Unit-3

Trusses

4.1 INTRODUCTION

The finite element analysis of truss structures is presented in this chapter. Two-dimensional trusses (or plane trusses) are treated in Section 4.2. In Section 4.3, this treatment is readily generalized to handle three-dimensional trusses. A typical plane truss is shown in Fig. 4.1. A truss structure consists only of two-force members. That is, every truss element is in direct tension or compression (Fig. 4.2). In a truss, it is required that all loads and reactions are applied only at the joints and that all members are connected together at their ends by frictionless pin joints. Every engineering student has, in a course on statics, analyzed trusses using the method of joints and the method of sections. These methods, while illustrating the fundamentals of statics, become tedious when applied to large-scale statically indeterminate truss structures. Further, joint displacements are not readily obtainable. The finite element method on the other hand is applicable to statically determinate or indeterminate structures alike. The finite element method also provides joint deflections. Effects of temperature changes and support settlements can also be routinely handled.

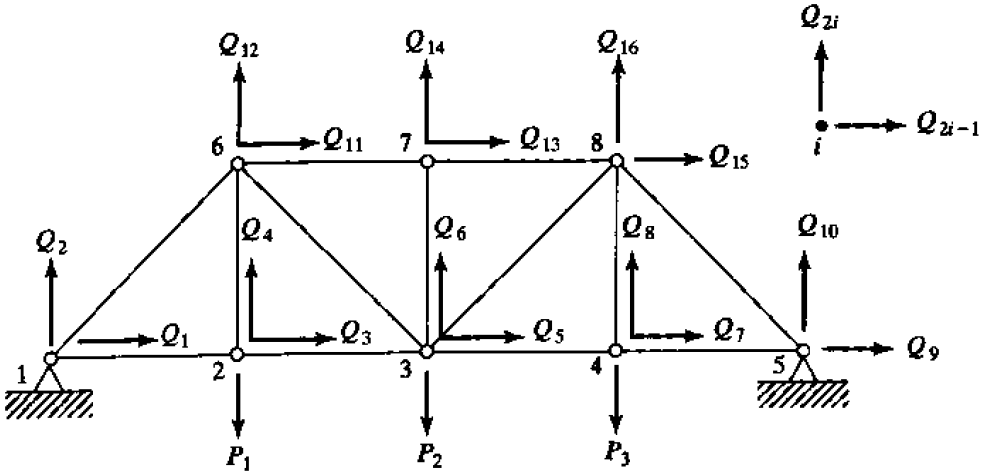


FIGURE 4.1 A two-dimensional truss.

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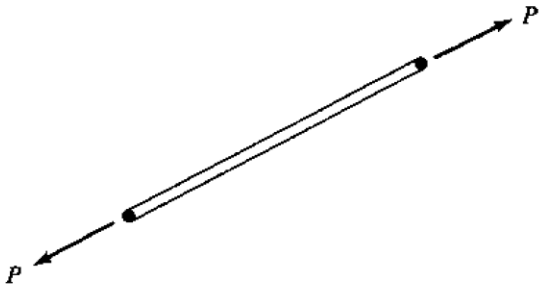


FIGURE 4.2 A two-force member.

4.2 PLANE TRUSSES

Modeling aspects discussed in Chapter 3 are now extended to the two-dimensional truss. The steps involved are discussed here.

Local and Global Coordinate Systems

The main difference between the one-dimensional structures considered in Chapter 3 and trusses is that the elements of a truss have various orientations. To account for these different orientations, **local** and **global** coordinate systems are introduced as follows:

A typical plane-truss element is shown in local and global coordinate systems in Fig. 4.3. In the local numbering scheme, the two nodes of the element are numbered 1 and 2. The local coordinate system consists of the x'-axis, which runs along the element from node 1 toward node 2. All quantities in the local coordinate system will be denoted by a prime ($'$). The global x-, y-coordinate system is fixed and does not depend on the orientation of the element. Note that x, y, and z form a right-handed coordinate system with the z-axis coming straight out of the paper. In the global coordinate system,

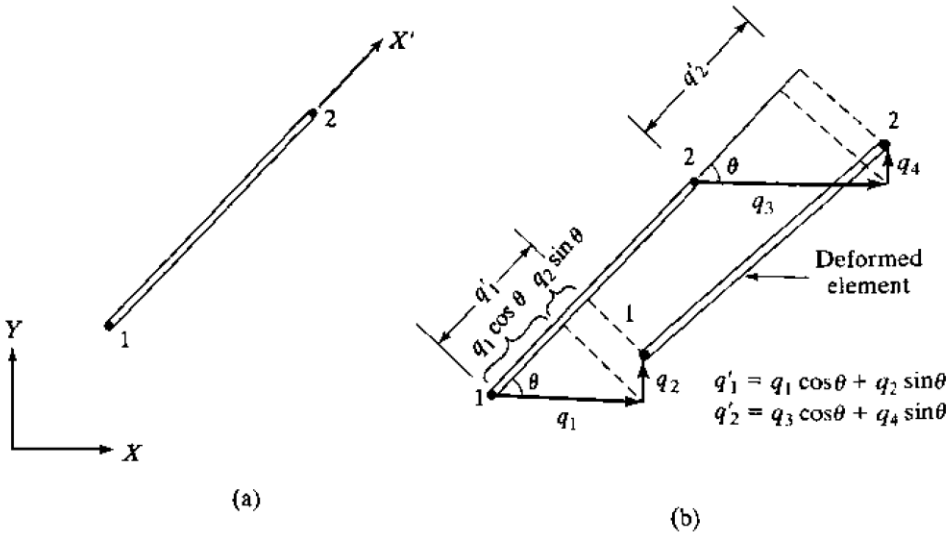


FIGURE 4.3 A two-dimensional truss element in (a) a local coordinate system and (b) a global coordinate system.

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every node has two degrees of freedom (dofs). A systematic numbering scheme is adopted here: A node whose global node number is j has associated with it dofs $2j - 1$ and $2j$. Further, the global displacements associated with node j are Q_{2j-1} and Q_{2j}, as shown in Fig. 4.1.

Let q'_1 and q'_2 be the displacements of nodes 1 and 2, respectively, in the local coordinate system. Thus, the element displacement vector in the local coordinate system is denoted by

$$\mathbf{q}' = [q'_1, q'_2]^{\mathrm{T}} \tag{4.1}$$

The element displacement vector in the global coordinate system is a (4×1) vector denoted by

$$\mathbf{q} = [q_1, q_2, q_3, q_4]^{\mathrm{T}} \tag{4.2}$$

The relationship between $\mathbf{q}'$ and $\mathbf{q}$ is developed as follows: In Fig. 4.3b, we see that q'_1 equals the sum of the projections of q_1 and q_2 onto the x'-axis. Thus,

$$q'_1 = q_1 \cos\theta + q_2 \sin\theta \tag{4.3a}$$

Similarly,

$$q'_2 = q_3 \cos\theta + q_4 \sin\theta \tag{4.3b}$$

At this stage, the **direction cosines** ℓ and m are introduced as $\ell = \cos\theta$ and $m = \cos\phi$ $(= \sin\theta)$. These direction cosines are the cosines of the angles that the local x'-axis makes with the global x-, y-axes, respectively. Equations 4.3a and 4.3b can now be written in matrix form as

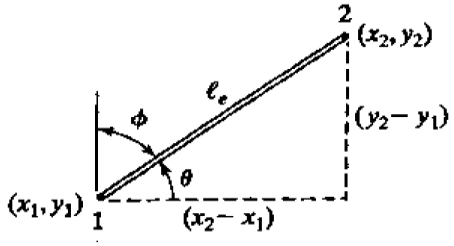
$$\mathbf{q}' = \mathbf{L}\mathbf{q} \tag{4.4}$$

where the **transformation matrix** $\mathbf{L}$ is given by

$$\mathbf{L} = \begin{bmatrix} \ell & m & 0 & 0 \\ 0 & 0 & \ell & m \end{bmatrix} \tag{4.5}$$

Formulas for Calculating ℓ and m

Simple formulas are now given for calculating the direction cosines ℓ and m from nodal coordinate data. Referring to Fig. 4.4, let (x_1, y_1) and (x_2, y_2) be the coordinates of nodes 1 and 2, respectively. We then have



$$\ell = \cos\theta = \frac{x_2 - x_1}{\ell_e}$$

$$m = \cos\phi = \frac{y_2 - y_1}{\ell_e} \; (= \sin\theta)$$

$$\ell_e = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

FIGURE 4.4 Direction cosines

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$$\ell = \frac{x_2 - x_1}{\ell_e} \qquad m = \frac{y_2 - y_1}{\ell_e} \tag{4.6}$$

where the length ℓ_e is obtained from

$$\ell_e = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{4.7}$$

The expressions in Eqs. 4.6 and 4.7 are obtained from nodal coordinate data and can readily be implemented in a computer program.

Element Stiffness Matrix

An important observation will now be made: *The truss element is a one-dimensional element when viewed in the local coordinate system.* This observation allows us to use previously developed results in Chapter 3 for one-dimensional elements. Consequently, from Eq. 3.26, the element stiffness matrix for a truss element in the local coordinate system is given by

$$\mathbf{k} = \frac{E_e A_e}{\ell_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{4.8}$$

where A_e is the element cross-sectional area and E_e is Young's modulus. The problem at hand is to develop an expression for the element stiffness matrix in the global coordinate system. This is obtainable by considering the strain energy in the element. Specifically, the element strain energy in local coordinates is given by

$$U_e = \tfrac{1}{2}\mathbf{q}'^{\mathrm{T}}\mathbf{k}'\mathbf{q}' \tag{4.9}$$

Substituting for $\mathbf{q}' = \mathbf{L}\mathbf{q}$ into Eq. 4.9, we get

$$U_e = \tfrac{1}{2}\mathbf{q}^{\mathrm{T}}[\mathbf{L}^{\mathrm{T}}\mathbf{k}'\mathbf{L}]\mathbf{q} \tag{4.10}$$

The strain energy in global coordinates can be written as

$$U_e = \tfrac{1}{2}\mathbf{q}^{\mathrm{T}}\mathbf{k}\mathbf{q} \tag{4.11}$$

where $\mathbf{k}$ is the element stiffness matrix in global coordinates. From the previous equation, we obtain the element stiffness matrix in global coordinates as

$$\mathbf{k} = \mathbf{L}^{\mathrm{T}}\mathbf{k}'\mathbf{L} \tag{4.12}$$

Substituting for $\mathbf{L}$ from Eq. 4.5 and for $\mathbf{k}'$ from Eq. 4.8, we get

$$\mathbf{k} = \frac{E_e A_e}{\ell_e}\begin{bmatrix} \ell^2 & \ell m & -\ell^2 & -\ell m \\ \ell m & m^2 & -\ell m & -m^2 \\ -\ell^2 & -\ell m & \ell^2 & \ell m \\ -\ell m & -m^2 & \ell m & m^2 \end{bmatrix} \tag{4.13}$$

The element stiffness matrices are assembled in the usual manner to obtain the structural stiffness matrix. This assembly is illustrated in Example 4.1. The computer logic for directly placing element stiffness matrices into global matrices for banded and skyline solutions is explained in Section 4.4.

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The derivation of the result $\mathbf{k} = \mathbf{L}^T\mathbf{k}'\mathbf{L}$ also follows from Galerkin's variational principle. The virtual work δW as a result of virtual displacement $\boldsymbol{\psi}'$ is

$$\delta W = \boldsymbol{\psi}'^T(\mathbf{k}'\mathbf{q}') \tag{4.14a}$$

Since $\boldsymbol{\psi}' = \mathbf{L}\boldsymbol{\psi}$ and $\mathbf{q}' = \mathbf{L}\mathbf{q}$, we have

$$\begin{aligned}\delta W &= \boldsymbol{\psi}^T[\mathbf{L}^T\mathbf{k}'\mathbf{L}]\mathbf{q} \\ &= \boldsymbol{\psi}^T\mathbf{k}\mathbf{q}\end{aligned} \tag{4.14b}$$

Stress Calculations

Expressions for the element stresses can be obtained by noting that a truss element in local coordinates is a simple two-force member (Fig. 4.2). Thus, the stress σ in a truss element is given by

$$\sigma = E_e\epsilon \tag{4.15a}$$

Since the strain ϵ is the change in length per unit original length,

$$\begin{aligned}\sigma &= E_e\frac{q_2' - q_1'}{\ell_e} \\ &= \frac{E_e}{\ell_e}[-1 \quad 1]\begin{Bmatrix} q_1' \\ q_2' \end{Bmatrix}\end{aligned} \tag{4.15b}$$

This equation can be written in terms of the global displacements $\mathbf{q}$ using the transformation $\mathbf{q}' = \mathbf{L}\mathbf{q}$ as

$$\sigma = \frac{E_e}{\ell_e}[-1 \quad 1]\mathbf{L}\mathbf{q} \tag{4.15c}$$

Substituting for $\mathbf{L}$ from Eq. 4.5 yields

$$\sigma = \frac{E_e}{\ell_e}[-\ell \quad -m \quad \ell \quad m]\mathbf{q} \tag{4.16}$$

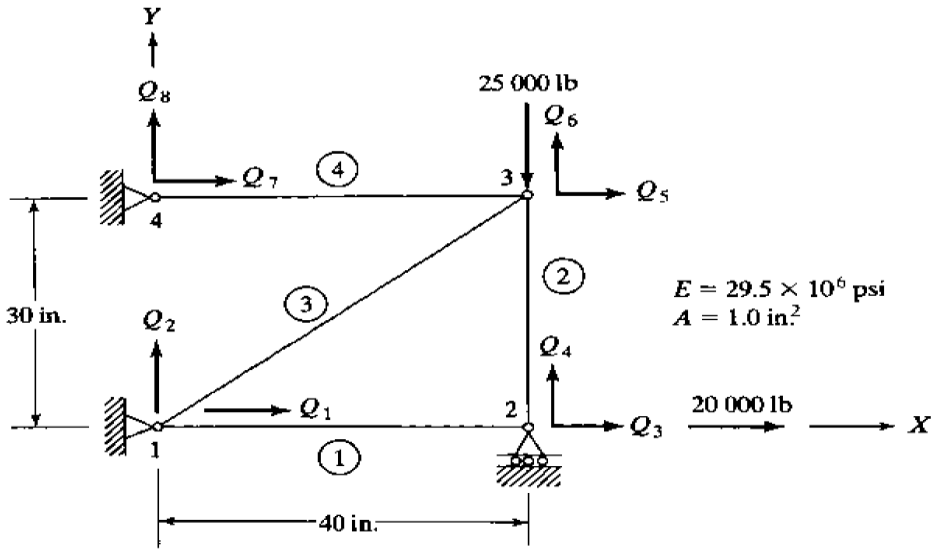
Once the displacements are determined by solving the finite element equations, the stresses can be recovered from Eq. 4.16 for each element. Note that a positive stress implies that the element is in tension and a negative stress implies compression.

Example 4.1

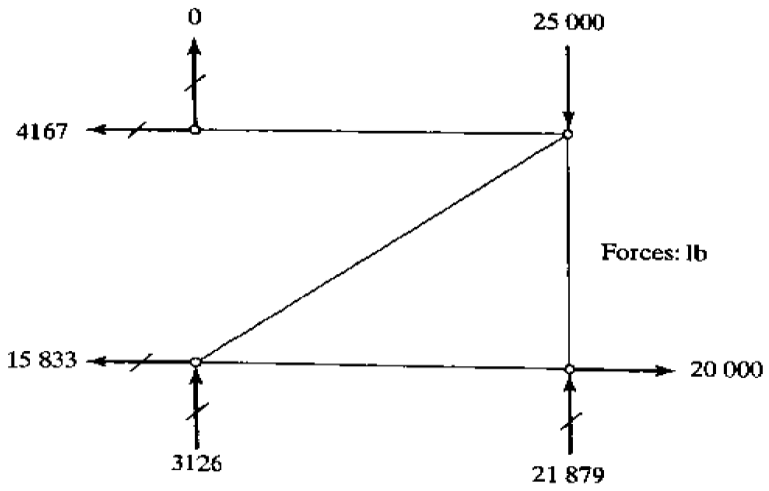
Consider the four-bar truss shown in Fig. E4.1a. It is given that $E = 29.5 \times 10^6$ psi and $A_e = 1$ in.^{2} for all elements. Complete the following:

(a) Determine the element stiffness matrix for each element.
(b) Assemble the structural stiffness matrix $\mathbf{K}$ for the entire truss.
(c) Using the elimination approach, solve for the nodal displacement.
(d) Recover the stresses in each element.
(e) Calculate the reaction forces.

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(a)



(b)

FIGURE E4.1

Solution

(a) It is recommended that a *tabular* form be used for representing nodal coordinate data and element information. The nodal coordinate data are as follows:

Node	x	y
1	0	0
2	40	0
3	40	30
4	0	30

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The element connectivity table is

Element	1	2
1	1	2
2	3	2
3	1	3
4	4	3

Note that the user has a choice in defining element connectivity. For example, the connectivity of element 2 can be defined as 2–3 instead of 3–2 as in the previous table. However, calculations of the direction cosines will be consistent with the adopted connectivity scheme. Using formulas in Eqs. 4.6 and 4.7, together with the nodal coordinate data and the given element connectivity information, we obtain the direction cosines table:

Element	ℓ_e	ℓ	m
1	40	1	0
2	30	0	−1
3	50	0.8	0.6
4	40	1	0

For example, the direction cosines of elements 3 are obtained as $\ell = (x_3 - x_1)/\ell_e = (40 - 0)/50 = 0.8$ and $m = (y_3 - y_1)/\ell_e = (30 - 0)/50 = 0.6$. Now, using Eq. 4.13, the element stiffness matrices for element 1 can be written as

$$\mathbf{k}^1 = \frac{29.5 \times 10^6}{40} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \leftarrow \text{Global dof} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \end{array}$$

The global dofs associated with element 1, which is connected between nodes 1 and 2, are indicated in $\mathbf{k}^1$ earlier. These global dofs are shown in Fig. E4.1a and assist in assembling the various element stiffness matrices.

The element stiffness matrices of elements 2, 3, and 4 are as follows:

$$\mathbf{k}^2 = \frac{29.5 \times 10^6}{30} \begin{array}{c} \begin{matrix} 5 & 6 & 3 & 4 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 3 \\ 4 \end{matrix} \end{array}$$

$$\mathbf{k}^3 = \frac{29.5 \times 10^6}{50} \begin{array}{c} \begin{matrix} 1 & 2 & 5 & 6 \end{matrix} \\ \begin{bmatrix} .64 & .48 & -.64 & -.48 \\ .48 & .36 & -.48 & -.36 \\ -.64 & -.48 & .64 & .48 \\ -.48 & -.36 & .48 & .36 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 5 \\ 6 \end{matrix} \end{array}$$

$$\mathbf{k}^4 = \frac{29.5 \times 10^6}{40} \begin{array}{c} \begin{matrix} 7 & 8 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 7 \\ 8 \\ 5 \\ 6 \end{matrix} \end{array}$$

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(b) The structural stiffness matrix **K** is now assembled from the element stiffness matrices. By adding the element stiffness contributions, noting the element connectivity, we get

$$\mathbf{K} = \frac{29.5 \times 10^6}{600}\begin{bmatrix} 22.68 & 5.76 & -15.0 & 0 & -7.68 & -5.76 & 0 & 0 \\ 5.76 & 4.32 & 0 & 0 & -5.76 & -4.32 & 0 & 0 \\ -15.0 & 0 & 15.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20.0 & 0 & -20.0 & 0 & 0 \\ -7.68 & -5.76 & 0 & 0 & 22.68 & 5.76 & -15.0 & 0 \\ -5.76 & -4.32 & 0 & -20.0 & 5.76 & 24.32 & 0 & 0 \\ 0 & 0 & 0 & 0 & -15.0 & 0 & 15.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix}$$

(c) The structural stiffness matrix **K** given above needs to be modified to account for the boundary conditions. The elimination approach discussed in Chapter 3 will be used here. The rows and columns corresponding to dofs 1, 2, 4, 7, and 8, which correspond to fixed supports, are deleted from the **K** matrix. The reduced finite element equations are given as

$$\frac{29.5 \times 10^6}{600}\begin{bmatrix} 15 & 0 & 0 \\ 0 & 22.68 & 5.76 \\ 0 & 5.76 & 24.32 \end{bmatrix}\begin{Bmatrix} Q_3 \\ Q_5 \\ Q_6 \end{Bmatrix} = \begin{Bmatrix} 20\,000 \\ 0 \\ -25\,000 \end{Bmatrix}$$

Solution of these equations yields the displacements

$$\begin{Bmatrix} Q_3 \\ Q_5 \\ Q_6 \end{Bmatrix} = \begin{Bmatrix} 27.12 \times 10^{-3} \\ 5.65 \times 10^{-3} \\ -22.25 \times 10^{-3} \end{Bmatrix} \text{in.}$$

The nodal displacement vector for the entire structure can therefore be written as

$$\mathbf{Q} = [0, 0, 27.12 \times 10^{-3}, 0, 5.65 \times 10^{-3}, -22.25 \times 10^{-3}, 0, 0]^T \text{ in.}$$

(d) The stress in each element can now be determined from Eq. 4.16, as shown below.

The connectivity of element 1 is 1 − 2. Consequently, the nodal displacement vector for element 1 is given by $\mathbf{q} = [0, 0, 27.12 \times 10^{-3}, 0]^T$, and Eq. 4.16 yields

$$\sigma_1 = \frac{29.5 \times 10^6}{40}[-1 \quad 0 \quad 1 \quad 0]\begin{Bmatrix} 0 \\ 0 \\ 27.12 \times 10^{-3} \\ 0 \end{Bmatrix}$$

$$= 20\,000.0 \text{ psi}$$

The stress in member 2 is given by

$$\sigma_2 = \frac{29.5 \times 10^6}{30}[0 \quad 1 \quad 0 \quad -1]\begin{Bmatrix} 5.65 \times 10^{-3} \\ -22.25 \times 10^{-3} \\ +27.12 \times 10^{-3} \\ 0 \end{Bmatrix}$$

$$= -21\,880.0 \text{ psi}$$

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Following similar steps, we get

$$\sigma_3 = -5208.0 \text{ psi}$$

$$\sigma_4 = 4167.0 \text{ psi}$$

(e) The final step is to determine the support reactions. We need to determine the reaction forces along dofs 1, 2, 4, 7, and 8, which correspond to fixed supports. These are obtained by substituting for $\mathbf{Q}$ into the original finite element equation $\mathbf{R} = \mathbf{KQ} - \mathbf{F}$. In this substitution, only those rows of $\mathbf{K}$ corresponding to the support dofs are needed, and $\mathbf{F} = \mathbf{0}$ for these dofs. Thus, we have

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_4 \\ R_7 \\ R_8 \end{Bmatrix} = \frac{29.5 \times 10^6}{600} \begin{bmatrix} 22.68 & 5.76 & -15.0 & 0 & -7.68 & -5.76 & 0 & 0 \\ 5.76 & 4.32 & 0 & 0 & -5.76 & -4.32 & 0 & 0 \\ 0 & 0 & 0 & 20.0 & 0 & -20.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -15.0 & 0 & 15.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 27.12 \times 10^{-3} \\ 0 \\ 5.65 \times 10^{-3} \\ -22.25 \times 10^{-3} \\ 0 \\ 0 \end{Bmatrix}$$

which results in

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_4 \\ R_7 \\ R_8 \end{Bmatrix} = \begin{Bmatrix} -15833.0 \\ 3126.0 \\ 21879.0 \\ -4167.0 \\ 0 \end{Bmatrix} \text{lb}$$

A free body diagram of the truss with reaction forces and applied loads is shown in Fig. E4.1b. ■

Temperature Effects

The thermal stress problem is considered here. Since a truss element is simply a one-dimensional element when viewed in the local coordinate system, the element temperature load in the local coordinate system is given by (see Eq. 3.103b)

$$\boldsymbol{\Theta}' = E_e A_e \epsilon_0 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} \tag{4.17}$$

where the initial strain ϵ_0 associated with a temperature change is given by

$$\epsilon_0 = \alpha \, \Delta T \tag{4.18}$$

in which α is the coefficient of thermal expansion, and ΔT is the average change in temperature in the element. It may be noted that the initial strain ϵ_0 can also be induced by forcing members into places that are either too long or too short, due to fabrication errors.

We will now express the load vector in Eq. 4.17 in the global coordinate system. Since the potential energy associated with this load is the same in magnitude whether measured in the local or global coordinate systems, we have

$$\mathbf{q}'^{\mathrm{T}} \boldsymbol{\Theta}' = \mathbf{q}^{\mathrm{T}} \boldsymbol{\Theta} \tag{4.19}$$

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where $\boldsymbol{\Theta}$ is the load vector in the global coordinate system. Substituting for $\mathbf{q}' = \mathbf{L}\mathbf{q}$ into Eq. 4.19, we get

$$\mathbf{q}^{\mathrm{T}}\mathbf{L}^{\mathrm{T}}\boldsymbol{\Theta}' = \mathbf{q}^{\mathrm{T}}\boldsymbol{\Theta} \tag{4.20}$$

Comparing the left and right sides of this equation, we obtain

$$\boldsymbol{\Theta} = \mathbf{L}^{\mathrm{T}}\boldsymbol{\Theta}' \tag{4.21}$$

Substituting for $\mathbf{L}$ from Eq. 4.5, we can write down the expression for the element temperature load as

$$\boldsymbol{\Theta}^e = E_e A_e \epsilon_0 \begin{Bmatrix} -\ell \\ -m \\ \ell \\ m \end{Bmatrix} \tag{4.22}$$

The temperature loads, along with other externally applied loads, are assembled in the usual manner to obtain the nodal load vector $\mathbf{F}$. Once the displacements are obtained by solving the finite element equations, the stress in each truss element is obtained from (see Eq. 3.100)

$$\sigma = E(\epsilon - \epsilon_0) \tag{4.23}$$

This equation for the element stress can be simplified by using Eq. 4.16 and noting that $\epsilon_0 = \alpha\,\Delta T$, to obtain

$$\sigma = \frac{E_e}{\ell_e}[-\ell \quad -m \quad \ell \quad m]\mathbf{q} - E_e\alpha\,\Delta T \tag{4.24}$$

Example 4.2

The four-bar truss of Example 4.1 is considered here, but the loading is different. Take $E = 29.5 \times 10^6$ psi and $\alpha = 1/150\,000$ per °F.

(a) There is an increase in temperature of 50°F in bars 2 and 3 only (Fig. E4.2a). There are no other loads on the structure. Determine the nodal displacements and element stresses as a result of this temperature increase. Use the elimination approach.

(b) A support settlement effect is considered here. Node 2 settles by 0.12 in. vertically down, and in addition, two point loads are applied on the structure (Fig. E4.2b). Write down (without solving) the equilibrium equations $\mathbf{KQ} = \mathbf{F}$, where $\mathbf{K}$ and $\mathbf{F}$ are the modified structural stiffness matrix and load vector, respectively. Use the penalty approach.

(c) Use the program TRUSS2 to obtain the solution to part (b).

Solution

(a) The stiffness matrix for the truss structure has already been developed in Example 4.1. Only the load vector needs to be assembled due to the temperature increase. Using Eq. 4.22, the temperature load as a result of temperature increases in elements 2 and 3 are, respectively,

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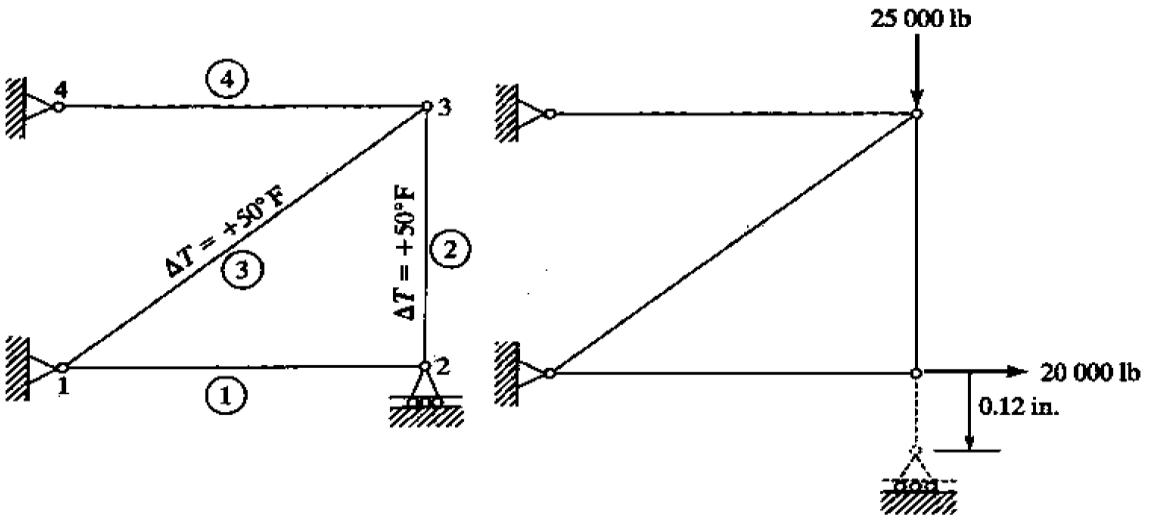


FIGURE E4.2

$$\Theta^2 = \frac{29.5 \times 10^6 \times 50}{150{,}000} \begin{Bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{Bmatrix} \begin{matrix} \leftarrow \text{Global dof} \\ 5 \\ 6 \\ 3 \\ 4 \end{matrix}$$

and

$$\Theta^3 = \frac{29.5 \times 10^6 \times 50}{150{,}000} \begin{Bmatrix} -0.8 \\ -0.6 \\ 0.8 \\ 0.6 \end{Bmatrix} \begin{matrix} 1 \\ 2 \\ 5 \\ 6 \end{matrix}$$

The Θ^2 and Θ^3 vectors contribute to the global load vector **F**. Using the elimination approach, we can delete all rows and columns corresponding to support dofs in **K** and **F**. The resulting finite element equations are

$$\frac{29.5 \times 10^6}{600} \begin{bmatrix} 15.0 & 0 & 0 \\ 0 & 22.68 & 5.76 \\ 0 & 5.76 & 24.32 \end{bmatrix} \begin{Bmatrix} Q_3 \\ Q_5 \\ Q_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 7866.7 \\ 15733.3 \end{Bmatrix}$$

which yield

$$\begin{Bmatrix} Q_3 \\ Q_5 \\ Q_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0.003951 \\ 0.01222 \end{Bmatrix} \text{in.}$$

The element stresses can now be obtained from Eq. 4.24. For example, the stress in element 2 is given as

$$\sigma_2 = \frac{29.5 \times 10^6}{30} [0 \quad 1 \quad 0 \quad -1] \begin{Bmatrix} 0.003951 \\ 0.01222 \\ 0 \\ 0 \end{Bmatrix} - \frac{29.5 \times 10^6 \times 50}{150{,}000}$$

$$= -8631.7 \text{ psi}$$

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The complete stress solution is

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 2183 \\ -3643 \\ 2914 \end{Bmatrix} \text{psi}$$

(b) Support 2 settles by 0.12 in. vertically down, and two concentrated forces are applied (Fig. E4.2b). In the penalty approach for handling boundary conditions (Chapter 3), recall that a large spring constant C is added to the diagonal elements in the structural stiffness matrix at those dofs where the displacements are specified. Typically, C may be chosen 10^4 times the largest diagonal element of the unmodified stiffness matrix (see Eq. 3.80). Further, a force Ca is added to the force vector, where a is the specified displacement. In this example, for dof 4, $a = -0.12$ in., and consequently, a force equal to $-0.12C$ gets added to the fourth location in the force vector. Consequently, the modified finite element equations are given by

$$\frac{29.5 \times 10^6}{600} \begin{bmatrix} 22.68 + C & 5.76 & -15.0 & 0 & -7.68 & -5.76 & 0 & 0 \\ & 4.32 + C & 0 & 0 & -5.76 & -4.32 & 0 & 0 \\ & & 15.0 & 0 & 0 & 0 & 0 & 0 \\ & & & 20.0 + C & 0 & -20.0 & 0 & 0 \\ & & & & 22.68 & 5.76 & -15.0 & 0 \\ & & & & & 24.32 & 0 & 0 \\ & & & & & & 15.0 + C & 0 \\ \text{Symmetric} & & & & & & & C \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 20\,000 \\ -0.12C \\ 0 \\ -25\,000.0 \\ 0 \\ 0 \end{Bmatrix}$$

(c) Obviously, the equations in (b) are too large for hand calculations. In the program TRUSS, that is provided, these equations are automatically generated and solved from the user's input data. The output from the program is

$$\begin{Bmatrix} Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{Bmatrix} = \begin{Bmatrix} 0.0271200 \\ -0.1200145 \\ 0.0323242 \\ -0.1272606 \end{Bmatrix} \text{in.}$$

and

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \end{Bmatrix} = \begin{Bmatrix} 20\,000.0 \\ -7\,125.3 \\ -29\,791.7 \\ 23\,833.3 \end{Bmatrix} \text{psi}$$

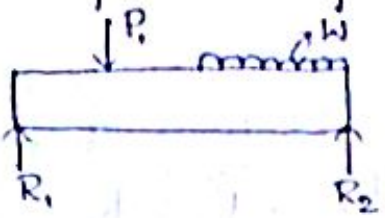
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&&&&&&&&&&&&&&&&&&&&&&&& The End &&&&&&&&&&&&&&&&&&&&&&&&

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Analysis of Beam :

Beam is a structural member having very long length compared to other two lateral dimensions such as width and height. It is another type of one-dimensional structural member in which the loads are applied in radial direction. That is, the beam is treated as a bar element subjected to transverse loads. The transverse load induces normal and shear stresses within the beam and displacements perpendicular to its longitudinal axis.

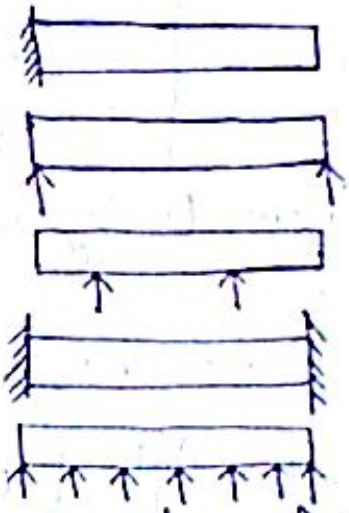


P_1 = Point load
W = Distributed load.
R_1, R_2 = Reaction forces.

Types of Beams :

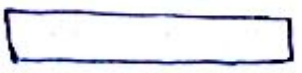
Depending on the nature of support for beams, the beams are classified as

(1) Cantilever Beam
(2) Simply Supported beam
(3) Overhanding beam
(4) Fixed beam
(5) Continuous beam



Depending on the Cross-section of Beam

(1) Beam of Uniform Cross-section

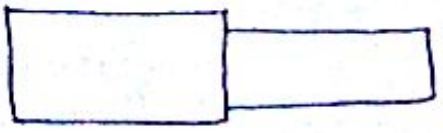


(2.) Beam of Varying Cross-section

(a) Uniformly Varied



(b) Stepped beam.



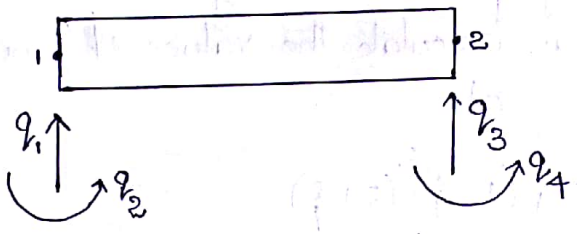
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Topic 2 : Beams

* Beams:

Beams are slender members that are used for supported transverse loading. Long horizontal members used in buildings and bridges, and shafts supported in bearings are some examples of beams.

In Beam structure, the beams are joined together by welding, so that both forces and moments can be transmitted between the beams.

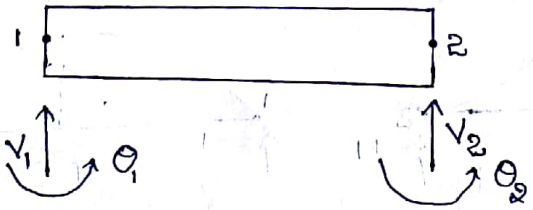


Each node has two degrees of freedom.

Typically, the degrees of freedom of Node 1 are q_1 & q_2. Here q_1 is the transverse displacement and q_2 is slope or rotation.

$\therefore Q = [q_1, q_2, q_3, q_4, \ldots]^T$.

We Can consider the beam in terms of Natural coordinate system, then.



Here, v_1, v_2 = Transverse displacements at Nodes 1 & 2.

θ_1, θ_2 = Angular displacements / slopes.

Where, $\theta_1 = \frac{dv_1}{d\xi}$

$\theta_2 = \frac{dv_2}{d\xi}$ in terms of Natural Coordinate system.

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Hermite Shape Functions:

$$H_i = a_i + b_i \xi + c_i \xi^2 + d_i \xi^3 \longrightarrow (I)$$

Where, $i = 1, 2, 3, 4$.

Considering the boundary conditions.

	H_1	H_1'	H_2	H_2'	H_3	H_3'	H_4	H_4'
$\xi = -1$	1	0	0	1	0	0	0	0
$\xi = +1$	0	0	0	0	1	0	0	1

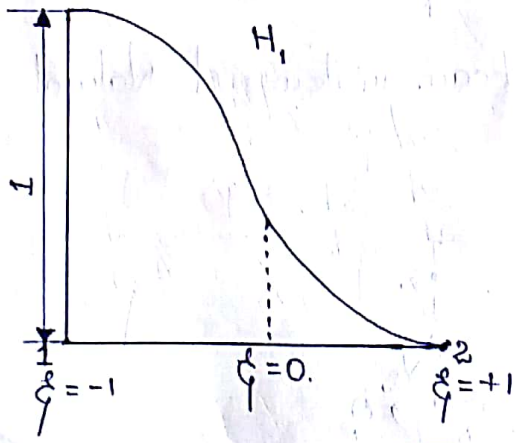
Now by applying the boundary conditions in the above equation (1), we can calculate the values of constants a_i, b_i c_i and d_i. Then we get.

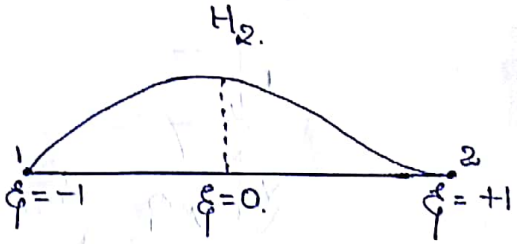
$$H_1 = \frac{1}{4}(1-\xi)^2(2+\xi)$$

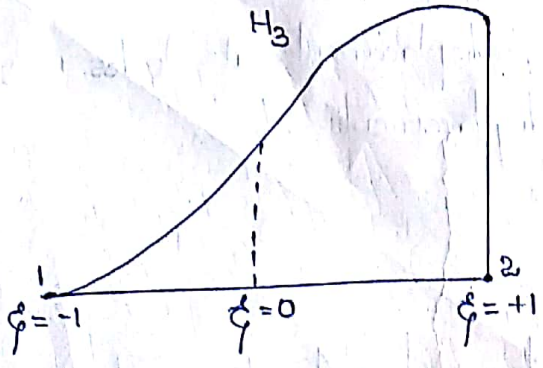
$$H_2 = \frac{1}{4}(1-\xi)^2(\xi+1)$$

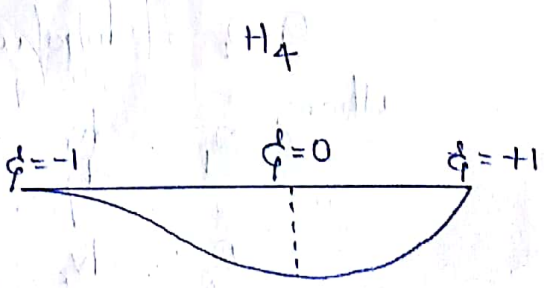
$$H_3 = \frac{1}{4}(1+\xi)^2(2-\xi)$$

$$H_4 = \frac{1}{4}(1+\xi)^2(\xi-1).$$









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Considering the displacement function in terms of Natural coordinate system.

$$v = H_1 v_1 + H_2 \theta_1 + H_3 v_2 + H_4 \theta_2$$

$$v = H_1 v_1 + H_2 \frac{dv_1}{d\xi} + H_3 v_2 + H_4 \frac{dv_2}{d\xi} \longrightarrow (1)$$

Let Consider the chain rule

$$\frac{dv}{d\xi} = \frac{dv}{dx} \cdot \frac{dx}{d\xi}$$

W.K.T

$$dx = \frac{l_e}{2} d\xi \Rightarrow \frac{dx}{d\xi} = \frac{l_e}{2}.$$

$$\therefore \frac{dv}{d\xi} = \frac{l_e}{2} \cdot \frac{dv}{dx} \quad - \text{in terms of Global}$$

$\therefore$ Eq. 1 can be written in terms of Global coordinate system as.

$$v = H_1 q_1 + \frac{l_e}{2} H_2 q_2 + H_3 q_3 + H_4 \frac{l_e}{2} q_4$$

where,

$$q_2 = \frac{dv_1}{dx} \qquad q_4 = \frac{dv_2}{dx}.$$

$$v = Hq$$

Where,

$$H = \left[H_1 , \frac{l_e}{2} H_2 , H_3 , \frac{l_e}{2} H_4 \right]$$

or

Simply in Matrix form.

$$H = \left[H_1 \quad \frac{l_e}{2} H_2 \quad H_3 \quad \frac{l_e}{2} H_4 \right]$$

Now, for Calculation of Stiffness Matrix; We should Consider the Hermite shape functions.

$$H_1 = \frac{1}{4}(1-\xi)^2(2+\xi) \qquad H_3 = \frac{1}{4}(1+\xi)^2(2-\xi)$$

$$H_2 = \frac{1}{4}(1-\xi)^2(\xi+1) \qquad H_4 = \frac{1}{4}(1+\xi)^2(\xi-1)$$

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Now Calculating the Single & double derivations of the Hermite shape functions as shown.

$$H_1 = \frac{1}{4}(1-\xi)^2(2+\xi)$$

$$= \frac{1}{4}(1+\xi^2-2\xi)(2+\xi)$$

$$= \frac{1}{4}\left[2+2\xi^2-4\xi+\xi+\xi^3-2\xi^2\right]$$

$$H_1 = \frac{1}{4}\left[\xi^3-3\xi+2\right]$$

$$\therefore H_1' = \frac{dH_1}{d\xi} = \frac{1}{4}\left[3\xi^2-3\right]$$

$$H_1'' = \frac{d^2H_1}{d\xi^2} = \frac{1}{4}\left[6\xi\right]$$

Similarly

$$H_2 = \frac{1}{4}(1-\xi)^2(\xi+1)$$

$$= \frac{1}{4}(1+\xi^2-2\xi)(\xi+1)$$

$$= \frac{1}{4}\left(\xi+\xi^3-2\xi^2+1+\xi^2-2\xi\right)$$

$$H_2 = \frac{1}{4}\left(\xi^3-\xi^2-\xi+1\right)$$

$$\therefore H_2' = \frac{dH_2}{d\xi} = \frac{1}{4}\left[3\xi^2-2\xi-1\right]$$

$$H_2'' = \frac{d^2H_2}{d\xi^2} = \frac{1}{4}\left[6\xi-2\right]$$

$$H_3 = \frac{1}{4}(1+\xi)^2(2-\xi)$$

$$= \frac{1}{4}(1+\xi^2+2\xi)(2-\xi)$$

$$= \frac{1}{4}\left(2+2\xi^2+4\xi-\xi-\xi^3-2\xi^2\right)$$

$$H_3 = \frac{1}{4}\left(-\xi^3+3\xi+2\right)$$

$$\therefore H_3' = \frac{dH_3}{d\xi} = \frac{1}{4}\left(-3\xi^2+3\right)$$

$$H_3'' = \frac{d^2H_3}{d\xi^2} = \frac{1}{4}\left[-6\xi\right]$$

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& $H_4 = \frac{1}{4}(1+\xi)^2(\xi-1)$

$= \frac{1}{4}(1+\xi^2+2\xi)(\xi-1)$

$= \frac{1}{4}(\xi+\xi^3+2\xi^2-1-\xi^2-2\xi)$

$= \frac{1}{4}(\xi^3+\xi^2-\xi-1)$

$H_4' = \frac{dH_4}{d\xi} = \frac{1}{4}[3\xi^2+2\xi-1]$

$H_4'' = \frac{d^2H_4}{d\xi^2} = \frac{1}{4}[6\xi+2]$

Now Considering the Strain Energy Term

$$U = 1/2 \int EI \frac{d^2v}{dx^2}.dx.$$

By solving we get.

$$U = 1/2\, q^T \cdot K_e\, q.$$

where,

$$K_e = \frac{EI}{l_e^3}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix}$$

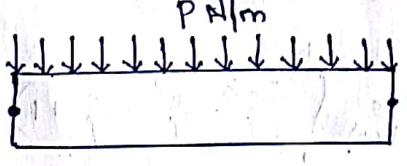
where,

K_e = Element Stiffness Matrix.

l_e = Element length.

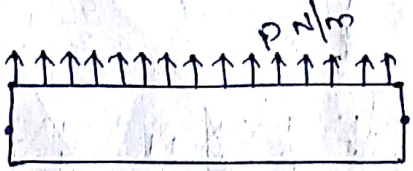
* Load Vector:

Let 'p' be the Uniformly distributed load over the length 'L'.



Downward.

(or)



Upward.

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Let

$$\Rightarrow \int P.V^T.dx$$

Where

$$V = Hq \Rightarrow V^T = H^T q^T$$

$$dx = \frac{l_e}{2} d\xi$$

$$\Rightarrow \int_{-1}^{+1} P.H^T q^T . \frac{l_e}{2} d\xi$$

$$\Rightarrow \frac{P l_e}{2} \int_{-1}^{+1} H^T d\xi . q^T$$

Let Consider the Hermite term.

$$\Rightarrow \int_{-1}^{+1} H^T d\xi$$

$$\Rightarrow \int_{-1}^{+1} \left[H_1 \quad \frac{l_e}{2} H_2 \quad H_3 \quad \frac{l_e}{2} H_4\right]^T d\xi$$

$$\Rightarrow \left[\int_{-1}^{+1} H_1 d\xi \quad \frac{l_e}{2}\int_{-1}^{+1} H_2 d\xi \quad \int_{-1}^{+1} H_3 d\xi \quad \frac{l_e}{2}\int_{-1}^{+1} H_4 d\xi\right]^T$$

Now Let

$$\int_{-1}^{+1} H_1 d\xi = \frac{1}{4}\int_{-1}^{+1} (\xi^3 - 3\xi + 2) d\xi$$

$$= \frac{1}{4}\left[\frac{\xi^4}{4} - 3\frac{\xi^2}{2} + 2\xi\right]_{-1}^{+1}$$

$$\Rightarrow \frac{1}{4}\left[\frac{1}{4} - \frac{3}{2} + 2 - \left[\frac{1}{4} - \frac{3}{4} - 2\right]\right]$$

$$\int_{-1}^{+1} H_2 d\xi = \frac{1}{4}\left[4\right] = 1$$

$$\frac{l_e}{2}\int_{-1}^{+1} H_2 d\xi = \frac{l_e}{4(2)}\int_{-1}^{+1} (\xi^3 - \xi^2 - \xi + 1) d\xi$$

$$= \frac{l_e}{8}\left[\left(\frac{\xi^4}{4} - \frac{\xi^3}{3} - \frac{\xi^2}{2} + \xi\right)\right]_{-1}^{+1}$$

$$= \frac{l_e}{8}\left[\frac{1}{4} - \frac{1}{3} - \frac{1}{2} + 1 + \frac{1}{4} + \frac{1}{3} - \frac{1}{2} + 1\right]$$

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$$= \frac{l_e}{8}\left[\frac{1}{4} - \frac{1}{3} - \frac{1}{2} + 1 - \left[\frac{1}{4} + \frac{1}{3} - \frac{1}{2} - 1\right]\right]$$

$$= \frac{l_e}{8}\left[\frac{1}{4} - \frac{1}{3} - \frac{1}{2} + 1 - \frac{1}{4} - \frac{1}{3} + \frac{1}{2} + 1\right]$$

$$= \frac{l_e}{8}\left[\frac{-2}{3} + 2\right]$$

$$\frac{l_e}{2}\int_{-1}^{+1} H_2\, d\xi = \frac{l_e}{8}\left[\frac{-2+6}{3}\right] = \frac{l_e}{8_2}\left[\frac{4}{3}\right] = \frac{l_e}{6}$$

$$\int_{-1}^{+1} H_3\, d\xi = \frac{1}{4}\int_{-1}^{+1}\left(-\xi^3 + 3\xi + 2\right) d\xi$$

$$= \frac{1}{4}\left[\frac{-\xi^4}{4} + \frac{3\xi^2}{2} + 2\xi\right]_{-1}^{+1}$$

$$= \frac{1}{4}\left[\frac{-1}{4} + \frac{3}{2} + 2 - \left[\frac{-1}{4} + \frac{3}{2} - 2\right]\right]$$

$$= \frac{1}{4}\left[\frac{-1}{4} + \frac{3}{2} + 2 + \frac{1}{4} - \frac{3}{2} + 2\right]$$

$$\int_{-1}^{+1} H_3\, d\xi = 1$$

&

$$\frac{l_e}{2}\int_{-1}^{+1} H_4\, d\xi = \frac{l_e}{8}\int_{-1}^{+1}\left(-\xi^3 + \xi^2 - \xi - 1\right) d\xi$$

$$= \frac{l_e}{8}\left[\frac{\xi^4}{4} + \frac{\xi^3}{3} - \frac{\xi^2}{2} - \xi\right]_{-1}^{+1}$$

$$= \frac{l_e}{8}\left[\frac{1}{4} + \frac{1}{3} - \frac{1}{2} - 1 - \left[\frac{1}{4} - \frac{1}{3} - \frac{1}{2} + 1\right]\right]$$

$$= \frac{l_e}{8}\left[\frac{1}{4} + \frac{1}{3} - \frac{1}{2} - 1 - \frac{1}{4} + \frac{1}{3} + \frac{1}{2} - 1\right]$$

$$= \frac{l_e}{8}\left[\frac{2}{3} - 2\right] \Rightarrow \frac{l_e}{8}\left[\frac{2-6}{3}\right] = \frac{l_e}{8_2}\left[\frac{-4}{3}\right]$$

$$\frac{l_e}{2}\int_{-1}^{+1} H_4\, d\xi = \frac{-l_e}{6}$$

$$\therefore \quad \Rightarrow \frac{P l_e}{2}\left[1 \quad \frac{l_e}{6} \quad 1 \quad \frac{-l_e}{6}\right]^T q^T$$

$$\Rightarrow \left[\frac{P l_e}{2} \quad \frac{P l_e^2}{12} \quad \frac{P l_e}{2} \quad \frac{-P l_e^2}{12}\right]^T q^T$$

$$\Rightarrow f_e q^T$$

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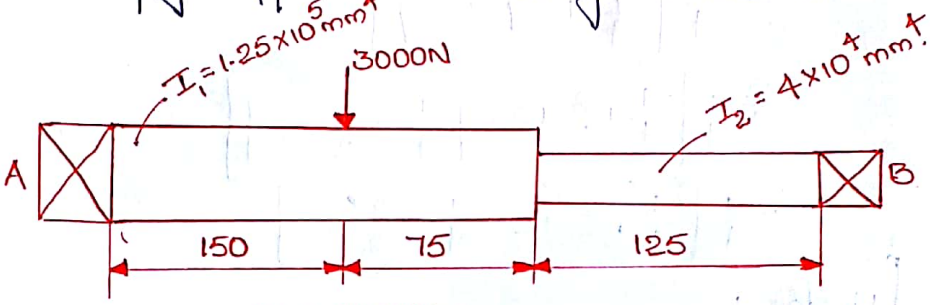
$$\therefore f_e = \left[\frac{Ple}{2} \quad \frac{Ple^2}{12} \quad \frac{Ple}{2} \quad -\frac{Ple^2}{12}\right]^T \text{ for Upward dirt.}$$

$$\& \quad f_e = \left[-\frac{Ple}{2} \quad -\frac{Ple^2}{12} \quad -\frac{Ple}{2} \quad \frac{Ple^2}{12}\right]^T \text{ for downward dirt.}$$

Problems:

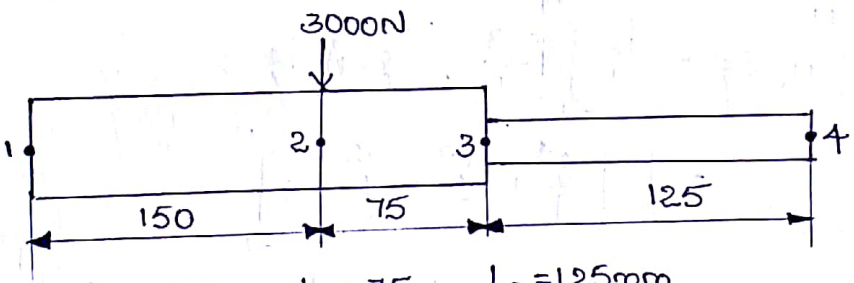
Model 1:

1. Find the deflection at the load and the slopes at the ends for the steel shaft shown in fig. Consider the shaft to be simply supported at bearings A and B.



E = 200GPas.

Sol: Given data:



$L_1 = 150 \quad L_2 = 75 \quad L_3 = 125mm$

$E = E_1 = E_2 = E_3 = 200GPas = 200 \times 10^9 N/m^2$

$= 200 \times 10^9 \times 10^{-6} N/mm^2$

$= 2 \times 10^5 N/mm^2$

$I_1 = 1.25 \times 10^5 mm^4$

$I_2 = 1.25 \times 10^5 mm^4$

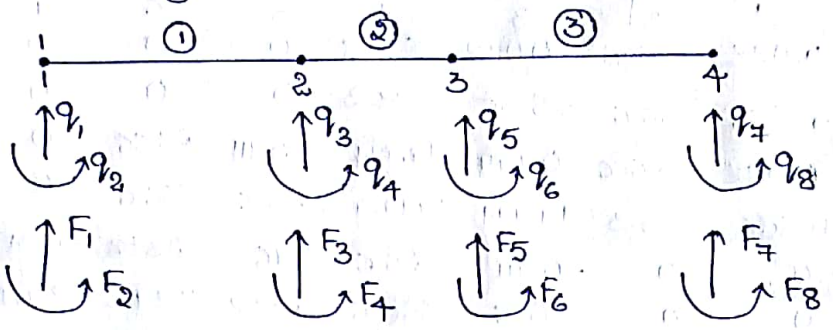
$I_3 = 4 \times 10^4 mm^4$

$P = 3000N.$

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Step 1: Discretization of Domain



Element	Nodes	
①	1	2
②	2	3
③	3	4

Step 2: Calculating element Stiffness Matrix.

$$k_1 = \frac{2\times10^5 \times 1.25\times10^5}{(150)^3}\begin{bmatrix} 12 & 6(150) & -12 & 6(150) \\ 6(150) & 4(150)^2 & -6(150) & 2(150)^2 \\ -12 & -6(150) & 12 & -6(150) \\ 6(150) & 2(150)^2 & -6(150) & 4(150)^2 \end{bmatrix}$$

$$= 10^6 \begin{bmatrix} \overset{1}{0.088} & \overset{2}{6.666} & \overset{3}{-0.088} & \overset{4}{6.666} \\ 6.666 & 666.6 & -6.66 & 333.33 \\ -0.088 & -6.666 & 0.088 & -6.666 \\ 6.666 & 333.3 & -6.66 & 666.6 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

$$k_2 = \frac{2\times10^5 \times 1.25\times10^5}{(75)^3}\begin{bmatrix} 12 & 6(75) & -12 & 6(75) \\ 6(75) & 4(75)^2 & -6(75) & 2(75)^2 \\ -12 & -6(75) & 12 & -6(75) \\ 6(75) & 2(75)^2 & -6(75) & 4(75)^2 \end{bmatrix}$$

$$= 10^6 \begin{bmatrix} \overset{3}{0.711} & \overset{4}{26.66} & \overset{5}{-0.711} & \overset{6}{26.6} \\ 26.66 & 1333.3 & -26.66 & 666.6 \\ -0.711 & -26.66 & 0.711 & -26.6 \\ 26.66 & 666.6 & -26.66 & 13333 \end{bmatrix} \begin{matrix} 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

$$k_3 = 10^6 \begin{bmatrix} \overset{5}{0.0491} & \overset{6}{3.072} & \overset{7}{-0.0491} & \overset{8}{3.072} \\ 3.072 & 256 & -3.072 & 128 \\ -0.0491 & -3.072 & 0.0491 & -3.072 \\ 3.072 & 128 & -3.072 & 256 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 7 \\ 8 \end{matrix}$$

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Step 3: Calculating Global Stiffness Matrix.

$$K = 10^6 \begin{bmatrix} 0.088 & 6.666 & -0.088 & 6.666 & 0 & 0 & 0 & 0 \\ 6.666 & 666.6 & -6.66 & 333.3 & 0 & 0 & 0 & 0 \\ -0.088 & -6.666 & 0.199 & 19.994 & -0.111 & 26.66 & 0 & 0 \\ 6.666 & 333.3 & 19.994 & 1999.9 & -26.6 & 666.6 & 0 & 0 \\ 0 & 0 & -0.111 & -26.6 & 0.16 & -23.58 & -0.049 & 3.012 \\ 0 & 0 & 26.6 & 666.6 & -23.58 & 1529.3 & -3.072 & 128 \\ 0 & 0 & 0 & 0 & -0.0491 & -3.012 & 0.049 & -3.012 \\ 0 & 0 & 0 & 0 & 3.012 & 128 & -3.012 & 256 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix}$$

Step 4: Calculating Global load vector.

$$F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -3000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Step 5: Considering Finite Element Equation.

$$F = KU$$

$$\begin{bmatrix} 0 \\ 0 \\ -3000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 10^6 \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix} \begin{bmatrix} & & & & & & & \\ & & & & & & & \\ & & & K_e & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{bmatrix}$$

Step 6: Applying Boundary Conditions.

Since Node 1 and Node 2 are fixed i.e., $q_1 = q_2 = q_7 = q_8 = 0$. By using Elimination approach, Eliminate 1, 2, 7, & 8 rows and Columns. Then the Final Matrix is.

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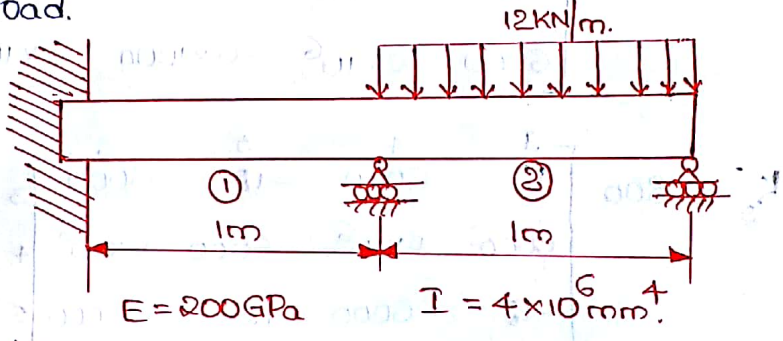
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$$\begin{bmatrix} -3000 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 10^6 \begin{bmatrix} 0.799 & 19.994 & -0.711 & 26.66 \\ 19.994 & 1999.9 & -26.6 & 666.6 \\ -0.711 & -26.6 & 0.76 & -23.58 \\ 26.6 & 666.6 & -23.58 & 1589.3 \end{bmatrix} \begin{bmatrix} q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix}$$

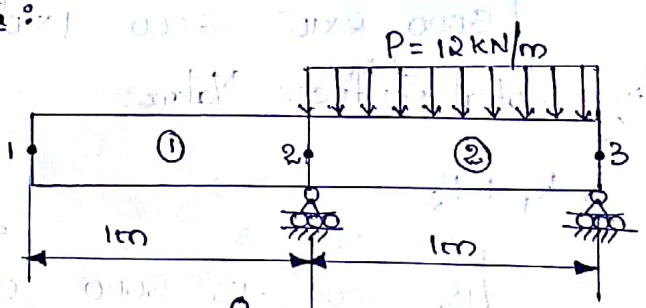
By solving the simultaneous equations, we get the values of q_3, q_4, q_5 & q_6.

Model 2 :-

2. For the beam and loading shown in fig. Determine
   (a) the slopes at 2 & 3.
   (b) the vertical deflection at the midpoint of the distributed load.



Sol:- Given data :-



$E = 200 \times 10^9 \text{ N/m}^2$

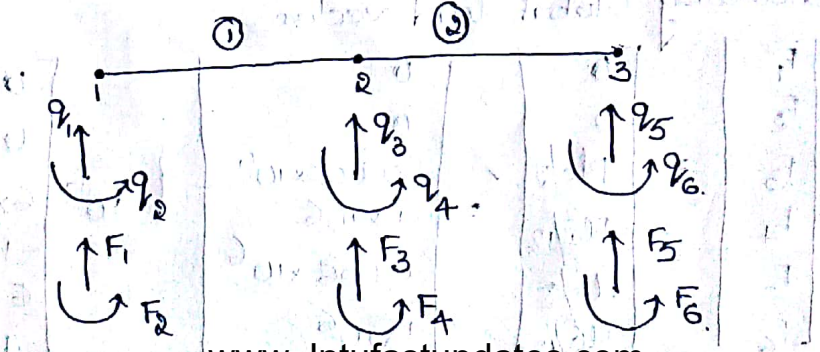
$= 200 \times 10^9 \times 10^{-6} \text{ N/mm}^2$

$\therefore E = 2 \times 10^5 \text{ N/mm}^2$.

$L_1 = L_2 = 1m = 1000mm.$

$P = 12kN/m = 12 N.$

Step 1 :- Discretization of Domain.



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Elements	Nodes	
①	1	2
②	2	3

Step 2: Calculating Element Stiffness Matrix.

$$K_1 = \frac{2\times10^5\times4\times10^6}{1000^3}\begin{bmatrix} 12 & 6\times1000 & -12 & 6\times1000 \\ 6\times1000 & 4\times1000^2 & -6\times1000 & 2\times1000^2 \\ -12 & -6\times1000 & 12 & -6\times1000 \\ 6\times1000 & 2\times1000^2 & -6\times1000 & 4\times1000^2 \end{bmatrix}$$

$$= 800\begin{bmatrix} \overset{1}{12} & \overset{2}{6000} & \overset{3}{-12} & \overset{4}{6000} \\ 6000 & 4\times10^6 & -6000 & 2\times10^6 \\ -12 & -6000 & 12 & -6000 \\ 6000 & 2\times10^6 & -6\times1000 & 4\times10^6 \end{bmatrix}\begin{matrix}1\\2\\3\\4\end{matrix}$$

$$K_2 = 800\begin{bmatrix} \overset{3}{12} & \overset{4}{6000} & \overset{5}{-12} & \overset{6}{6000} \\ 6000 & 4\times10^6 & -6000 & 2\times10^6 \\ -12 & -6000 & 12 & -6000 \\ 6000 & 2\times10^6 & -6000 & 4\times10^6 \end{bmatrix}\begin{matrix}3\\4\\5\\6\end{matrix}$$

Step 3: Calculating Global Stiffness Matrix:

$$K = K_1 + K_2$$

$$= 800\begin{bmatrix} \overset{1}{12} & \overset{2}{6000} & \overset{3}{-12} & \overset{4}{6000} & \overset{5}{0} & \overset{6}{0} \\ 6000 & 4\times10^6 & -6000 & 2\times10^6 & 0 & 0 \\ -12 & -6000 & 24 & 0 & -12 & 6000 \\ 6000 & 2\times10^6 & 0 & 8\times10^6 & -6000 & 2\times10^6 \\ 0 & 0 & -12 & -6000 & 12 & -6000 \\ 0 & 0 & 6000 & 2\times10^6 & -6000 & 4\times10^6 \end{bmatrix}\begin{matrix}1\\2\\3\\4\\5\\6\end{matrix}$$

Step 4: Calculating Global load vector.

$$F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -Ple/2 \\ -Ple^2/12 \\ Ple/2 \\ Ple^2/12 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -0.006\times10^6 \\ -1\times10^6 \\ -0.006\times10^6 \\ 1\times10^6 \end{bmatrix} = 10^6\begin{bmatrix} 0 \\ 0 \\ 6\times10^{-3} \\ -1 \\ 6\times10^{-3} \\ 1 \end{bmatrix}$$

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Step5:- Considering Finite Element Equation.

$$F=KQ \text{ or } F=KQ$$

$$10^6\begin{bmatrix}0\\0\\6\times10^{-3}\\-1\\6\times10^{3}\\1\end{bmatrix}=800\begin{bmatrix}12 & 6000 & -12 & 6000 & 0 & 0\\ 6000 & 4\times10^6 & -6000 & 2\times10^6 & 0 & 0\\ -12 & -6000 & 24 & 0 & -12 & 6000\\ 6000 & 2\times10^6 & 0 & 8\times10^6 & -6000 & 2\times10^6\\ 0 & 0 & -12 & -6000 & 12 & -6000\\ 0 & 0 & 6000 & 2\times10^6 & -6000 & 4\times10^6\end{bmatrix}\begin{bmatrix}q_1\\q_2\\q_3\\q_4\\q_5\\q_6\end{bmatrix}$$

Step6:- Applying Boundary Conditions.

Since there is a Roller support at Nodes 2 and 3, therefore $q_3=0$ & $q_5=0$ & $q_1=0, q_2=0$ since fixed

So Eliminating 1, 2, 3, & 5 rows and columns using Elimination Approach, we get.

$$10^6\begin{bmatrix}-1\\1\end{bmatrix}=800\begin{bmatrix}8\times10^6 & 2\times10^6\\ 2\times10^6 & 4\times10^6\end{bmatrix}\begin{bmatrix}q_4\\q_6\end{bmatrix}$$

$$q_4=-2.67\times10^{-4}$$

$$q_6=4.46\times10^{-4}$$

(b) $V(\xi)=H_1q_1+\frac{l_e}{2}H_2q_2+H_3q_3+\frac{l_e}{2}H_4q_4$

Here Since the Uniformly distributed load is for Element (2)

$$V(\xi)=H_1q_3+\frac{l_e}{2}H_2q_4+H_3q_5+\frac{l_e}{2}H_4q_6.$$

Here $q_3=q_5=0$.

$$V(\xi)=\frac{l_e}{2}H_2q_4+\frac{l_e}{2}H_4q_6.$$

At $\xi=0$, $H_2=\frac{1}{4}(1-\xi)^2(\xi+1)=1/4$ & $H_4=1/2(1+\xi)^2(\xi-1)=-1/4$

$$V(\xi=0)=\frac{l_e}{2}(1/4)q_4+l_e/2(-1/4)q_6$$

$$=\frac{1000}{2}\times1/4\left(-2.67\times10^{-4}\right)+\frac{1000}{2}\left(-1/4\right)\left(4.46\times10^{-4}\right)$$

$$=-0.0893\,mm$$

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