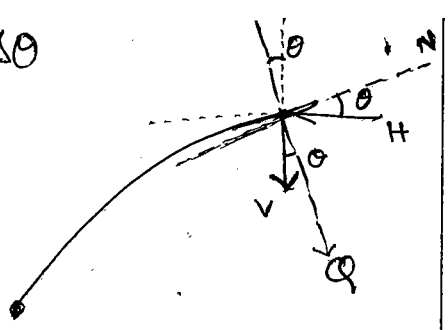
If a, b are constant a+b is also const.

$\frac{L_1}{\sqrt{h_1}} = \frac{L_1+L_2}{\sqrt{h_1}+\sqrt{h_2}} \Rightarrow L_1 = \frac{L\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}$; $L_2 = \frac{L\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}$

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N & Q :- $N = V\sin\theta + H\cos\theta$

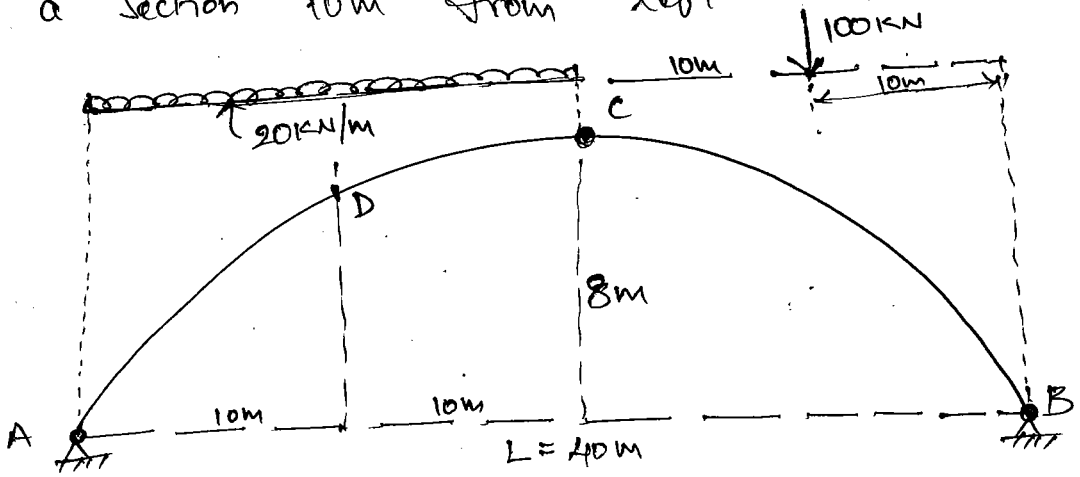
$Q = V\cos\theta - H\sin\theta$



prob 1:- TSO

prob 1:- A 3H circular arch hinged at the springing and crown points has a span of 40m. and a central rise of 8m. It carries a UDL of 20 KN/m over the left half of the span together with a point load of 100 KN at the right Quarter span point. Find the reactions at the supports, normal thrust and shear at a section 10m from left support ?

sol:-



from FBD of ACB :- $\Sigma M_B = 0$

$V_A \times 40 - 20\times20\times30 - 100\times10 = 0$

$$\boxed{V_A = 325\ KN}$$

$\Sigma F_y = 0$ $\quad V_A + V_B = 20\times20 + 100$

$$\boxed{V_B = 175\ KN}$$

from FBD of AC :- applying $\Sigma M_C = 0$

$V_A \times 20 - 20\times20\times10 - H\times8 = 0$

$$\boxed{H = 312.5\ KN}$$

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from the property of circle:- $\frac{L}{2} \times \frac{L}{2} = h(2R-h)$

$20 \times 20 = 8(2R-8)$

$\therefore R = 29\,m$

for point 'D' on the circle:-

$\sin\theta = \frac{10}{29} \Rightarrow \theta = 20.17^\circ$

Vertical shear, V at D $= V_A - 20 \times 10$

$= 325 - 200 = 125\,KN$

At 10m from left support:-

$\therefore$ N (Normal thrust) $= V\sin\theta + H\cos\theta$

$= 125 \sin 20.17^\circ + 312.5 \cos 20.17^\circ$

$= 336.437\,KN$

Q (Radial shear) $= V\cos\theta - H\sin\theta$

$= 125\cos 20.17^\circ - 312.5 \sin 20.17^\circ$

$= 9.575\,KN$

Prob 2:- TS ②

A circular arch to span 25m with a central rise 5m is hinged at the crown and springing. It carries a point load of 100 KN at 6m from the left support. Calculate reactions at the supports and at crown. Also cal. moment at 5m from left support?

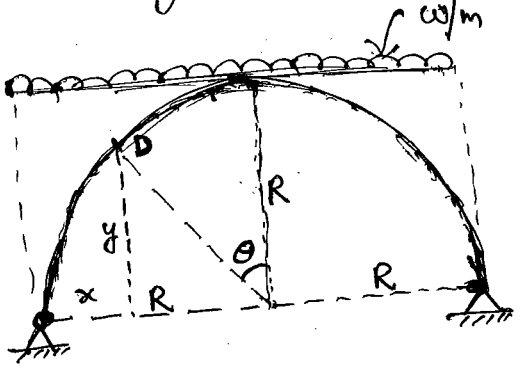
Sol:-

$V_A = 76\,KN$ $\quad V_B = 24\,KN$ $\quad H = 60\,KN$

$R = 18.125\,m$ $\quad \theta = 24.443^\circ$; $y_D = 3.375\,m$.

$M_D = 177.5\,KNm$

(3) Prob:- A Three Hinged Semi-circular arch of radius R carries a UDL of w/m, over its entire horizontal span. Determine the reactions of supports and maximum bending moment in the arch?



Sol:-

$$V_A = V_B = \frac{w \times 2R}{2} = wR$$

$$H \times R - V_A \times R + wR \cdot \frac{R}{2} = 0$$

$$\boxed{H = \frac{wR}{2}}$$

$$x = R - R\sin\theta \qquad y = R\cos\theta$$

$$M_x = V_A \cdot x - Hy - \frac{wx^2}{2}$$

$$M_x = \frac{wR^2}{2}\left(1 - \cos\theta - \sin^2\theta\right)$$





$$\frac{dM_x}{d\theta} = \sin\theta - 2\sin\theta\cos\theta = 0$$

$$\sin\theta\left(1 - 2\cos\theta\right) = 0$$

$\sin\theta = 0 \Rightarrow \theta = 0^\circ$; point 'C'.

$1 - 2\cos\theta = 0 \Rightarrow \cos\theta = 1/2 \quad \theta = 60^\circ$

$\therefore x = R - R\sin 60^\circ$; $M_{max} = -\frac{wR^2}{8}$

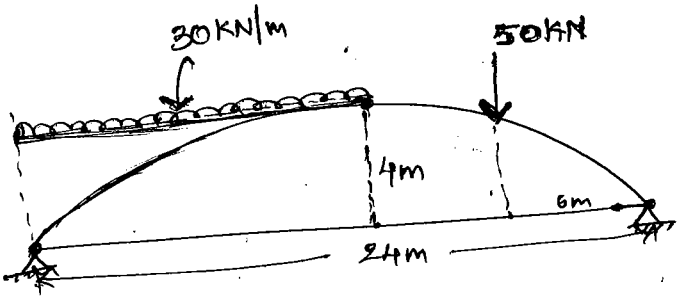
* "funicular" is derived from the latin word "rope".

* A funicular structure is said to be a structure which can achieve a mechanism of a 'right' form (shape/Geometry) corresponding to the applied load.

Prob: (4)

A Three Hinged parabolic arch hinged at the Support and at the crown has a span of 24m. and a central rise of 4m. It carries a point Load of 50 KN at 18m from left Support and a UDL of 30KN/m over the left half portion. Determine the moment, thrust and radial shear at a Section 6m from left Support?

Sol:-



$\Sigma M_B = 0 \Rightarrow V_A = 289.5\ KN$

$\Sigma F_y = 0 \quad V_B = 127.5\ KN$

$\Sigma M_c = 0 \quad H = 307.5\ KN$

at 6m, $y = 3m \Rightarrow y = \frac{4hx(L-x)}{L^2} = 3m$

$BM = 232.5\ KNm$

$y = \frac{4hx(L-x)}{L^2} \Rightarrow y = \frac{4hxL}{L^2} - \frac{4hx^2}{L^2} = \frac{4hx}{L} - \frac{4hx^2}{L^2}$

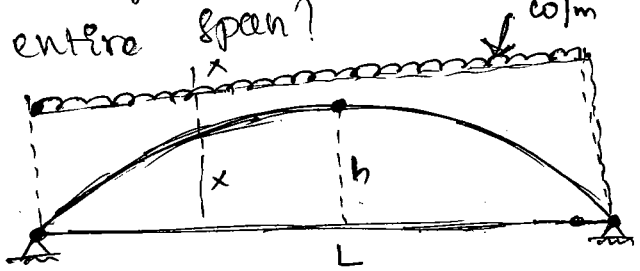
$\frac{dy}{dx} = \frac{4h}{L} - \frac{8hx}{L^2} \Rightarrow$ at 6m, $\frac{dy}{dx} = \tan\theta$

$\theta = 18.435^\circ$

$N = 324.133\ KN ; \quad Q = 0$

Prob: (5)

Show that the parabolic shape is a funicular shape for a three hinged arch subjected to a UDL over its entire span?



Sol:-

$V_A = V_B = \frac{wL}{2}$

$H = \frac{wL^2}{8h}$

$y = \frac{4hx(L-x)}{L^2}$; $M_x = \frac{wL}{2}x - \frac{wL^2}{8h}(y) - \frac{wx^2}{2}$

$= \frac{wL}{2}x - \frac{wL^2}{8h}\left(\frac{4hx}{L} - \frac{4hx^2}{L^2}\right) - \frac{wx^2}{2}$

$= \frac{wL}{2}x - \frac{wLx}{2} + \frac{wLx^2}{2L} - \frac{wx^2}{2}$

$BM_x = 0$

Two Hinged Arches

* Two hinged arches are indeterminate structures
* They have degree of indeterminacy as one.
* Commonly used geometry for two hinged arches are circular and parabolic.
* Horizontal force 'H' may be taken as the redundant force. This can be found by theorem of Castigliano (or) unit load method.
* Arches are structural members curved in elevation used to support heavy loads on large spans.

From Castigliano's first theorem:-

$$M_x = M' - Hy \quad (\because M' = \text{beam moment})$$

$$U = \int \frac{M_x^2}{2EI}\, ds.$$

$$\frac{\partial U}{\partial H} = 0 \Rightarrow \int M_x \cdot \left(\frac{\partial M_x}{\partial H}\right) \frac{ds}{EI} = 0$$

$$\int (M' - Hy)(-y)\, \frac{ds}{EI} = 0$$

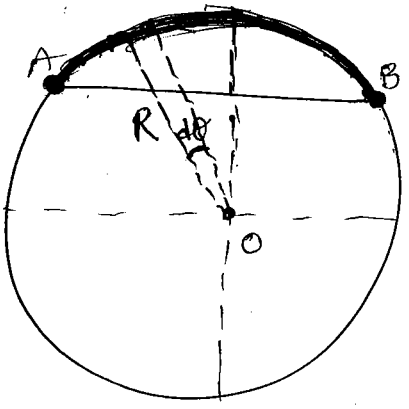
$$\int M' y\, \frac{ds}{EI} = \int \frac{Hy^2\, ds}{EI}$$

$$\therefore H = \frac{\int M' y\, \frac{ds}{EI}}{\int y^2\, \frac{ds}{EI}}$$

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$ds = R\,d\theta$

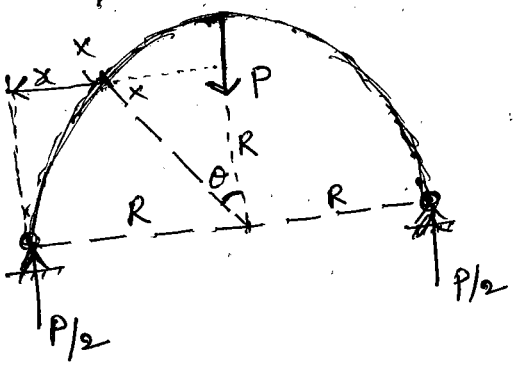
$y = h - R(1-\cos\theta)$



prob. (1) Determine horizontal thrust developed in a semi-circular arch of radius R subjected to a concentrated load 'P' at the crown?

sol:-

$$H = \frac{\int M' y\,ds}{\int y^2\,ds}$$



$$M' = \frac{P}{2}x = \frac{PR}{2}(1-\sin\theta)$$

$$\int M' y\,ds = \int \left[\frac{PR}{2}(1-\sin\theta)\right](R\cos\theta)(R\,d\theta)$$

$$= \frac{PR^3}{2}\int_0^{\pi/2}(1-\sin\theta)\cos\theta\,d\theta$$

Let $1-\sin\theta = t$; $-\cos\theta\,d\theta = dt$

$$= PR^3\int_0^{\pi/2} t\,(-dt) = -PR^3\left(\frac{t^2}{2}\right) = -PR^3\times\left(\frac{t^2}{2}\right)_1^0$$

$$= -PR^3\left[0-\frac{1}{2}\right] = \frac{PR^3}{2}$$

$$\int y^2\,ds = \int (R\cos\theta)^2 R\,d\theta = 2\int_0^{\pi/2} R^2\,\frac{(1+\cos 2\theta)}{2}\,d\theta$$

$$= R^2\times R\left[(\theta)_0^{\pi/2} + \left(\frac{\sin 2\theta}{2}\right)_0^{\pi/2}\right]$$

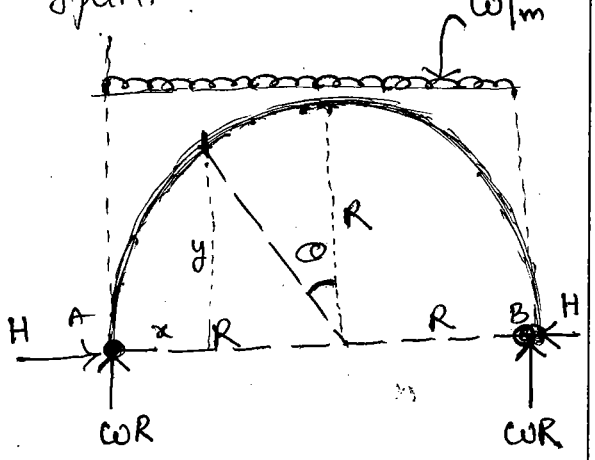
$$= R^2\times R\left[\frac{\pi}{2}+0\right] = \frac{\pi R^3}{2}$$

$\because H = \left(\frac{PR^3}{2}\right)\Big/\left(\frac{\pi R^3}{2}\right)$

$$\boxed{H = \frac{P}{\pi}}$$

(∵ neglecting self wt of arch).

② prob: Determine horizontal thrust in a semi circular arch subjected to UDL w/m over its horizontal span.?



sol:-

$x = R(1-\sin\theta)$

$y = R\cos\theta$

$ds = R\,d\theta$

$M' = wRx - \frac{wx^2}{2}$

$= wR \cdot R(1-\sin\theta) - \frac{wR^2}{2}(1-\sin\theta)^2$

$= \frac{wR^2}{2}\left[2 - 2\sin\theta - 1 - \sin^2\theta + 2\sin\theta\right]$

$= \frac{wR^2}{2}\left[\cos^2\theta\right]$

$\because t = \sin\theta$
$dt = \cos\theta\, d\theta \rightarrow \left(\frac{t^3}{3}\right)$

$\int M' y\, ds = \int \frac{wR^2}{2}(\cos^2\theta) \times R\cos\theta\, R\, d\theta$

$= \frac{wR^4}{2} \times 2\int_0^{\pi/2} (1-\sin^2\theta)\cos\theta\, d\theta$

$= wR^4\left[\int_0^{\pi/2} \cos\theta\, d\theta - \int_0^{\pi/2} \sin^2\theta\cos\theta\, d\theta\right]$

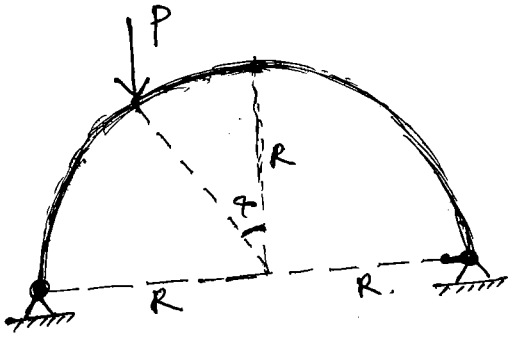
$= wR^4\left[(\sin\theta)_0^{\pi/2} - \left(\frac{t^3}{3}\right)_0^1\right]$

$= wR^4\left[1 - \frac{1}{3}\right] = \frac{2}{3}wR^4.$

$\int y^2\, ds = \frac{\pi}{2}(R^3) \Rightarrow \because H = \left(\frac{2wR^4}{3}\right) \Big/ \frac{\pi R^3}{2}$

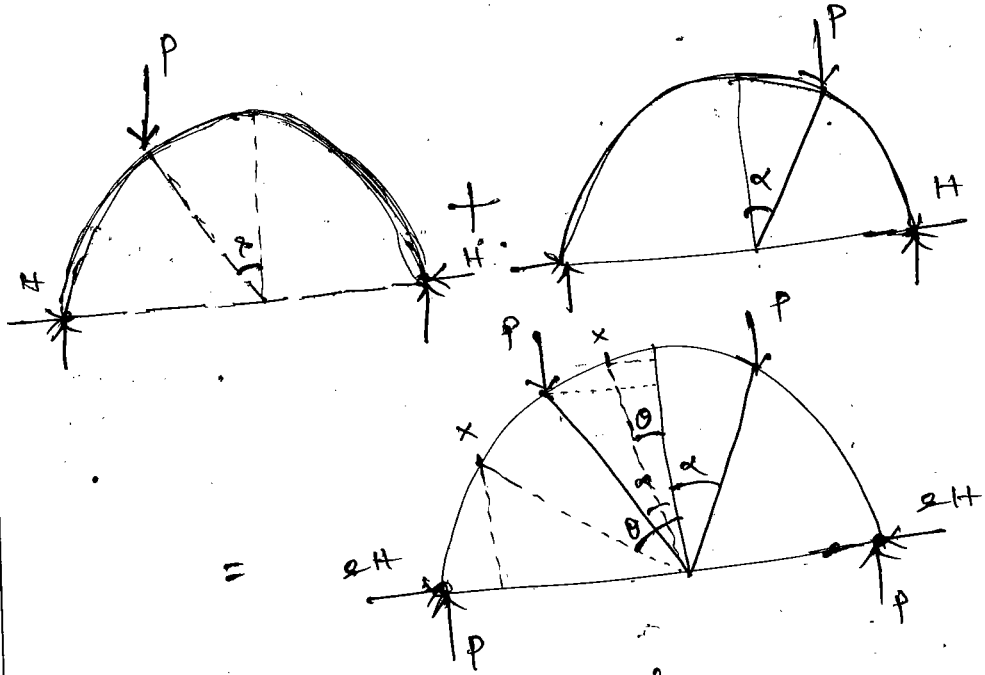
$$\boxed{H = \frac{4wR}{3\pi}}$$

(3) prob: Determine horizontal reaction in a semi circular two hinged arch when a load 'P' acts at a point 'D'?



Sol:- To make the given problem as symmetric loading, add a force 'P' at an angle 'α' to the right side.

When the horizontal thrust developed is 2H.



$1-\sin\theta = t$

$-\cos\theta\, d\theta = dt$

Now solving symmetrical arch,

for $\theta = 0$ to α $\quad M' = Px - P(R\sin\alpha - R\sin\theta)$

$M' = P(R - R\sin\theta) - PR(\sin\alpha - \sin\theta)$

$= PR - PR\sin\theta - PR\sin\alpha + PR\sin\theta = PR(1-\sin\alpha)$

for $\theta = \alpha$ to $\pi/2$; $M' = Px = P \times R(1-\sin\theta)$

$$\int M'y\,ds = 2\int_0^{\alpha} PR(1-\sin\alpha)\,R\cos\theta\,R\,d\theta + 2\int_{\alpha}^{\pi/2} PR(1-\sin\theta)R\cos\theta\,R\,d\theta$$

$$= 2PR^3(1-\sin\alpha)(\sin\theta)_0^{\alpha} + 2PR^3(-1)\left(\frac{t^2}{2}\right)$$

$$= 2PR^3(1-\sin\alpha)\sin\alpha + 2PR^3(-1)\left(\frac{(1-\sin\theta)^2}{2}\right)_{\alpha}^{\pi/2}$$

$= 2PR^3(\sin\alpha - \sin^2\alpha) + PR^3(1-\sin\alpha)$

$= 2PR^3\sin\alpha - 2PR^3\sin^2\alpha + PR^3(1+\sin^2\alpha - 2\sin\alpha)$

$= 2PR^3\sin\alpha - 2PR^3\sin^2\alpha + PR^3 + PR^3\sin^2\alpha - 2PR^3\sin\alpha$

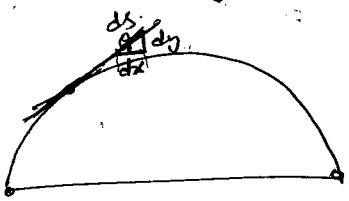
$= PR^3 - PR^3\sin^2\alpha = PR^3(\cos^2\alpha)$

$$\int y^2 ds = \frac{\pi R^3}{2} \Rightarrow H = \frac{PR^3(\cos^2\alpha)\times 2}{\pi R^3}$$

$$\boxed{H = \frac{P\cos^2\alpha}{\pi}}$$

Two Hinged parabolic Arches $H = \dfrac{\int M' y\, ds}{\int y^2 ds}$.

$$H = \frac{\int M' y \left(\frac{dx}{\cos\theta}\right)}{\int y^2 \frac{dx}{\cos\theta}}$$

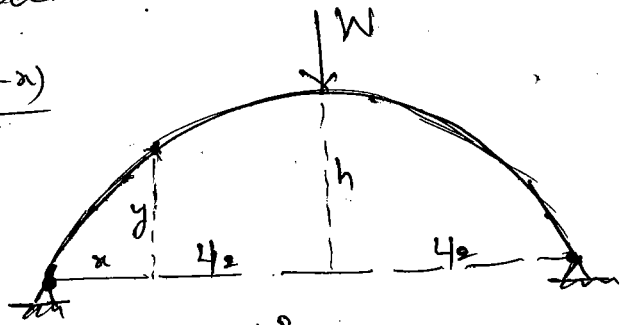


$\cos\theta = \dfrac{dx}{ds}$

$\boxed{ds = \dfrac{dx}{\cos\theta}}$

$$\therefore \boxed{H = \frac{\int M' y\, dx}{\int y^2 dx}}$$

Prob Determine the horizontal thrust in a two hinged parabolic arch subjected to a point Load at the crown?



Sol $M' = \dfrac{W}{2}x \;;\; y = \dfrac{4hx(L-x)}{L^2}$

$$\therefore \int M' y\, dx$$

$$= 2\int_0^{l/2} \frac{Wx}{2}\left(\frac{4hx}{l^2}\right)(L-x)\,dx = \frac{5\,Whl^2}{48}$$

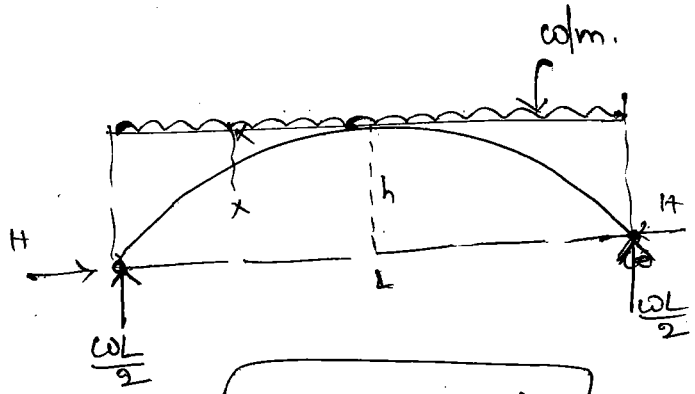
$$\int y^2 dx = \frac{8}{15} lh^2$$

$$\therefore \boxed{H = \frac{25}{98}\frac{WL}{h}}$$

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Prob:- Determine the horizontal thrust in a two hinged parabolic arch subjected to a UDL of w/m on its horizontal span?

Sol:- $H = \dfrac{\int M'y\,dx}{\int y^2\,dx}$



Numr:- $\int M'y\,dx$

$= \int \left(\frac{wL}{2}x - \frac{wx^2}{2}\right) \frac{4hx}{L^2}(L-x)\,dx$ $\quad \left[M' = \frac{wL}{2}x - \frac{wx^2}{2}\right]$

$= \frac{4hw}{L^2}\int \left(\frac{Lx}{2} - \frac{x^2}{2}\right)(Lx - x^2)\,dx$

$= \frac{4hw}{2L^2}\int (Lx - x^2)^2\,dx$

$= \frac{4hw}{2L^2}\int_0^L (L^2x^2 + x^4 - 2Lx^3)\,dx$

$= \frac{4hw}{2L^2}\left[L^2\left(\frac{x^3}{3}\right)_0^L + \left(\frac{x^5}{5}\right)_0^L - 2L\left(\frac{x^4}{4}\right)_0^L\right]$

$= \frac{4hw}{2L^2}\left[\frac{1}{3} + \frac{1}{5} - 2\times\frac{1}{4}\right]$ $\quad \left(\frac{10+6-15}{30} = \frac{1}{30}\right)$

$= \frac{2hwL^5}{2L^2}\left[\frac{1}{3} + \frac{1}{5} - \frac{1}{2}\right]$

$= 2hwL^3\left(\frac{1}{3} + \frac{1}{5} - \frac{1}{2}\right)$

$= \frac{2hwL^3}{30} = \frac{hwL^3}{15}$.

Den $= \frac{8Lh^2}{15}$ $\quad \therefore H = \frac{hwL^3}{15} \Big/ \frac{8Lh^2}{15} = \frac{wL^2}{8h}$

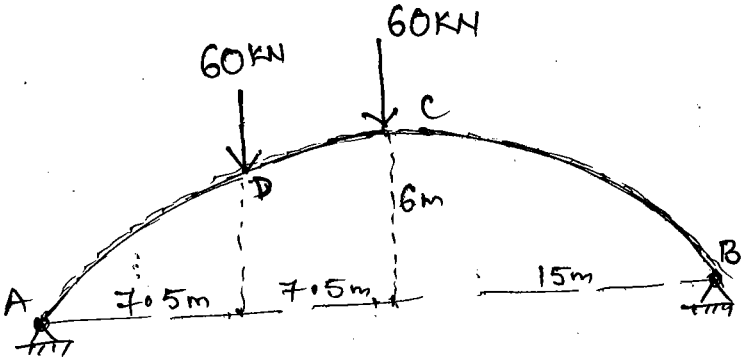
prob. Determine horizontal thrust developed in a two hinged parabolic arch subjected to a point load 'W' at the crown?

Sol:- $$H = \frac{5\,WhL^2}{48} \Big/ \frac{8Lh^2}{15} = \frac{25\,WL}{128h}$$

prob:- Two-hinged parabolic arch of span 30m and rise 6m carries two point loads each 60 KN at a distance 7.5m and 15m from left. Determine horizontal thrust and maximum the -ve moments in the arch?

Sol:- $V_A = 75$ KN

$V_B = 45$ KN



for $x = 0$ to 7.5 ; $M' = 75x$

$x = 7.5$ to 15 : $M' = 75x - 60(x-7.5) = 15x + 450$

$x = 0$ to 15 from B : $M'_m = 45x$

$$y = \frac{4hx(L-x)}{L^2} = \frac{4 \times 6x\,(30-x)}{30^2}$$

$$\int M'y\,dx = 577798.87 \; ; \; \int y^2 dx = 576$$

$$\therefore \quad H = 100.345 \text{ KN}$$

$BM_D = 110.95$ KNm, $BM_C = 78.93$ KNm.

max +ve BM = 110.95 KNm

$M_x = 45x - 100 \cdot 345\,y$

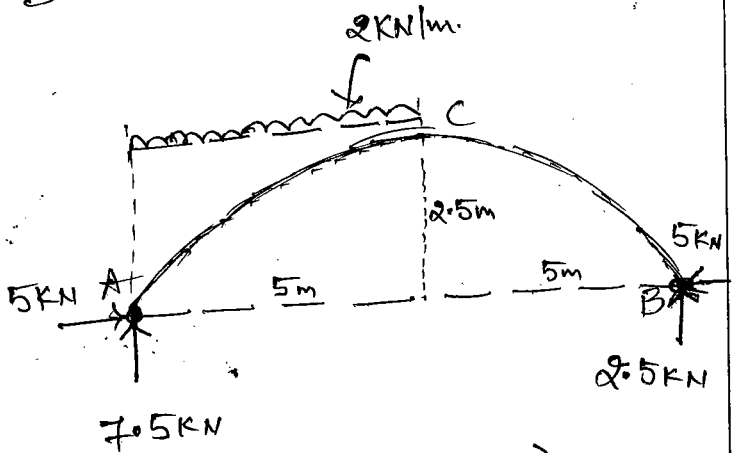
$\frac{dM_x}{dx} = 0 \Rightarrow x = 6.591\,m.$

$M_{min} = -116.25\,KNm.$

Nov-15 Prob:- A two hinged parabolic arch has a span of 10m has a central rise of 2.5m. It is loaded with UDL 2KN/m over a half of the span from left support. Determine end reactions, horizontal thrust, max & min. BM in the arch?

Sol:- $H = \frac{\int M' y\,dx}{\int y^2\,dx}$

$= \frac{166.665}{33.334} = 5KN$



AC:- $M_x = 7.5x - x^2 - 5\left(\frac{4 \times 2.5x}{100}(10-x)\right)$

$= 2.5x - 0.5x^2$

$\frac{dM_x}{dx} = 0 \Rightarrow x = 2.5\,m.$ $M_{max/min} = 3.125\,KNm$

BC:- $M_x = 2.5x - 5\left(\frac{4 \times 2.5x}{100}(10-x)\right)$

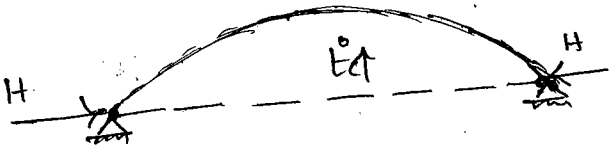
$\frac{dM_x}{dx} = 0 \Rightarrow x = 2.5\,m$

$M_{max/min} = -3.125\,KNm.$

$M_C = 0$

SA-II
I-(10)

Effect of temperature:- Let temperature of an arch is increased by $t^\circ C$. Supports exert presence on the arch.



$M_x = -Hy$

$$U = \int \frac{M_x^2\, dx}{2EI}$$

$$\frac{\partial U}{\partial H} = L\alpha t = \frac{\partial \left(\int \frac{M_x^2\, dx}{2EI}\right)}{\partial H} = \frac{1}{2}\int \frac{\partial (-Hy)^2}{\partial H}\, dx.$$

$$= \frac{1}{2EI} \int 2(-Hy)(-y)\, dx = \int \frac{Hy^2\, dx}{EI}$$

$$L\alpha t = H \int \frac{y^2\, dx}{EI} \Rightarrow \boxed{H_t = \frac{L\alpha t}{\int \frac{y^2\, dx}{EI}}}$$

Rib Shortening:- (Due to normal thrust)

$M_x = M' - Hy$

$N = V\sin\theta + H\cos\theta$

$$U = \int \frac{N_x^2}{2AE}\, ds \Rightarrow \frac{\partial U}{\partial H} = 0$$

$$\therefore \int \frac{(M' - Hy)(-y)\, ds}{EI} + \int \frac{(V\sin\theta + H\cos\theta)\cos\theta \cdot ds}{AE} = 0$$

$$\Rightarrow -\int \frac{M'y\, ds}{EI} + \int \frac{Hy^2\, ds}{EI} + \int \frac{H\cos^2\theta\, ds}{AE}$$

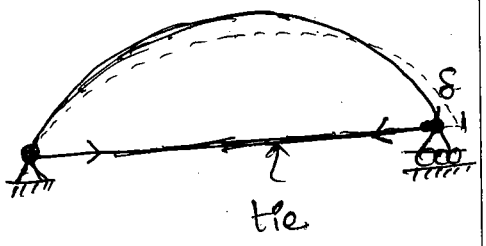
neglecting shear

$$+ \int \frac{V\sin\theta\cos\theta\, ds}{AE} = 0$$

$$H = \left(\int \frac{M'y\, ds}{EI}\right) \Big/ \int \frac{y^2\, ds}{EI} + \int \frac{\cos^2\theta\, ds}{AE}$$

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Pied Arch:- $\delta = \frac{HL}{AE}$

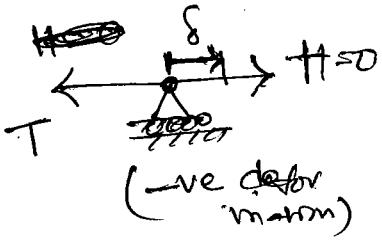


$M_x = M' - Hy$

$= M' - Hy \cdot 1$

$\frac{\partial U}{\partial H} = \frac{HL}{AE} \Rightarrow \frac{\partial\left(\int \frac{M_x^2\, dx}{2EI}\right)}{\partial H} = \frac{HL}{AE}$

$\int \frac{(M' - Hy)(-y)\, dx}{EI} = \frac{HL}{AE}$



$-\int \frac{M'y\, dx}{EI} + \int \frac{Hy^2\, dx}{EI} = \frac{-HL}{AE}$

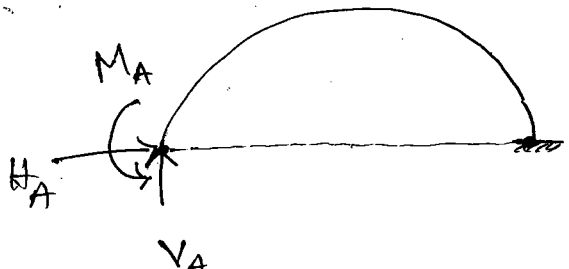
$\int \frac{Hy^2\, dx}{EI} + \frac{HL}{AE} = \int \frac{M'y\, dx}{EI}$

$H = \frac{\int \frac{M'y\, dx}{EI}}{\left(\int \frac{y^2\, dx}{EI} + \frac{L}{AE}\right)}$

Fixed Arch:- (Hingeless arch) Analysis methods

(i) CDM (ii) Elastic centre method

$\Delta V, \Delta H, \theta_A = 0$



Let M' be the moment due to given load,

m_1, m_2 and m_3 be the moments due to unit loading in the directions of H_A, V_A & M_A.

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(i) due to given loading, $\Delta V_{A0} = \int \frac{M' m_1 \, ds}{EI}$

$\Delta H_{A0} = \int \frac{M' m_2 \, ds}{EI}$; $\Delta \theta_{A0} = \int \frac{M' m_3 \, ds}{EI}$.

(ii) Due to V_A alone, $\Delta V_{A1} = \int (V_A x) \, m_1 \frac{ds}{EI}$.

$\Delta H_{A1} = \int V_A x \, m_2 \frac{ds}{EI}$; $\Delta \theta_{A1} = \int V_A x \cdot m_3 \frac{ds}{EI}$

(iii) Due to H_A alone $\Delta V_{A2} = \int H_A y \cdot m_1 \frac{ds}{EI}$

$\Delta H_{A2} = \int (H_A \cdot y) \, m_2 \frac{ds}{EI}$; $\Delta \theta_{A2} = \int H_A y \cdot m_3 \frac{ds}{EI}$.

(iv) Due to M_A alone $\Delta V_{A3} = \int M_A \cdot m_1 \frac{ds}{EI}$

$\Delta H_{A3} = \int M_A \cdot m_2 \frac{ds}{EI}$; $\Delta \theta_{A3} = \int M_A \cdot m_3 \frac{ds}{EI}$.

Consistency conditions:-

$\Delta V_A = 0$; $\Delta H_A = 0$ $\Delta \theta_A = 0$

$\therefore$ eq^n (1) $\Rightarrow \Delta V_{A0} + \Delta V_{A1} + \Delta V_{A2} + \Delta V_{A3} = 0$

eq^n (2) $\Rightarrow$ " " = 0

eq (3) $\Rightarrow$ " " = 0

Solving eq^{ns} (1), (2) & (3)

we get V_A, H_A & M_A

Problems (JNTUK)

Oct-16

1 (a). How are arches classified based on shape and end conditions? (8M)

(b) State and prove Eddy's theorem? (8M)

2. ~~A three hinged parabolic~~ Derive the expression for normal thrust, radial shear and horizontal thrust for a two hinged arch? (16M)

Nov-15

3. What is the ~~effect~~ of rib shortening on two hinged arch? (3M)

4. What is the ~~effect~~ of temperature on two hinged arch? (3M)

5. Find the horizontal thrust of a two hinged semi circular arch of radius R' carries a concentrated load of W? (4M)

May-15 (R10)

6. A two-hinged parabolic arch of span 14m and rise 2.2m. Carries two concentrated loads of 30KN each placed at the crown and at the right Quarter Span Section. Find the horizontal thrust and BM at loaded points? (15M)

may-15 R10.

7. A two hinged parabolic ~~arch~~ of span 35m and rise 5m carries a UDL of 40KN/m on the left half span. Determine horizontal thrust and BM at the crown? (15M)

may-15

8. A two hinged parabolic arch of span 30m and rise 6m carries a triangular load covering left half of the span, the intensity of load varying uniformly from 0 to 10KN/m. Evaluate the ~~Hori~~ horizontal thrust?

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