Code No: R19BS1203 **R19** SET - 1

I B. Tech II Semester Regular Examinations, December - 2020
MATHEMATICS-III
(Com. to CE, EEE, ECE, CSE, Chem E, EIE, IT, Auto E, Min E, Pet E)

Time: 3 hours Max. Marks: 75

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

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1. a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) in the direction 2i –j – 2k. (8M)

   b) If $\bar{r}$ is the position vector of the point (x, y, z), prove that $\nabla^2(r^n) = (n+1)r^{n-2}$. (7M)

Or

2. a) Use Gauss divergence theorem to evaluate $\iint_S (yz^2\bar{i} + zx^2\bar{j} + 2z^2\bar{k}).ds$, where S is the closed surface bounded by the xy - plane and the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ above this plane. (8M)

   b) Using Green's theorem evaluate $\int_C (2xy - x^2)dx + (x^2 + y^2)dy$ where C is the closed curve of the region bounded $y = x^2$ and $y^2 = x$. (7M)

3. a) Using Laplace transform evaluate $\int_0^\infty e^t t^2 \sin atdt$. (7M)

   b) Find $L^{-1}\left[\cot^{-1}\left(\frac{s+a}{b}\right)\right]$. (8M)

Or

4. a) Using convolution theorem, evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^3}\right]$. (6M)

   b) Solve $(D^2 + 2D + 1)y = 3te^{-t}$ given that $y(0) = 4, y'(0) = 2$. (9M)

5. a) Find the Fourier series of $f(x) = \begin{cases} \frac{-1}{2}(\pi - x), \text{for } -\pi < x < 0 \\ \frac{1}{2}(\pi - x), \text{for } 0 < x < \pi \end{cases}$ (8M)

   b) Find the Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$. (7M)

Or

6. a) Find the half – range cosine series for the function $f(x) = (x-1)^2$ in the interval. Hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ... = \frac{\pi^2}{8}$. (8M)

   b) Using Fourier integral, show that $\int_0^\infty \frac{1-\cos\pi\lambda}{\lambda}\sin x\lambda = \begin{cases} \frac{\pi}{2}, & if\ 0 < x < \pi \\ 0, & if\ x > \pi \end{cases}$. (7M)

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7. a) Form a partial differential equation by eliminating a and b from $\log(az-1) = x + ay + b.$ (7M)

   b) Solve $px(z-2y^2) = (z-qy)(z-y^2-2x^3)$. (8M)

Or

8. a) Solve $z^2(p^2+q^2) = x^2 + y^2$. (7M)

   b) Solve $\frac{x^2}{p} + \frac{y^2}{q} = z$. (8M)

9. a) Solve $(D^2 - 4DD' + 4D'^2)z = e^{2x+y}$. (8M)

   b) Solve $(D^2 - 3DD' + 2D'^2)z = sin(x - 2y)$. (7M)

Or

10. a) Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-3x}$ by method of separation of variables. (6M)

    b) A tightly stretched string with fixed end points x = 0 and x = L is initially in a position given by $y = y_0 \sin^3 \frac{\pi x}{l}$. If it is released from rest from this position; find the displacement y(x, t). (9M)

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Code No: R19BS1203 **R19** SET - 2

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Time: 3 hours Max. Marks: 75

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1. a) Find the angle of intersection of the spheres $x^2 + y^2 + z^2 = 39$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z + 52 = 0$ at the point (4, -3, 2). (8M)

   b) Find the constants a, b, c so that $(x + 2y + az)\bar{i} + (bx - 3y - z)\bar{j} + (4x + cy + 2z)\bar{k}$ is irrotational. Also find ϕ (scalar potential). (7M)

Or

2. Verify Stoke's theorem for $\bar{F} = (x^2 - y^2)\bar{i} + 2xy\,\bar{j}$ over the box bounded by the planes x = 0, x = a, y = 0, y = b, z = 0, z = c. (15M)

3. a) Find $L\left[e^{-3t}\int_0^t \frac{1-\cos t}{t^2}dt\right]$. (7M)

   b) Find $L^{-1}\left[\frac{e^{-2s}}{s^2+4s+5}\right]$. (8M)

Or

4. a) Solve $y''' - 3y'' + 3y' - y = t^2e^t$ given that $y = 1, y' = 0, y'' = -2$ at t = 0. (9M)

   b) Using convolution theorem, evaluate $L^{-1}\left[\frac{s^2}{(s^2+a^2)^2}\right]$. (6M)

5. a) Find the Fourier series for $f(x) = 2x - x^2$ in $0 < x < 3$. (8M)

   b) Find the Fourier cosine transform of $f(x) = \frac{e^{-ax}}{x}$. (7M)

Or

6. a) Find the half – range cosine series for the function $f(x) = (x-1)^2$ in the interval. (8M)
   Hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ... = \frac{\pi^2}{8}$.

   b) Using Fourier integral, show that $e^{-x}\cos x = \frac{2}{\pi}\int_0^\infty \frac{\lambda^2+2}{\lambda^4+4}\cos\lambda x d\lambda$. (7M)

7. a) Find the partial differential equation arising from $\phi(x + y + z, x^2 + y^2 + z^2) = 0$. (7M)

   b) Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$. (8M)

Or

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8. a) Solve $\left(\frac{p}{2}+x\right)^2+\left(\frac{q}{2}+y\right)^2=1.$ (7M)

b) Solve $(x+pz)^2+(y+qz)^2=1.$ (8M)

9. a) Solve $\left(D^2+3DD'+D'^2\right)z=e^{2x+3y}$ (7M)

b) Solve $\left(D^2+DD'-6D'^2\right)z=cos\left(3x+y\right).$ (8M)

Or

10. a) Solve $u_{xt}=e^{-t}\cos x$ with $u(x,0)=0$ and $u(0,t)=0$ by method of separation of variables. (6M)

b) A rightly stretch of length 20 cms., fastened at both ends is displaced from its position of equilibrium, by imparting to each of its points an initial velocity given by: $V(x)=\begin{cases} x & , 0\le x\le 10 \\ 20-x, & 10\le x\le 20 \end{cases}$. X being the distance from one end. Determine the displacement at any subsequent time. (9M)

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Code No: R19BS1203 **R19** SET - 3

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Time: 3 hours Max. Marks: 75

Answer any five Questions one Question from Each Unit
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1. a) Find the values of a and b so that the surfaces $ax^2 - byz = (a+2)x$ and $4x^2y + z^3 = 4$ may intersect orthogonally at the point (1, – 1, 2). (7M)

   b) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) in the direction 2i –j – 2k. (8M)

Or

2. Verify Gauss divergence theorem for $\overline{F} = x^3\bar{i} + y^3\bar{j} + z^3\bar{k}$ taken over the cube bounded by x = 0, x = a, y = 0, y = a, z = 0, z = a. (15M)

3. a) Find $L\left[e^{-3t}\int_0^t \frac{1-\cos t}{t^2}dt\right]$. (7M)

   b) Find $L^{-1}\left[\frac{s+1}{s^2-2s+2}\right]$. (8M)

Or

4. a) Find $L^{-1}\left\{\frac{s^2}{s^4+4a^4}\right\}$ using convolution theorem. (7M)

   b) Solve $(D^2+6D+9)y = \sin t$ given that $y(0)=1, y'(0)=0$. (8M)

5. a) Find the Fourier series of $f(x) = \begin{cases} 0, \text{for } -\pi < x < 0 \\ x^2, \text{for } 0 < x < \pi \end{cases}$. (9M)

   b) If F(p) is the complex Fourier transform of f(x), then the complex Fourier transform of f(x) cos ax is $\frac{1}{2}[F(p+a)+F(p-a)]$. (6M)

Or

6. a) Obtain half range Fourier sine series for e^x in $0 < x < 1$. (8M)

   b) Find the finite Fourier sine transform of $f(x) = x^3$ in $(0,\pi)$. (7M)

7. a) From the partial differential equation by eliminating the arbitrary functions f and g from $z = xf(ax+by) + g(ax+by)$. (6M)

   b) Solve $(x^2 - y^2 - yz)p + (x^2 - y^2 - zx)q = z(x-y)$. (9M)

Or

8. a) Solve $p^2x^2 + q^2y^2 = z^2$. (8M)

   b) Solve $z^2(p^2+q^2) = x^2 + y^2$. (7M)
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9. a) Solve $\left(D+D'\right)^2 z = e^{x-y}$. (7M)

b) Solve $\left(D^2 - D'^2\right)z = \cos 2x \cos 3y$. (8M)

Or

10. a) Solve $4u_x + u_y = 3u$ and $u(0, y) = e^{-5y}$ by method of separation of variables. (6M)

b) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set to vibrate by giving each of its points a velocity $\lambda x(l - x)$, find the displacement of the string at any distance x from one end at any time t. (9M)

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Time: 3 hours Max. Marks: 75

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

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1. a) Find the constants a, b, c so that $(x+2y+az)\bar{i}+(bx-3y-z)\bar{j}+(4x+cy+2z)\bar{k}$ is irrotational. Also find ϕ (scalar potential). (8M)

   b) Prove that div. (grad r^m) = m (m + 1) r^{m-2}. (7M)

Or

2. Verify Gauss divergence theorem for $\bar{F}=x^2\bar{i}+y^2\bar{j}+z^2\bar{k}$, over the cube formed by the planes x = 0, x = a, y = 0, y = b, z = 0, z = c. (15M)

3. a) Evaluate $\int_0^\infty \frac{\cos 6t-\cos 4t}{t}dt$ using Laplace transforms. (8M)

   b) Find the inverse Laplace transform of $\frac{s+1}{s^2-2s+2}$. (7M)

Or

4. a) Using convolution theorem, evaluate $L^{-1}\left\{\frac{s^2}{(s^2+a^2)^3}\right\}$. (6M)

   b) Using Laplace transform, solve $(D^2 + 5D - 6)y = x^2 e^{-x}$, y(0) = a, $y'(0)=b$. (9M)

5. a) Find the Fourier series of $f(x)=\left(\frac{\pi-x}{2}\right)^2$ in the interval $0<x<2\pi$. (8M)

   b) Using Fourier integral, show that $\int_0^\infty \frac{1-\cos\pi\lambda}{\lambda}\sin x\lambda=\begin{cases}\frac{\pi}{2}, & if\ 0<x<\pi\\ 0, & if\ x>\pi\end{cases}$. (7M)

Or

6. a) Find the Fourier cosine series for $f(x)=2x-x^2$, in $0<x<3$ (8M)

   b) Find the finite Fourier cosine transform of $f(x)=\sin ax$ in $(0,\pi)$. (7M)

7. a) Form the partial differential equation by eliminating the arbitrary function f from $xyz=f(x^2+y^2+z^2)$. (6M)

   b) Solve $x^2(z-y)p + y^2(x-z)q = z^2(y-x)$. (9M)

Or

8. a) Solve $q^2=z^2p^2(1-p^2)$. (8M)

   b) Solve $(x+pz)^2+(y+qz)^2=1$. (7M)

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9. a) Solve $\left(4D^2+12DD'+9D'^2\right)z = e^{3x-2y}$. (7M)

   b) Solve $\left(D^3-7DD'^2-6D'^3\right)z = \sin(x+2y)$. (8M)

Or

10. a) Solve $3u_x + 2u_y = 0$ with $u(x,0) = 4e^{-x}$ by method of separation of variables. (6M)

    b) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3 \frac{\pi x}{l}$. If it is released from rest from this position, find displacement $y(x,t)$. (9M)

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