UNIT-III

1. Analysis of Trusses

Truss is a structural member constructed by no. of bars and 'L' angles and are connected eachother firmly at their ends by means of bolts or rivets. The truss elements such bars or angles transmit force only and they cannot move individually or relative to each other. The truss is mainly used to sustain the load of building roof and railway bridges etc. A two-dimensional truss is shown in fig.

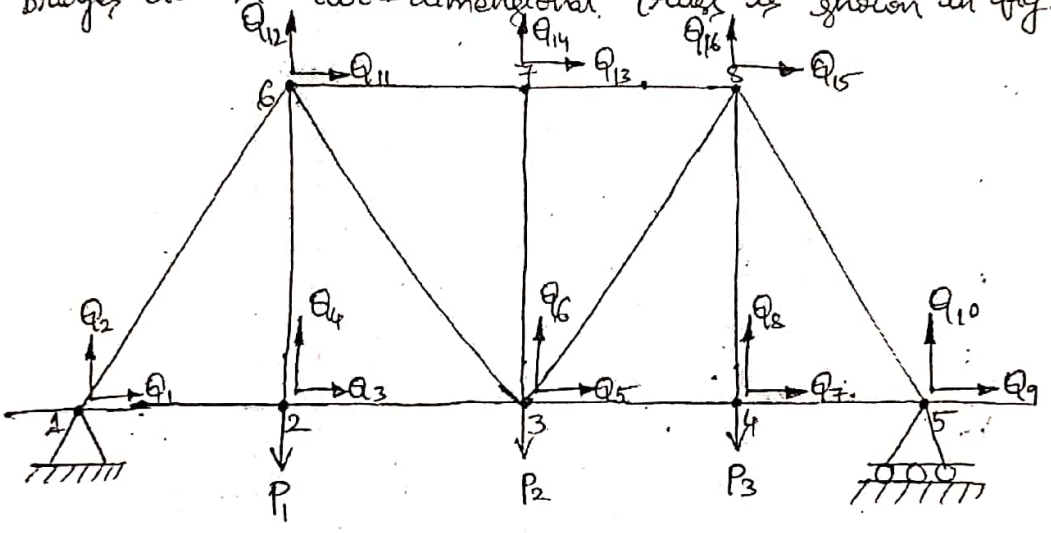


Fig. Two-dimensional truss

A Truss structure consists only of two force members i.e, every truss element is in direct tension or compression (axial forces) and can deform only in the axial direction. It will not be able to carry transverse loads (lateral loads) or bending moments. In a truss it is required that all loads and reactions are applied only at the joints and that all members are connected together at their ends by frictionless pin joints. The following assumptions are made while finding the forces in a truss.

1. The truss is loaded only at the joints.
2. All the members are pin jointed.
3. Self weight of the members are neglected unless stated.

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In a 2 dimensional truss analysis, each of the truss nodes can have a components of displacement parallel to X and Y axis.

In a 3-Dimensional trusses analysis each node can have displacement components in X, Y and Z axis. A two force member is represented as



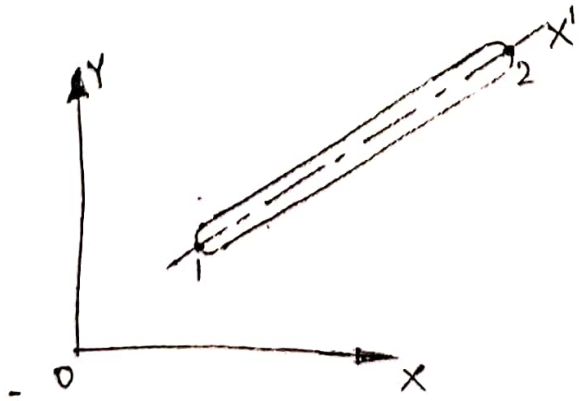
Two-force member

The two-dimensional bar joint is constructed by no. of bars jointed in different directions as shown in fig.

Local and Global Coordinate System:

The main difference between one-dimensional structures and trusses is that the elements of a truss have various orientations. To account for this different orientations local and Global Coordinate systems are introduced as follows.

A typical plane truss is shown in local and global coordinate systems in the fig below.

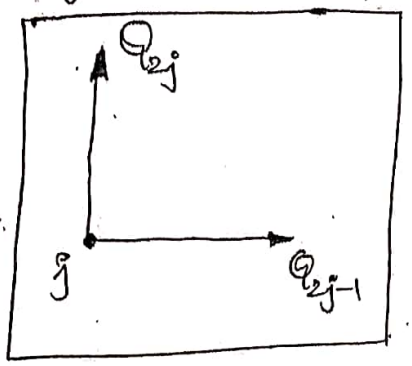


In local numbering scheme the two nodes of the element are numbered The local coordinate system consists of the X' axis 1 & 2.

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which runs along the element from node-1 towards node-2. All quantities in local Co-ordinate system will be devided by a prime (').

The Global X and Y Coordinate system is fixed and does not depend on Orientation of the element. In Global Coordinate system every node has 2 DOF. A systematic numbering scheme is adapted. A node whose global node number is (j) has associated with it degrees of freedom 2j-1 and 2j for that global displacements associated with node-j are Q_{2j-1} & Q_{2j} as shown in the fig.



Let q_1' & q_2' be the displacements at nodes 1 & 2 in the local coordinate system. Thus the element displacement vector in the local coordinate system is denoted by

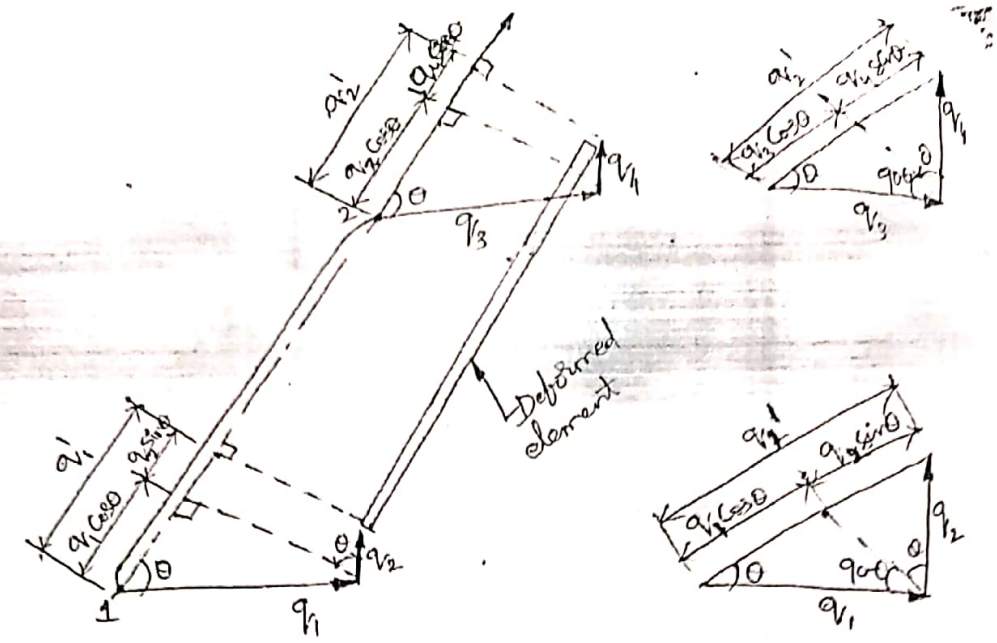
$$\{q'\} = \{q_1' \; q_2'\}^T.$$

The element displacement vector in the Global Coordinate system is a (4×1) vector (or) matrix, denoted by

$$\{q\} \text{ or } \{\delta\} = \{Q_1 \; Q_2 \; Q_3 \; Q_4\}^T \text{ or } \{q_1 \; q_2 \; q_3 \; q_4\}^T$$

The relationship between $\{q'\}$ & $\{q\}$ or $\{\delta\}$ is developed as follows.

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In fig. q_1' equals the sum of the projections of q_1 & q_2 onto the x'-axis. Thus $q_1' = q_1\cos\theta + q_2\sin\theta$. Similarly $q_2' = q_3\cos\theta + q_4\sin\theta$. At this stage the direction Cosines l & m are introduced as $l = \cos\theta$ and $m = \cos\phi$ i.e $\sin\theta$. These direction Cosines are the Cosines of the angles that the local x'-axis makes with the global x and Y axis respectively.

The above equations can be written as $q_1' = q_1 l + q_2 m$

$$q_2' = q_3 l + q_4 m$$

The above equations can be written in matrix form as

$$\{q'\} = [L]\{q\} \quad (\text{or}) \quad \{q'\} = [L]\{\delta\}$$

where, q' = The element displacement vector as per local Coordinate system. $= \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$

$\{q\}$ or $\{\delta\}$ = element displacement vector as per Global Coordinate System.

$$= \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix}.$$

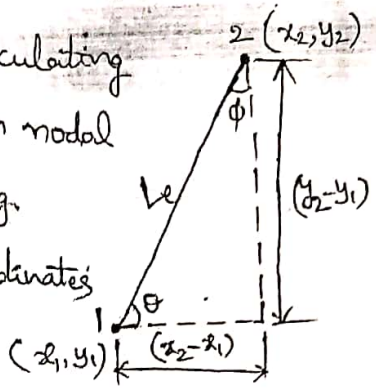
$[L]$ = Transformation matrix (or) rotation matrix.

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$$[L] = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix}$$

Formulas for Calculating direction Cosine l & m.

Simple formulas are given for Calculating the direction Cosines l & m from nodal Coordinate data. Refering to fig. Let (x_1, y_1) & (x_2, y_2) be the Coordinates of nodes 1 & 2 respectively.



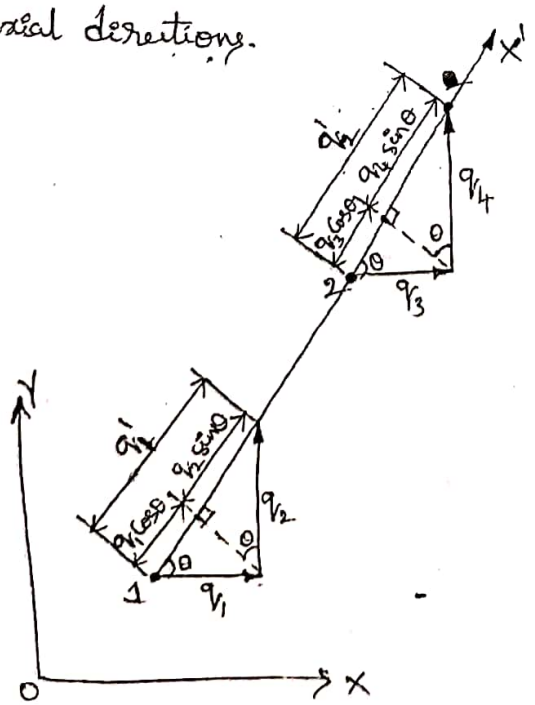
We have, $l = \cos\theta = \frac{x_2-x_1}{Le}$

$$m = \sin\theta = \frac{y_2-y_1}{Le}$$

where, $Le = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Stiffness matrix [K] for truss element

Consider a two noded bar element as shown in fig. for the analysis of trusses. This element is subjected to only axial forces. So the displacements are only in axial directions.



Two noded bar element

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Let q_1' & q_2' = displacements of the nodes 1 & 2 in local coordinate system (along the element axis).

q_1, q_2 & q_3, q_4 = Components of displacements q_1' & q_2' denoted in Global coordinate system (along X & Y axis)

From fig we have $q_1' = q_1\cos\theta + q_2\sin\theta$

$$q_2' = q_3\cos\theta + q_4\sin\theta$$

Consider l & m are the direction cosines, so $l = \cos\theta$ & $m = \sin\theta$. then the above equation becomes.

$$q_1' = q_1 l + q_2 m$$

$$q_2' = q_3 l + q_4 m$$

The above equations can be written in the matrix form as

$$\begin{Bmatrix} q_1' \\ q_2' \end{Bmatrix} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix}$$

i.e. $\{q'\} = [L]\{q\}$ (or) $\{q'\} = [L]\{\delta\}$

[Note:- The problems related to truss is one-dimensional when viewed (Considered) in local coordinate systems and two dimensional when Considered in Global Coordinate System]

Since, the truss is treated as one-dimensional element in the local coordinate system. The element stiffness matrix for the truss on the basis of one-dimensional can be given by

$$[K']_e = \frac{A_e E_e}{L_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad \text{——} (1)$$

where, A_e = Element Cross-sectional area

E_e = Young's modulus

L_e = Length of the truss element

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To develop element stiffness matrix in global coordinate system, the strain energy concept may be adapted. This is obtainable by considering the strain energy in local coordinate system and is given by.

$$U_e = \frac{1}{2}\{q'\}^T [K'] \{q'\}$$

Substituting for $\{q'\} = [L]\{q\}$

$$U_e = \frac{1}{2}([L]\{q\})^T [K'] ([L]\{q\}) \quad (\because (AB)^T = B^T A^T)$$

$$U_e = \frac{1}{2} \{q\}^T [L]^T [K'] ([L]\{q\})$$

The strain energy in global coordinate can be written as

$$U_e = \frac{1}{2}\{q\}^T [K]_e \{q\}.$$

where, $[K]_e$ is the element stiffness matrix in global coordinate system and is given as $[K]_e = [L]^T [K'][L]$. substituting for $[L]$ & $[K']$

$$\therefore [K]_e = \begin{bmatrix} l & 0 \\ m & 0 \\ 0 & l \\ 0 & m \end{bmatrix} \frac{A_e E_e}{L_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix}$$

$$[K]_e = \frac{A_e E_e}{L_e} \begin{bmatrix} l & 0 \\ m & 0 \\ 0 & l \\ 0 & m \end{bmatrix}_{4\times 2} \begin{bmatrix} l & m & -l & -m \\ -l & -m & l & m \end{bmatrix}_{2\times 4}$$

$$[K]_e = \frac{A_e E_e}{L_e} \begin{bmatrix} l^2 & lm & -l^2 & -ml \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}_{4\times 4}$$

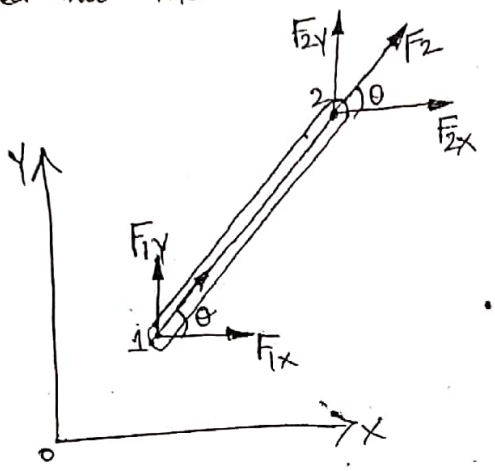
It may be noted that stiffness matrix properties are satisfied.

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i.e., $[K]_e$ is symmetric and the sum of elements in any column is equal to zero.

Formulation of Finite element equation

Similar to nodal displacements, nodal forces can also be resolved into their X and Y components



If F_1 & F_2 are the nodal forces acting at the nodes 1 & 2 along the element as shown in fig. Then we can write as F_{1X} & F_{1Y} as X & Y components of F_1.

And F_{2X} & F_{2Y} as X & Y components of F_2.

Combining the element stiffness, nodal displacement and nodal forces, the finite element equation for the truss element can be written in matrix form as $\{F\} = [K]\{\delta\}$

i.e.,

$$\begin{Bmatrix} F_{1x} \\ F_{1Y} \\ F_{2X} \\ F_{2Y} \end{Bmatrix} = \frac{A_e E_e}{L_e} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix}$$

For the given structure consisting of two & more no. of elements, the element stiffness matrices for all the elements are derived individually and then by combining them in suitable format, the Global stiffness matrix is formulated and the nodal displacements and forces can

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be determined from the suitable global finite element equation.

Stress Calculation

Expressions for element stresses can be obtain by noting the truss element in local coordinate is a simple two force member. Thus the stress (σ) in a truss element is given by $\sigma = E_e \epsilon$. Since, the strain (ϵ) is the change in length per unit original length.

$$\epsilon = \frac{\text{change in length}}{\text{original length}}$$

$$\epsilon = \frac{q_2' - q_1'}{L_e}$$

$$\therefore \text{stress } (\sigma) = E_e \left(\frac{q_2' - q_1'}{L_e} \right)$$

$$\sigma = \frac{E_e}{L_e} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{Bmatrix} q_1' \\ q_2' \end{Bmatrix}$$

$$\sigma = \frac{E_e}{L_e} \begin{bmatrix} -1 & 1 \end{bmatrix} \{q'\}$$

This equation can be written in terms of Global displacements $\{q\}$ using transformation, $\{q'\} = [L]\{q\}$

$$\text{i.e, } \{q'\} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix}$$

$$\sigma = \frac{E_e}{L_e} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \{q\}$$

$$\sigma = \frac{E_e}{L_e} \begin{bmatrix} -l & -m & l & m \end{bmatrix} \{q\}$$

Using this equation stresses are calculated in all the elements or members if forces are required the stresses may be

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multiplied by cross sectional area, positive value indicates tension, negative value indicates Compression.

Analysis procedure for Truss-element

1. Divide the given truss assemble into suitable no. of truss elements and name them as ①, ② & ③.. etc. in the Counterclockwise direction from left side (usually from left bottom of the assembly).

2. Name the nodal points (or) nodes as 1, 2, 3.. etc. from left bottom end of truss assembly in Counter-clockwise direction.

3. For any element locate its angle of inclination 'θ' at the first node & measure the magnitude of angle Counter clockwise from global +ve x-axis.

4. For analysing the selected element write the X & Y Coordinates of axial displacements q_1' & q_2' as Q_1, Q_2, Q_3 & Q_4.

5. Similarly mention the X & Y Coordinates of nodal forces F_1', F_2. as F_{1X}, F_{1Y} & F_{2X}, F_{2Y} etc.

6. Find the element stiffness matrix for the i^{th} element in the global Coordinate system such as

$$[K]_i = \frac{A_i E_i}{L_i} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

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where, $l = \cos\theta$, $m = \sin\theta$

sometimes $\cos\theta$ & $\sin\theta$ are directly calculated using the trigonometric relationships between elements & its locational distance in X & Y axis.

7. using stiffness matrices of various elements of truss from the Global stiffness matrix [K]

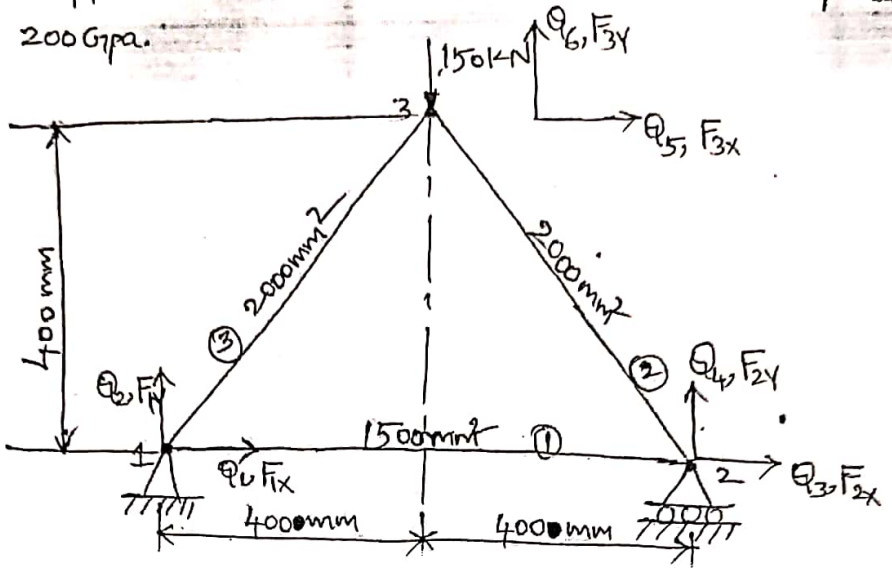
8. Write the Global finite element equation for the entire truss element assembly as $\{F\} = [K]\{S\}$

9. Apply the boundary Conditions, in global finite element equation for example, $U_i = 0$, $Q_i = 0$, reject i^{th} row & i^{th} Column of the global stiffness matrix.

10. From the remaining items of the Finite element equation determine the nodal displacements, element stresses, support reactions using proper formulas and guidelines. If temperature is included Consider thermal effect during analysis.

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1. For the 3 bar truss shown in fig. Determine the nodal displacements & stress in each member. Find the support reactions. Also take modulus of Elasticity as 200 Gpa.



A) The given truss is a 3 bar structure element members, node numbers & global displacements are as shown in fig.

I.

Taking node-1 as origin, the Coordinates of various nodes (nodal Coordinates data) are as follows.

Nodal Coordinates table:-

Nodes	Nodal Coordinates	
	x	y
1	0	0
2	800	0
3	400	400

Element Connectivity table :- (Nodal Connectivity details are given in table)

Element No.	Element Node 1	Element Node 2
1	1	2
2	2	3
3	1	3

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Length of element ①, $L_1 = 800mm$

" " ②, $L_2 = 400\sqrt{2} = 565.565mm$.

" " ③, $L_3 = \sqrt{(400-0)^2+(400-0)^2} = 565.565mm$

Direction Cosine table :- $l = \cos\theta$, $m = \sin\theta$

Element No.	L_e (mm)	θ	l	m
1	800	0.	1	0
2	565.565	135	0.707	0.707
3.	565.565	45	0.707	0.707

given, $A_1 = 1500 mm^2$; $A_2 = 2000mm^2$; $A_3 = 2000mm^2$

$E_1 = E_2 = E_3 = E = 2\times10^5 N/mm^2$

The element stiffness matrix is given by

$$[K]_e = \frac{A_e E_e}{L_e}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

Element stiffness matrix for element ① Can be written as

$$[K]_1 = \frac{1500\times200\times10^3}{800}\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{matrix}1\\2\\3\\4\end{matrix} \leftarrow \text{Global DOF (or) nodes.}$$

$$[K]_1 = 10^3\begin{bmatrix} 1 & 2 & 3 & 4 \\ 375 & 0 & -375 & 0 \\ 0 & 0 & 0 & 0 \\ -375 & 0 & 375 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{matrix}1\\2\\3\\4\end{matrix} \leftarrow \text{nodes}$$

Element stiffness matrix for element ② Can be written as:

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$$[K]_2 = \frac{2000 \times 200 \times 10^3}{565.565}$$

	3	4	5	6	nodes
3	0.5	-0.5	-0.5	0.5	3
4	-0.5	0.5	0.5	-0.5	4
5	-0.5	0.5	0.5	-0.5	5
6	0.5	-0.5	-0.5	0.5	6

$$[K]_2 = 10^3$$

	3	4	5	6	nodes
3	353.62	-353.62	-353.62	353.62	3
4	-353.62	353.62	353.62	-353.62	4
5	-353.62	353.62	353.62	-353.62	5
6	353.62	-353.62	-353.62	353.62	6

Element stiffness matrix for element ③

$$[K]_3 = \frac{2000 \times 200 \times 10^3}{565.565}$$

	1	2	5	6
1	0.5	0.5	-0.5	-0.5
2	0.5	0.5	-0.5	-0.5
5	-0.5	-0.5	0.5	0.5
6	-0.5	-0.5	0.5	0.5

$$[K]_3 = 10^3$$

	1	2	5	6	nodes
1	353.62	353.62	-353.62	-353.62	1
2	353.62	353.62	-353.62	-353.62	2
5	-353.62	-353.62	353.62	353.62	5
6	-353.62	-353.62	353.62	353.62	6

The Global stiffness matrix $[K]$ is assembled from the element stiffness matrices $[K]_1$, $[K]_2$ & $[K]_3$. By adding the element stiffness contributions by noting the element connectivity, we get,

$$[K] = 10^3$$

	1	2	3	4	5	6
1	(375+353.62)	353.62	-375	0	-353.62	-353.62
2	353.62	353.62	0	0	-353.62	-353.62
3	-375	0	(375+353.62)	353.62	-353.62	353.62
4	0	0	-353.62	353.62	353.62	-353.62
5	-353.62	-353.62	-353.62	353.62	(353.62+353.62)	(-353.62 353.62)
6	-353.62	-353.62	353.62	-353.62	0	(353.62+353.62)

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$$[K] = 10^3 \begin{bmatrix} 728.62 & 353.62 & -375 & 0 & -353.62 & -353.62 \\ 353.62 & 353.62 & 0 & 0 & -353.62 & -353.62 \\ -375 & 0 & 728.62 & 353.62 & -353.62 & 353.62 \\ 0 & 0 & -353.62 & 353.62 & 353.62 & -353.62 \\ -353.62 & -353.62 & -353.62 & 353.62 & 707.24 & 0 \\ -353.62 & -353.62 & 353.62 & -353.62 & 0 & 707.24 \end{bmatrix}$$

Global displacement vector $\{\delta\}$ or $\{Q\} = \{Q_1\ Q_2\ Q_3\ Q_4\ Q_5\ Q_6\}^T$

Global load vector $\{F\} = \{F_{1x}\ F_{1y}\ F_{2x}\ F_{2y}\ F_{3x}\ F_{3y}\}^T$

The Global finite element equation or equilibrium equation in matrix form is $\{F\} = [K]\{\delta\}$.

$$\begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{Bmatrix} = 10^3 \begin{bmatrix} 728.62 & 353.62 & -375 & 0 & -353.62 & -353.62 \\ 353.62 & 353.62 & 0 & 0 & -353.62 & -353.62 \\ -375 & 0 & 728.62 & 353.62 & -353.62 & 353.62 \\ 0 & 0 & -353.62 & 353.62 & 353.62 & -353.62 \\ -353.62 & -353.62 & -353.62 & 353.62 & 707.24 & 0 \\ -353.62 & -353.62 & 353.62 & -353.62 & 0 & 707.24 \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{Bmatrix}$$

The given boundary conditions are $Q_1 = 0$, $Q_2 = 0$ & $Q_4 = 0$ and $F_{2x} = 0$, $F_{3x} = 0$; $F_{3y} = -150 \times 10^3 N$. Applying the boundary conditions, then the global finite element equation becomes

$$\begin{Bmatrix} F_{1x} \\ F_{1y} \\ 0 \\ F_{2y} \\ 0 \\ -150 \times 10^3 \end{Bmatrix} = 10^3 \begin{bmatrix} 728.62 & 353.62 & -375 & 0 & -353.62 & -353.62 \\ 353.62 & 353.62 & 0 & 0 & -353.62 & -353.62 \\ -375 & 0 & 728.62 & 353.62 & -353.62 & 353.62 \\ 0 & 0 & -353.62 & 353.62 & 353.62 & -353.62 \\ -353.62 & -353.62 & -353.62 & 353.62 & 707.24 & 0 \\ -353.62 & -353.62 & 353.62 & -353.62 & 0 & 707.24 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ Q_3 \\ 0 \\ Q_5 \\ Q_6 \end{Bmatrix} \quad (1)$$

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The rows & Columns Corresponding to DOF 1, 2 & 4. which corresponds to fixed supports are neglected. The reduced finite element equations are

$$\begin{Bmatrix} 0 \\ 0 \\ -150\times10^3 \end{Bmatrix} = 10^3 \begin{bmatrix} 728.62 & -353.62 & 353.62 \\ -353.62 & 707.24 & 0 \\ 353.62 & 0 & 707.24 \end{bmatrix} \begin{Bmatrix} Q_3 \\ Q_5 \\ Q_6 \end{Bmatrix}$$

$$0 = 10^3 [728.62Q_3 - 353.62Q_5 + 353.62Q_6] \quad \text{---(2)}$$

$$0 = 10^3 [-353.62Q_3 + 707.24Q_5] \quad \text{---(3)}$$

$$-150\times10^3 = 10^3 [353.62Q_3 + 707.24Q_6] \quad \text{---(4)}$$

$$-353.62Q_3 + 707.24Q_5 = 0$$

$$353.62Q_3 = 707.24Q_5$$

$$Q_3 = 2Q_5 \quad \text{---(5)}$$

$$-150 = 353.62(Q_3) + 707.24Q_6 \quad \text{---(6)}$$

$$-150 = 707.24Q_5 + 707.24Q_6 \quad \text{---(7)}$$

$$0 = 728.62(2Q_5) - 353.62Q_5 + 353.62Q_6$$

$$0 = 1457.24Q_5 - 353.62Q_5 + 353.62Q_6$$

$$0 = 1103.62Q_5 + 353.62Q_6 \quad \text{---(8)}$$

from (6) & (7) $\Rightarrow$

$$Q_5 = 0.1\,mm$$

$$Q_6 = -0.312\,mm$$

$$Q_3 = 0.2\,mm$$

(v) Stress in each element can be obtained from the Connectivity of element (1) is 1–2 Consequently the nodal displacement vector for element (1) is given by.

$$\{q_1\} = \{Q_1 \;\; Q_2 \;\; Q_3 \;\; Q_4\}^T$$

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∴ The stress in element ①, $\sigma_1 = \frac{E_1}{L_1}\begin{bmatrix}-l & -m & l & m\end{bmatrix}\begin{Bmatrix}Q_1\\Q_2\\Q_3\\Q_4\end{Bmatrix}$

$$\sigma_1 = \frac{200\times10^3}{800}\begin{bmatrix}-1 & 0 & 1 & 0\end{bmatrix}\begin{Bmatrix}0\\0\\0.2\\0\end{Bmatrix}$$

$$\sigma_1 = 250\,(0+0+0.2+0)$$

$\sigma_1 = 50\ N/mm^2$ (Tensile) $\quad (\because \sigma = \frac{P}{A})$

$P_1 = \sigma_1 \times A_1 = 50 \times 1500 = 75\ KN$ (Tensile)

Stress in element ②, $\sigma_2 = \frac{E_2}{L_2}\begin{bmatrix}-l & -m & l & m\end{bmatrix}\{q_2\}$

$\{q_2\} = \{Q_3\ Q_4\ Q_5\ Q_6\}^T$

$$\sigma_2 = \frac{200\times10^3}{565.565}\begin{bmatrix}0.707 & -0.707 & -0.707 & 0.707\end{bmatrix}\begin{Bmatrix}0.2\\0\\0.1\\-0.312\end{Bmatrix}$$

$$\sigma_2 = 353.62\left[0.707(0.2) - 0.707(0) - 0.707(0.1) - 0.707(0.312)\right]$$

$\sigma_2 = -53\ N/mm^2$ (or) $53\ N/mm^2$ (Compressive)

$\therefore P_2 = \sigma_2 \times A_2 = -53 \times 2000 = -106\ KN$

$P_2 = 106\ KN$ (Compressive)

Connectivity of element ③ is 1–3 Consequently the nodal displacement vector for element ③ is given by

$\{q_3\} = \{Q_1\ Q_2\ Q_5\ Q_6\}^T$

The stress in element ③ is, $\sigma_3 = \frac{E_3}{L_3}\begin{bmatrix}-l & -m & l & m\end{bmatrix}\begin{Bmatrix}Q_1\\Q_2\\Q_5\\Q_6\end{Bmatrix}$

$$\sigma_3 = \frac{200\times10^3}{565.565}\begin{bmatrix}-0.707 & -0.707 & 0.707 & 0.707\end{bmatrix}\begin{Bmatrix}0\\0\\0.1\\-0.312\end{Bmatrix}$$

$$\sigma_3 = 353.62\left[0 - 0 + (0.707\times0.1) + (0.707)(-0.312)\right]$$

$\sigma_3 = -53\ N/mm^2$ (or) $53\ N/mm^2$ (Compressive)

$\therefore P_3 = \sigma_3 A_3 = -53 \times 2000 = -106\ KN$

$P_3 = 106\ KN$ (Compressive)

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iii) To find support reactions

We need to find reaction forces along the DOF 1, 2 & 4 which correspond to fixed supports. From the Global finite element eqn ① we have

$$F_{1X} = 10^3(0+0-375Q_3+0-353.62Q_5-353.62Q_6)$$

$$F_{1Y} = 10^3(0+0+0+0-353.62Q_5-353.62Q_6)$$

$$F_{2Y} = 10^3(0+0-353.62Q_3+0+353.62Q_5-353.62Q_6)$$

$$F_{1X} = 10^3[(-375\times0.2)-353.62(0.1)+353.62(0.312)]$$

$$F_{1X} = -32.56N$$

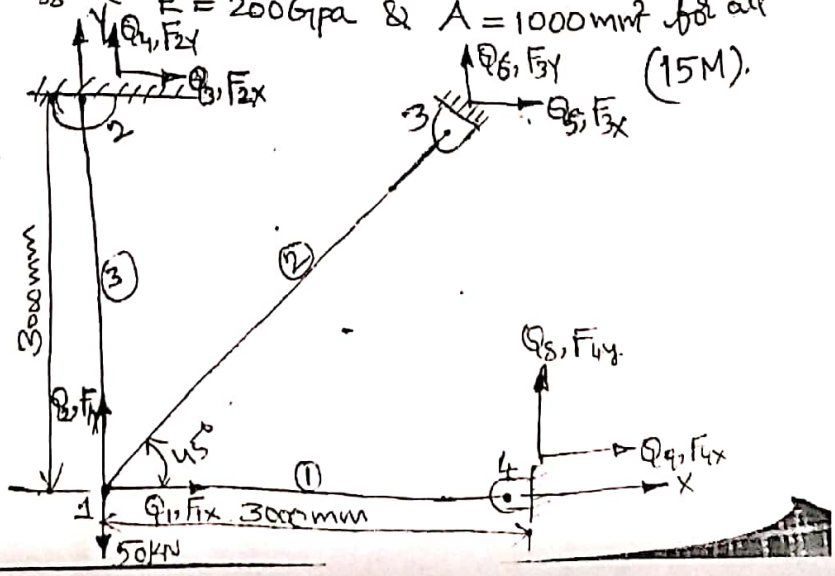
$$F_{1Y} = 10^3[-353.62(0.1)+353.62(0.312)]$$

$$F_{1Y} = 74.96 \simeq 75KN$$

$$F_{2Y} = 10^3[-353.62(0.2)+353.62(0.1)+353.62(0.312)]$$

$$F_{2Y} = 74.96 \simeq 75KN$$

2. For the plane truss composed of 3 elements shown in fig subjected to a downward force of 50KN applied at node 1 Determine the X and Y displacement at node 1 and stresses in each element assume E = 200Gpa & A = 1000mm² for all elements. (15M).



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The given truss is a 3 element structure, node numbers and global displacements are shown in Fig.

Taking node 1 as the origin, the Coordinates of various nodes i.e, nodal Coordinate data are as follows.

Nodal Coordinates table

Nodes	Nodal Coordinates	
	x	y
1	0	0
2	0	3000
3	3000	3000
4	3000	0

Element Connectivity table i.e, nodal Connectivity Coordinates areas follows

Element Connectivity table

Element no	Element node 1	Element node 2
1	1	4
2	1	3
3	1	2

Length of element ①, $L_1 = 3000\,mm$

" " " ②, $L_2 = \sqrt{(3000-0)^2 + (3000-0)^2}$

$L_2 = 4242.64\,mm$

" " " ③, $L_3 = 3000\,mm$

Direction Cosines table:

$l = \cos\theta$, $m = \sin\theta$

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Element no.	L_e (mm)	θ	l	m
1	3000	0	1	0
2	4242.64	45°	0.707	0.707
3	3000	90°	0	1

$A = 1000 mm^2$ & $E_1 = E_2 = E_3 = 200 GPa = 200 \times 10^3 N/mm^2$

The element stiffness matrix is given by

$$[K]_e = \frac{A_e E_e}{L_e} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

Element stiffness matrix for element (1) ← nodes

$$[K]_1 = \frac{1000 \times 200 \times 10^3}{3000} \begin{bmatrix} 1 & 2 & 7 & 8 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \downarrow \text{nodes} \\ 1 \\ 2 \\ 7 \\ 8 \end{matrix}$$

$$[K]_1 = 10^3 \begin{bmatrix} 1 & 2 & 7 & 8 \\ 66.67 & 0 & -66.67 & 0 \\ 0 & 0 & 0 & 0 \\ -66.67 & 0 & 66.67 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 7 \\ 8 \end{matrix}$$

Element stiffness matrix for element (2)

$$[K]_2 = \frac{1000 \times 200 \times 10^3}{4242.64} \begin{bmatrix} 1 & 2 & 5 & 6 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix} \begin{matrix} \downarrow \text{nodes} \\ 1 \\ 2 \\ 5 \\ 6 \end{matrix}$$

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$$[K]_2 = 10^3 \begin{bmatrix} 1 & 2 & 5 & 6 \leftarrow \text{nodes} \\ 23.57 & 23.57 & -23.57 & -23.57 \\ 23.57 & 23.57 & -23.57 & -23.57 \\ -23.57 & -23.57 & 23.57 & 23.57 \\ -23.57 & -23.57 & 23.57 & 23.57 \end{bmatrix} \begin{matrix} \downarrow \\ 1 \\ 2 \\ 5 \\ 6 \end{matrix}$$

Element stiffness matrix for element ③,

$$[K]_3 = \frac{1000 \times 200 \times 10^3}{3000} \begin{bmatrix} 1 & 2 & 3 & 4 \leftarrow \text{nodes} \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{matrix} \downarrow \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

$$= 10^3 \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 66.67 & 0 & -66.67 \\ 0 & 0 & 0 & 0 \\ 0 & -66.67 & 0 & 66.67 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

The Global stiffness matrix [K] is assembled from element stiffness matrices. By adding the element stiffness matrices, noting the element Connectivity, we get

$$[K] = 10^3 \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 90.24 & 23.57 & 0 & 0 & -23.57 & -23.57 & -66.67 & 0 \\ 23.57 & 90.24 & 0 & -66.67 & -23.57 & -23.57 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -66.67 & 0 & 66.67 & 0 & 0 & 0 & 0 \\ -23.57 & -23.57 & 0 & 0 & 23.57 & 23.57 & 0 & 0 \\ -23.57 & -23.57 & 0 & 0 & 23.57 & 23.57 & 0 & 0 \\ -66.67 & 0 & 0 & 0 & 0 & 0 & 66.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix}$$

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Global displacement vector $\{\delta\}$ or $\{Q\} = \{Q_1\ Q_2\ Q_3\ Q_4\ Q_5\ Q_6\ Q_7\ Q_8\}$

Global load vector $\{F\} = \{F_{1X}\ F_{1Y}\ F_{2X}\ F_{2Y}\ F_{3X}\ F_{3Y}\ F_{4X}\ F_{4Y}\}^T$

The Global finite element equation or equilibrium equation in matrix form is $\{F\} = [K]\{\delta\}$

$$\begin{Bmatrix} F_{1X} \\ F_{1Y} \\ F_{2X} \\ F_{2Y} \\ F_{3X} \\ F_{3Y} \\ F_{4X} \\ F_{4Y} \end{Bmatrix} = 10^3 \begin{bmatrix} 90.24 & 23.57 & 0 & 0 & -23.57 & -23.57 & -66.67 & 0 \\ 23.57 & 90.24 & 0 & -66.67 & -23.57 & -23.57 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -66.67 & 0 & 66.67 & 0 & 0 & 0 & 0 \\ -23.57 & -23.57 & 0 & 0 & 23.57 & 23.57 & 0 & 0 \\ -23.57 & -23.57 & 0 & 0 & 23.57 & 23.57 & 0 & 0 \\ -66.67 & 0 & 0 & 0 & 0 & 0 & 66.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{Bmatrix}$$

The given boundary conditions are $Q_3=0$, $Q_4=0$, $Q_5=0$, $Q_6=0$ & $Q_7=0$, $Q_8=0$ and $F_{1X}=0$, $F_{1Y}=-50\,KN$; Applying the boundary conditions, then the global finite element equation becomes

$$\begin{Bmatrix} 0 \\ -50\times10^3 \\ F_{2X} \\ F_{2Y} \\ F_{3X} \\ F_{3Y} \\ F_{4X} \\ F_{4Y} \end{Bmatrix} = 10^3 \begin{bmatrix} 90.24 & 23.57 & 0 & 0 & -23.57 & -23.57 & -66.67 & 0 \\ 23.57 & 90.24 & 0 & -66.67 & -23.57 & -23.57 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -66.67 & 0 & 66.67 & 0 & 0 & 0 & 0 \\ -23.57 & -23.57 & 0 & 0 & 23.57 & 23.57 & 0 & 0 \\ -23.57 & -23.57 & 0 & 0 & 23.57 & 23.57 & 0 & 0 \\ -66.67 & 0 & 0 & 0 & 0 & 0 & 66.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad -(1)$$

The rows & Columns Corresponding to DOF 3, 4, 5, 6, 7 & 8 which Corresponds to fixed supports are neglected. The reduced finite element equation becomes

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$$\begin{Bmatrix} 0 \\ -50\times10^3 \end{Bmatrix} = 10^3 \begin{bmatrix} 90.24 & 23.57 \\ 23.57 & 90.24 \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix}$$

$$0 = 10^3 [90.24 Q_1 + 23.57 Q_2] \quad \text{——— (2)}$$

$$-50\times10^3 = 10^3 [23.57 Q_1 + 90.24 Q_2] \quad \text{——— (3)}$$

$$Q_1 = 0.155 \text{ mm}$$

$$Q_2 = -0.594 \text{ mm}$$

ii) Stress in each element can be obtained from the connectivity of element (1) is 1–4 Consequently the nodal displacement vector for element (1) is given by. $\sigma_1 = \frac{E_1}{L}[-l\ -m\ l\ m]\{q_1\}$

$$\{q_1\} = \{Q_1\ Q_2\ Q_7\ Q_8\}^T$$

∴ The stress in element (1), $\sigma_1 = \frac{E_1}{L_1}[-l\ -m\ l\ m] \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_7 \\ Q_8 \end{Bmatrix}$

$$\sigma_1 = \frac{200\times10^3}{3000}[-1\ \ 0\ \ 1\ \ 0] \begin{Bmatrix} 0.155 \\ -0.594 \\ 0 \\ 0 \end{Bmatrix}$$

$$\sigma_1 = 66.666 (-0.155)$$

$$\sigma_1 = -10.33 \text{ N/mm}^2 \text{ (or) } 10.33 \text{ N/mm}^2 \text{ (Compressive)}$$

The stress in element (2), $\sigma_2 = \frac{E_2}{L_2}[-l\ -m\ l\ m]\{q_2\}$

$$\{q_2\} = \{Q_1\ Q_2\ Q_5\ Q_6\}^T$$

$$\sigma_2 = \frac{200\times10^3}{4242.64}[-0.707\ \ -0.707\ \ 0.707\ \ 0.707] \begin{Bmatrix} 0.155 \\ -0.594 \\ 0 \\ 0 \end{Bmatrix}$$

$$\sigma_2 = 47.140[(-0.707\times0.155) + (0.707\times0.594)]$$

$$\sigma_2 = 47.140(-0.5295)$$

$$\sigma_2 = -14.63$$

$$\sigma_2 = 14.63 \text{ N/mm}^2 \text{ (Tensile)}$$

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The stress in element ③, $\sigma_3 = \frac{E_3}{L_3}[-l \; -m \; l \; m]\{q_3\}$

$\{q_3\} = \{q_1 \; q_2 \; q_3 \; q_4\}$

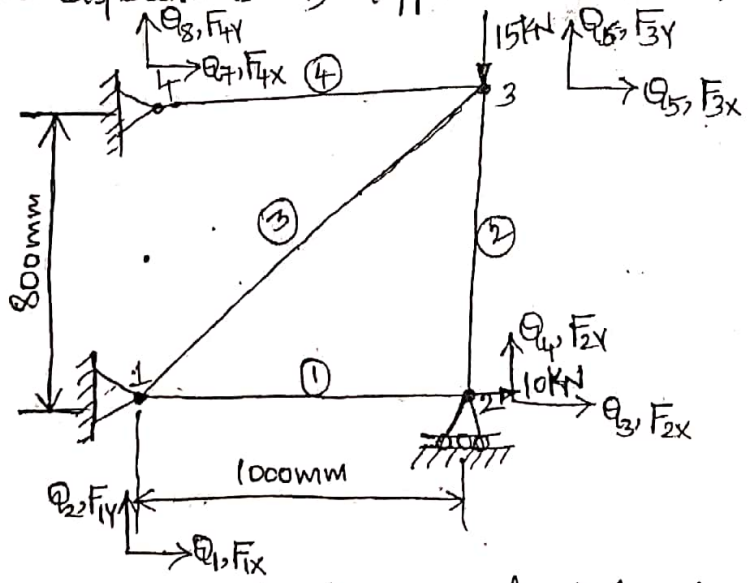
$$\sigma_3 = \frac{200\times10^3}{3000}\begin{bmatrix}0 & -1 & 0 & 1\end{bmatrix}\begin{Bmatrix}0.155\\-0.594\\0\\0\end{Bmatrix}$$

$\sigma_3 = 66.666(0.594)$

$\sigma_3 = 39.59 \simeq 39.6 \text{ N/mm}^2$ (Tensile)

26-12-2020

3. Consider a 4bar truss as shown in fig. It is given that $E = 200\text{Gpa}$, $A = 500\text{mm}^2$ for all the elements. Determine i) Nodal displacements ii) Support reactions iii) element stress



A) The given truss structure is divided into 4 elements Element numbers, Node numbers and global displacements are as shown in fig.

Let Q_1, Q_2 displacements along global axis X and Y at node-1. Q_3, Q_4 displacements along global axis X and Y at node-2. Q_5, Q_6 displacements along global axis X and Y at node-3. Q_7, Q_8 displacements along global axis X and Y at node-4.

F_{1X}, F_{1Y} nodal forces along global axis X and Y at node-1.

F_{2X}, F_{2Y} nodal forces along global axis X, Y at node-2.

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F_{3x}, F_{3y} nodal forces along global axis X, Y at node-3.

F_{4x}, F_{4y} nodal forces along global axis X, Y at node-4.

Taking node-1 is the origin the Coordinates of various nodes are follows.

Nodal Coordinates Table:-

Node No.	Nodal Coordinates (mm)	
	x	y.
1	0	0
2	1000	0
3	1000	800
4	0	800

Element Connectivity table:-

Element	Element Node 1	Element Node-2
1	1	2
2	2	3
3	1	3
4	4	3

} Global numbers.

Length of element ①, $L_1 = 1000$ mm

" " ②, $L_2 = 800$ mm

" " ③, $L_3 = \sqrt{(1000-0)^2 + (800-0)^2} = 1280.624$ mm

" " ④, $L_4 = 1000$ mm.

Directions Cosine table, where, $l = \cos\theta$, $m = \sin\theta$

Element No.	Le (mm)	θ	l	m
1	1000	0	1	0
2	800	90	0	1
3	1280.624	38.65	0.780	0.624
4	1000	0	1	0

$\tan\theta = \frac{800}{1000}$

$\theta = 38.65°$

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7) Nodal displacements

$A_1 = A_2 = A_3 = A_4 = A = 500 mm^2$

$E_1 = E_2 = E_3 = E_4 = E = 200 Gpa = 200 \times 10^3 N/mm^2$

Element stiffness matrix, for element is given by

$$[K]_e = \frac{A_e E_e}{L_e} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

Element stiffness matrix for element ①

$$[K]_1 = \frac{500 \times 200 \times 10^3}{1000} \begin{matrix} \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \begin{matrix} \begin{matrix} 1 & 2 & 3 & 4 \leftarrow \text{nodes} \end{matrix} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \begin{matrix} \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

$$= 10^5 \begin{matrix} \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \begin{matrix} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \begin{matrix} \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

Element stiffness matrix for element ②

$$[K]_2 = \frac{500 \times 200 \times 10^3}{800} \begin{matrix} \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \begin{matrix} \begin{matrix} 3 & 4 & 5 & 6 \leftarrow \text{nodes} \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{matrix} \begin{matrix} \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

$$= 10^5 \begin{matrix} \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \begin{matrix} \begin{matrix} 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1.25 & 0 & -1.25 \\ 0 & 0 & 0 & 0 \\ 0 & -1.25 & 0 & 1.25 \end{bmatrix} \end{matrix} \begin{matrix} \leftarrow \text{nodes} \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

Element stiffness matrix for element ③

$$[K]_3 = \frac{500 \times 200 \times 10^3}{1280.624} \begin{matrix} \\ 1 \\ 2 \\ 5 \\ 6 \end{matrix} \begin{matrix} \begin{matrix} 1 & 2 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 0.6084 & 0.48 & -0.6084 & -0.48 \\ 0.48 & 0.389 & -0.48 & -0.389 \\ -0.6084 & -0.48 & 0.6084 & 0.48 \\ -0.48 & -0.389 & 0.48 & 0.389 \end{bmatrix} \end{matrix} \begin{matrix} \\ 1 \\ 2 \\ 5 \\ 6 \end{matrix}$$

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$$[K]_3 = \frac{0.78}{75} \times 10^5$$

	1	2	5	6
1	0.475	0.374	−0.475	−0.374
2	0.374	0.303	−0.374	−0.303
5	−0.475	−0.374	0.475	0.374
6	−0.374	−0.303	0.374	0.303

Element stiffness matrix for element (4).

$$[K]_4 = \frac{500 \times 200 \times 10^3}{1000}$$

	5	6	7	8
5	1	0	−1	0
6	0	0	0	0
7	−1	0	1	0
8	0	0	0	0

$$= 10^5$$

	5	6	7	8
5	1	0	−1	0
6	0	0	0	0
7	−1	0	1	0
8	0	0	0	0

The Global stiffness matrix [K] is assembled from the element stiffness matrices By adding the element stiffness matrices, noting the element Connectivity, we get

$$[K] = 10^5$$

	1	2	3	4	5	6	7	8
1	1.475	0.374	−1	0	−0.475	−0.374	0	0
2	0.374	0.303	0	0	−0.374	−0.303	0	0
3	−1	0	1	0	0	0	0	0
4	0	0	0	1.25	0	−1.25	0	0
5	−0.475	−0.374	0	0	0.475	0.374	−1	0
6	−0.374	−0.303	0	−1.25	0.374	1.553	0	0
7	0	0	0	0	−1	0	1	0
8	0	0	0	0	0	0	0	0

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Global displacement vector $\{Q\}$ or $\{\delta\} = \{Q_1\ Q_2\ Q_3\ Q_4\ Q_5\ Q_6\ Q_7\ Q_8\}$

Global load vector $\{F\} = \{F_{1x}\ F_{1Y}\ F_{2x}\ F_{2Y}\ F_{3x}\ F_{3Y}\ F_{4x}\ F_{4Y}\}^T$

The Global finite element equation or equilibrium equation in matrix form is $\{F\} = [K]\{\delta\}$

$$\begin{Bmatrix} F_{1x} \\ F_{1Y} \\ F_{2x} \\ F_{2Y} \\ F_{3x} \\ F_{3Y} \\ F_{4x} \\ F_{4Y} \end{Bmatrix} = 10^5 \begin{bmatrix} 1.475 & 0.374 & -1 & 0 & -0.475 & -0.374 & 0 & 0 \\ 0.374 & 0.303 & 0 & 0 & -0.374 & -0.303 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.25 & 0 & -1.25 & 0 & 0 \\ -0.475 & -0.374 & 0 & 0 & 1.475 & 0.374 & -1 & 0 \\ -0.374 & -0.303 & 0 & -1.25 & 0.374 & 1.553 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{Bmatrix}$$

The known boundary conditions are, $Q_1 = Q_2 = 0$, $Q_4 = 0$, $Q_7 = Q_8 = 0$

i.e. $F_{2x} = 10 \times 10^3 N$, $F_{3x} = 0$, $F_{3Y} = -15kN = -15 \times 10^3 N$

The Global finite element equation after applying the boundary conditions is

$$\begin{Bmatrix} F_{1x} \\ F_{1Y} \\ 10\times10^3 \\ F_{2Y} \\ 0 \\ -15\times10^3 \\ F_{4x} \\ F_{4Y} \end{Bmatrix} = 10^5 \begin{bmatrix} 1.475 & 0.374 & -1 & 0 & -0.475 & -0.374 & 0 & 0 \\ 0.374 & 0.303 & 0 & 0 & -0.374 & -0.303 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.25 & 0 & -1.25 & 0 & 0 \\ -0.475 & -0.374 & 0 & 0 & 1.475 & 0.374 & -1 & 0 \\ -0.374 & -0.303 & 0 & -1.25 & 0.374 & 1.553 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ Q_3 \\ 0 \\ Q_5 \\ Q_6 \\ 0 \\ 0 \end{Bmatrix}$$

The rows & Column Corresponding to DOF 1, 2, 4, 7, & which corresponds to fixed support are neglected.

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The reduced finite element equations are

$$10\times10^{3} = [1\times Q_3]\times10^{5} \quad \text{——} (1)$$

$$Q_3 = 0.1\,mm$$

$$0 = 10^{5}\left[1.475\,Q_5 + 0.374\,Q_6\right] \quad \text{——} (2)$$

$$-15\times10^{3} = 10^{5}\left[0.374(Q_5) + 1.553(Q_6)\right] \text{——}$$

$$-0.15 = 0.374\,Q_5 + 1.553\,Q_6 \quad \text{——} (3)$$

Solve eq (2) & (3), $Q_5 = 0.026\,mm$

$$Q_6 = -0.102\,mm.$$

$\therefore$ Nodal displacement vector $\{\delta\} = \{0 \;\; 0 \;\; 0.1 \;\; 0 \;\; 0.026 \;\; -0.102 \;\; 0 \;\; 0\}^{T}$ mm

ii) Support reactions,

from the Global finite element equation.

$$F_{1x} = 10^{5}\left[0+0-Q_3+0-0.475\,Q_5-0.374\,Q_6+0+0\right]$$

$$F_{1x} = 10^{5}\left[-0.1-0.475(0.026)+0.374(0.102)\right]$$

$$F_{1x} = -7420.2\,N \text{ (or) } 7.420\,KN \text{ (Compression)}$$

$$F_{1y} = 10^{5}\left[0+0+0+0-0.374\,Q_5-0.303\,Q_6\right]$$

$$= 10^{5}\left[-0.374(0.026)+0.303(0.102)\right]$$

$$F_{1y} = 2118.2\,N \text{ (or) } 2.118\,KN \text{ (Tensile)}$$

$$F_{2y} = \left[-1.25\,Q_6\right]10^{5}$$

$$= \left[-1.25\times(-0.102)\right]10^{5}$$

$$F_{2y} = 12750\,N \text{ (or) } 12.75\,KN \text{ (Tensile)}$$

$$F_{4x} = 10^{5}\left[-0.026\right]$$

$$F_{4x} = -2600\,N \text{ (or) } 2.6\,KN \text{ (Compressive)}$$

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$F_{4y} = 10^5 [0] = 0$

Nodal force vector is $\{F\} = \{-7.142 \;\; 2.118 \;\; 10 \;\; 12.75 \;\; 0 \;\; -15 \;\; -2.6 \;\; 0\}^T$ KN

(iii) Element stresses :-

The stress in each element can be determine using

$$\sigma = \frac{Ee}{Le} [-l \;\; -m \;\; l \;\; m] \{q\}$$

The Connectivity of element (1) is 1-2 Consequently the nodal displacement vector for element (1) is given by

$$\{q\}_1 = \{Q_1 \;\; Q_2 \;\; Q_3 \;\; Q_4\}^T$$

∴ Stress in element (1),

$$\sigma_1 = \frac{200 \times 10^3}{1000} [-1 \;\; 0 \;\; 1 \;\; 0] \begin{Bmatrix} 0 \\ 0 \\ 0.1 \\ 0 \end{Bmatrix}$$

$$\sigma_1 = 200(0.1) = 20 \text{ N/mm}^2 \quad \text{(Tensile)}$$

Connectivity of element (2) is 2-3 Consequently the nodal displacement vector for element (2) is given by

$$\{q_2\} = \{Q_3 \;\; Q_4 \;\; Q_5 \;\; Q_6\}^T$$

∴ Stress in element (2)

$$\sigma_2 = \frac{200 \times 10^3}{800} [0 \;\; -1 \;\; 0 \;\; 1] \begin{Bmatrix} 0.1 \\ 0 \\ 0.026 \\ -0.102 \end{Bmatrix}$$

$$\sigma_2 = 250(-0.102)$$

$$\sigma_2 = -25.5 \text{ N/mm}^2 \text{ (or) } 25.5 \text{ N/mm}^2 \text{ (Compressive)}$$

Connectivity of element (3) is 1-3 Consequently the nodal displacement vector for element (3) is given by

$$\{q\}_3 = \{Q_1 \;\; Q_2 \;\; Q_5 \;\; Q_6\}^T$$

∴ Stress in element (3)

$$\sigma_3 = \frac{200 \times 10^3}{1280.624} [-0.780 \;\; -0.624 \;\; 0.780 \;\; 0.624] \begin{Bmatrix} 0 \\ 0 \\ 0.026 \\ -0.102 \end{Bmatrix}$$

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$\sigma_3 = 156.17\left[0.780(0.026) - 0.624(0.102)\right]$

$\sigma_3 = -6.77$ (or) $6.77\ N/mm^2$ (Compressive)

Connectivity of element (4) is 4-3 Consequently the nodal displacement vector for element (4) is

$$\{q\}_4 = \{Q_7\ Q_8\ Q_5\ Q_6\}$$

$\therefore$ Stress in element (4),

$$\sigma_4 = \frac{200\times10^3}{1000}\begin{bmatrix}-1 & 0 & 1 & 0\end{bmatrix}\begin{Bmatrix}0\\0\\0.026\\-0.102\end{Bmatrix}$$

$$\sigma_4 = 200(0.026)$$

$$\sigma_4 = 5.2\ N/mm^2\ (Tensile)$$

Results:-

1. Nodal displacements

$Q_1 = Q_2 = Q_4 = Q_7 = Q_8 = 0$; $Q_3 = 0.1mm$

$Q_5 = 0.026mm$

$Q_6 = -0.102mm$

2. Support reactions.

$$\{F\} = \{-7.42\ \ 2.118\ \ 10\ \ 12.75\ \ 0\ \ -15\ \ -2.6\ \ 0\}^T\ KN$$

3. Element stresses.

$\sigma_1 = 20\ N/mm^2$

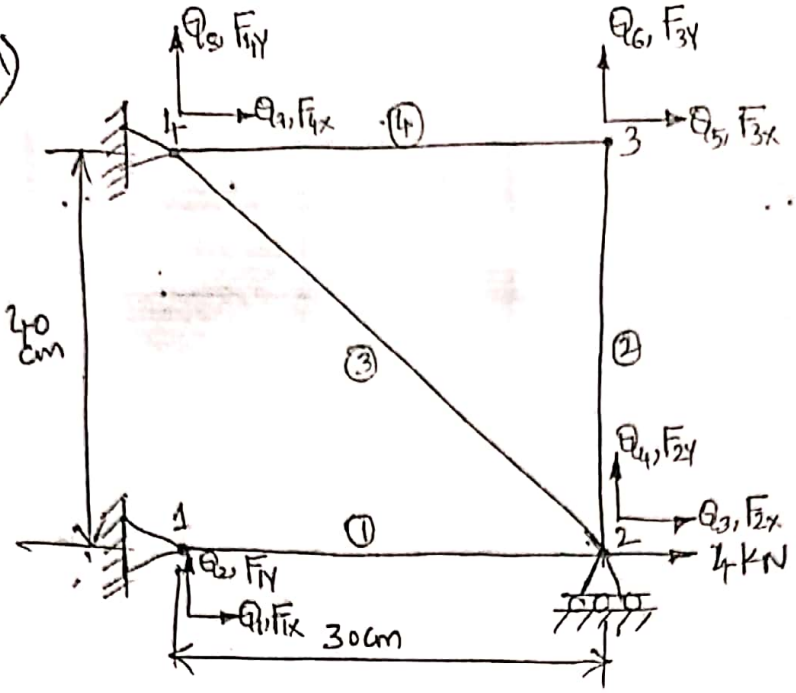
$\sigma_2 = -25.5\ N/mm^2$

$\sigma_3 = -6.77\ N/mm^2$

$\sigma_4 = 5.2\ N/mm^2$

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15M)



Assume, $E = 200 Gpa$ (for mild steel)

$A = 200\ mm^2$

A) Take node-1 as the origin, the coordinates of various nodes are follows.

Nodal Coordinates table:

Node No.	Nodal Coordinates (mm)	
	x	y
1	0	0
2	300	0
3	300	400
4	0	400

Element Connectivity table

Element	Element Node 1	Element Node 2
1	1	2
2	2	3
3	1	4
4	4	3

Length of element (1), $L_1 = 300mm$

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Length of element ②, $L_2 = 400\,mm$

Length of element ③, $L_3 = \sqrt{(0-300)^2 + (400-0)^2} = 500\,mm$

Length of element ④, $L_4 = 300\,mm$.

Directions Cosine table, where $l = \cos\theta$, $m = \sin\theta$

Element No.	Le(mm)	θ	l	m
1	300	0	1	0
2	400	90	0	1
3	500	53.13	0.6	0.799≈0.8
4	300	0	1	0

$\tan\theta = \frac{400}{300}$

$\theta = 53.13°$

1) Nodal displacements

$A_1 = A_2 = A_3 = A_4 = 200\,mm^2$

$E_1 = E_2 = E_3 = E_4 = 200\times10^3\,N/mm^2$

Element stiffness matrix is given by

$$[K] = \frac{A_e E_e}{L_e}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

Element stiffness matrix for element ① is

$$[k]_1 = \frac{200\times200\times10^3}{300}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{matrix}1\\2\\3\\4\end{matrix}$$

$$[K]_1 = 10^3\begin{bmatrix} 133.33 & 0 & -133.33 & 0 \\ 0 & 0 & 0 & 0 \\ -133.33 & 0 & 133.33 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{matrix}1\\2\\3\\4\end{matrix}$$

Element stiffness matrix for element ② is

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$$[K]_2 = \frac{200\times200\times10^3}{400}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$$[K]_2 = 10^3\begin{bmatrix} 3 & 4 & 5 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & -100 \\ 0 & 0 & 0 & 0 \\ 0 & -100 & 0 & 100 \end{bmatrix}\begin{matrix}3\\4\\5\\6\end{matrix}$$

Element stiffness matrix for element (3) is

$$[K]_3 = \frac{200\times200\times10^3}{500}\begin{bmatrix} 0.36 & 0.48 & -0.36 & -0.48 \\ 0.48 & 0.64 & -0.48 & -0.64 \\ -0.36 & -0.48 & 0.36 & 0.48 \\ -0.48 & -0.64 & 0.48 & 0.64 \end{bmatrix}$$

$$[K]_3 = 10^3\begin{bmatrix} 3 & 4 & 7 & 8 \\ 28.8 & 38.4 & -28.8 & -38.4 \\ 38.4 & 51.2 & -38.4 & -51.2 \\ -28.8 & -38.4 & 28.8 & 38.4 \\ -38.4 & -51.2 & 38.4 & 51.2 \end{bmatrix}\begin{matrix}3\\4\\7\\8\end{matrix}$$

Element stiffness matrix for element (4) is

$$[K]_4 = \frac{200\times200\times10^3}{300} = [K]_1 = 10^3\begin{bmatrix} 5 & 6 & 7 & 8 \\ 133.33 & 0 & -133.33 & 0 \\ 0 & 0 & 0 & 0 \\ -133.33 & 0 & 133.33 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{matrix}5\\6\\7\\8\end{matrix}$$

The global stiffness matrix [K] is obtained by assembling the element stiffness matrices.

$$[K] = 10^3\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 133.33 & 0 & -133.33 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -133.33 & 0 & 162.13 & 38.4 & 0 & 0 & -28.8 & -38.4 \\ 0 & 0 & 38.4 & 151.2 & 0 & -100 & -38.4 & -51.2 \\ 0 & 0 & 0 & 0 & 133.33 & 0 & -133.33 & 0 \\ 0 & 0 & 0 & -100 & 0 & 100 & 0 & 0 \\ 0 & 0 & -28.8 & -38.4 & -133.33 & 0 & 162.13 & 38.4 \\ 0 & 0 & -38.4 & -51.2 & 0 & 0 & 38.4 & 51.2 \end{bmatrix}\begin{matrix}1\\2\\3\\4\\5\\6\\7\\8\end{matrix}$$

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The Global finite element equation or equilibrium equation in matrix form is $\{F\} = [K]\{\delta\}$.

$$\begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \\ F_{4x} \\ F_{4y} \end{Bmatrix} = 10^3 \begin{bmatrix} 133.33 & 0 & -133.33 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -133.33 & 0 & 162.13 & 38.4 & 0 & 0 & -28.8 & -38.4 \\ 0 & 0 & 38.4 & 151.2 & 0 & -100 & -38.4 & -51.2 \\ 0 & 0 & 0 & 0 & 133.33 & 0 & -133.33 & 0 \\ 0 & 0 & 0 & -100 & 0 & 100 & 0 & 0 \\ 0 & 0 & -28.8 & -38.4 & -133.33 & 0 & 162.13 & 38.4 \\ 0 & 0 & -38.4 & -51.2 & 0 & 0 & 38.4 & 51.2 \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{Bmatrix}$$

The known boundary conditions are, $Q_1 = Q_2 = 0$, $Q_4 = 0$, $Q_7 = Q_8 = 0$.

$F_{2x} = 4 \times 10^3 N$; $F_{3x} = F_{3y} = 0$,

$$\begin{Bmatrix} F_{1x} \\ F_{1y} \\ 4\times10^3 \\ F_{2y} \\ 0 \\ 0 \\ F_{4x} \\ F_{4y} \end{Bmatrix} = 10^3 \begin{bmatrix} 133.33 & 0 & -133.33 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -133.33 & 0 & 162.13 & 38.4 & 0 & 0 & -28.8 & -38.4 \\ 0 & 0 & 38.4 & 151.2 & 0 & -100 & -38.4 & -51.2 \\ 0 & 0 & 0 & 0 & 133.33 & 0 & -133.33 & 0 \\ 0 & 0 & 0 & -100 & 0 & 100 & 0 & 0 \\ 0 & 0 & -28.8 & -38.4 & -133.33 & 0 & 162.13 & 38.4 \\ 0 & 0 & -38.4 & -51.2 & 0 & 0 & 38.4 & 51.2 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \\ Q_3 \\ 4 \\ Q_5 \\ Q_6 \\ 7 \\ 8 \end{Bmatrix}$$

The rows & Colums Corresponding to DOF 1, 2, 4, 7, 8 which Correspond to fixed support are neglected. The reduced finite element equations are, $4 \times 10^3 = 10^3 [162.13 Q_3]$

$$Q_3 = 0.0246 mm$$

$$0 = (133.33 Q_5) 10^3$$

$$Q_5 = 0$$

$$0 = 100 Q_6$$

$$Q_6 = 0$$

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ii) Support reactions

$F_{1x} = 10^3[-133.33(0.0246)]$

$\therefore F_{1x} = -3279\ N$ (or) $3.279\ KN$ (Compressive)

$F_{2Y} = 10^3[38.4(0.0246)]$

$F_{2Y} = 944.64\ N$ (or) $0.944\ KN$ (Tensile)

$F_{4x} = 10^3[-28.8(0.0246)]$

$= -708.4\ N$ (or) $0.708\ KN$ (Compressive)

$F_{4Y} = 10^3[-38.4(0.0246)] = -944.64\ N$ (or) $0.944\ KN$ (Compressive)

iii) Element stresses

The stress in each element can be determine using

$$\sigma = \frac{E_e}{L_e}[-l\ -m\ l\ m]\{q\}$$

The connectivity of element ① is 1-2

$\{q\}_1 = \{Q_1\ Q_2\ Q_3\ Q_4\}^T$

∴ stress in element ①

$$\sigma_1 = \frac{200\times10^3}{300}[-1\ 0\ 1\ 0]\begin{Bmatrix}0\\0\\0.0246\\0\end{Bmatrix}$$

$\sigma_1 = 16.4\ N/mm^2$ ✓

The connectivity of element ② is 2-3.

$\{q\}_2 = \{Q_3\ Q_4\ Q_5\ Q_6\}^T$

∴ stress in element ②

$$\sigma_2 = \frac{200\times10^3}{400}[0\ -1\ 0\ 1]\begin{Bmatrix}0.0246\\0\\0\\0\end{Bmatrix}$$

$\sigma_2 = 500(0) = 0$ ✓

The connectivity of element ③ is 2-4

$\{q\}_3 = \{Q_3\ Q_4\ Q_7\ Q_8\}$

$$\sigma_3 = \frac{200\times10^3}{500}[-0.6\ -0.8\ 0.6\ 0.8]\begin{Bmatrix}0.0246\\0\\0\\0\end{Bmatrix}$$

$\sigma_3 = 400[-0.6\times0.0246]$

$\sigma_3 = -5.904\ N/mm^2$ ✓

$\sigma_3 = 5.904\ N/mm^2$ (Compressive)

The connectivity of element ④ is 4-3

$\{q\}_4 = \{Q_5\ Q_6\ Q_7\ Q_8\}^T$

$\sigma_4 = 0$ ✓

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