Gyroscope

Acceleration: rate of change of velocity with respect to time

Angular acceleration

- tangential acceleration $\alpha_t = \frac{d\omega}{dt}$ — due to change of magnitude
- normal acceleration $\alpha_c = \omega \cdot \frac{d\theta}{dt}$ — due to change in direction

ω = const — axis of spin is changing

$\alpha_t = 0$; $\alpha_c \neq 0$

Acceleration produced due to change in axis of spin ← Gyroscopic acceleration

⇓

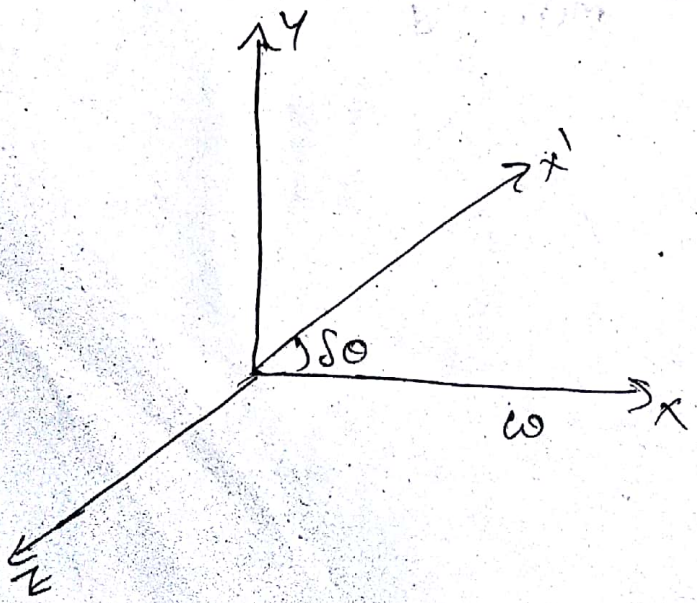
Gyroscopic couple

⇓

Reactive Gyroscopic couple

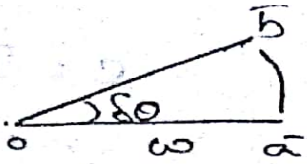
⇓

effect produced is Gyroscopic effect



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$$\frac{\text{change in velocity}}{\delta t} = \frac{\delta O.\omega}{\delta t}$$



$\overline{ab} = \delta O.\omega$

$$\alpha_c = \frac{\delta O\,\omega}{\delta t} = \omega_p.\omega \qquad \frac{\delta\theta}{\delta t} = \omega_p$$

Torque $T = I.\alpha$

$$\boxed{C = I.\omega.\omega_p}$$

ω_p – Pre Angular velocity

1 A disc with radius of gyration of 60mm and mass of 4kg is mounted centrally on a horizontal axis of 80mm length between the bearings. it spins about the axle at 800 rpm counter clockwise when viewed from the right hand side of bearing. The axle precesses above the vertical axis at 50 rpm in clockwise direction when viewed from above determine the resultant reactions at each bearing due to mass and gyroscopic effect.

Sol

$k = 60mm = 60\times10^{-3}\ m$

$m = 4kg$

$N = 800 rpm$

$\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times800}{60} = 83.77 rad/s$

$N_p = 50 rpm$

$\omega_p = 5.23 rad/s$

$$I = \frac{mr^2}{2} = mk^2$$

$$= 4\times(60\times10^{-3})^2$$

$$I = 0.0144\ kg/m^2$$

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C = I · ω · ωp

= 0.0140 × 83.77 × 5.03

= 6.305



Reaction force at A due to gyroscope

$= \frac{C}{OA} = \frac{6.305}{0.086} = 76.805$ (downwards)

Force at bearing B due to gyro effect

= 76.805 (downwards)

Force at each bearing due to weight = $\frac{mg}{2}$

$= \frac{4 \times 9.81}{2} = 19.62$

Reactions at each bearing due to weight

= 19.62 (upwards)

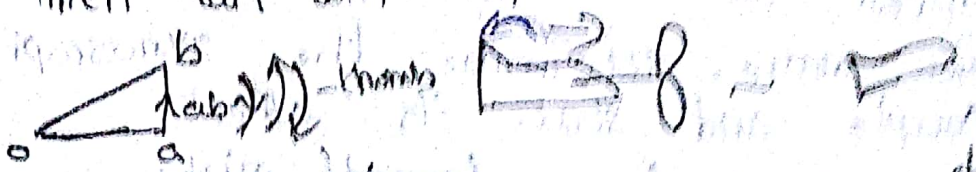
Reaction at bearing A = 76.805 − 19.62 = 57.185 (downwards)

" " " B = 76.805 + 19.62 = 96.425 (downwards)

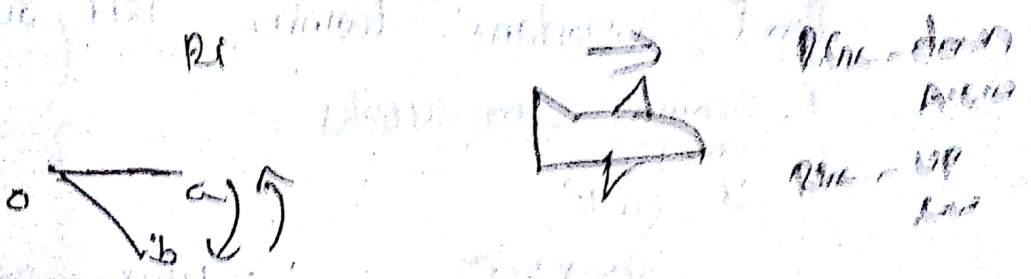
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Gyroscopic effect on aeroplanes

1) When engine rotates in c.w while seeing from back
2) Aeroplane turning left
3) Then tail will dip and nose will start

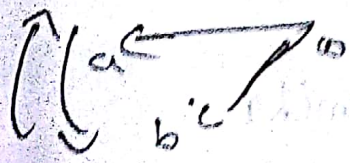


Seen back in c.w (Right)



R.G - c.w
nose-dip
tail stall

Engine A.c.w left turn



R.G.C - c.w
nose-dip
tail stall

Engine A.c.w Right turn

R.G.C - A.c.w
nose - stalls
tail - dips

Use only right hand

An aeroplane flying at 240 km/hr turns towards left completes a circle of 60m radius. The mass of the engine and propeller of plane is 450kg with radius of gyration of 320mm. The engine speed is 2000 rpm clockwise when viewed from rear. determine the gyroscopic couple and state its effect

① aeroplane turns towards right

② Engine rotating clockwise when viewed from front, aeroplane turning left, right

5) $k = 320mm = 0.32m$, $m = 450kg$

$I = mk^2$

$= 450 \times 320^2$ $= 450 \times 0.32^2$

$= 0.046 \times 10^9$ kg m^2 $= 46.08$ kg-m^2

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2000}{60} = 209.43$ rad/sec

$C = I \cdot \omega \cdot \omega_p$

ω_p: $V_p = 240$ km/hr

$= \frac{240 \times 1000}{3600} = 66.66$ m/s

$\frac{km}{h} = \frac{5}{18}$ m/s

$V_p = r \cdot \omega_p$

$\omega_p = \frac{V_p}{r} = \frac{66.66}{60} = 1.1$ rad/s

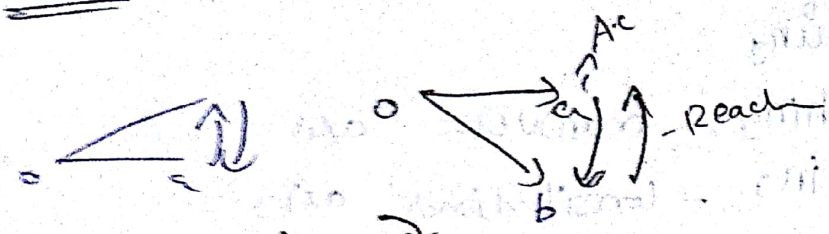
$C = I \cdot \omega \cdot \omega_p$

$= \frac{0.046 \times 10^9}{46.08} \times 209.43 \times 1.1$

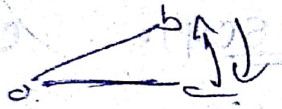
$= 10615.58$ N.m

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c) aeroplane turning right







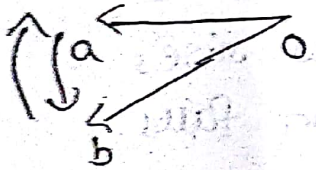
① R.G.C ⇒ C.ω

Nose dips

tail raise

d) Engine rotating clockwise when viewed front. aeroplane turning left, right

i) aeroplane turning right Left



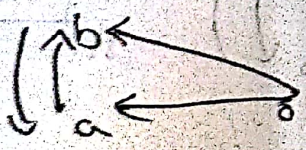


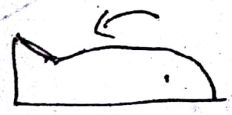
R.G.C = C.ω

nose fall

tail rai

ii) aeroplane turning right





R.G.C = Ac.ω

nose-rai

tail dip

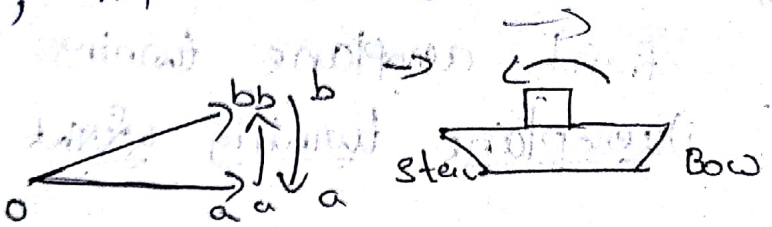
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Gyroscopic effect on ships

1) steering
2) pitching - transverse axis
3) Rolling - longitudinal axis

Gyroscopic effect during steering operation on a ship

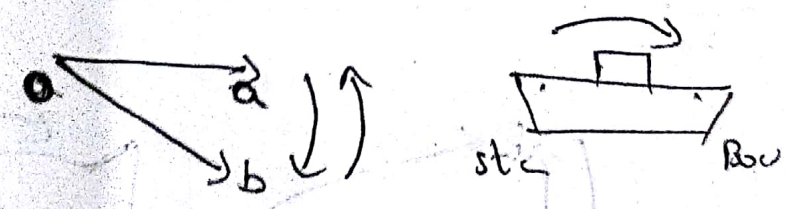
1) Engine rotating clockwise seeing from stern side, ship turning left



Reactive Gyroscopic couple - A.C.W

Bow rises
stern falls

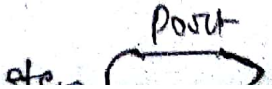
Seeing from back/front rotating clockwise turning left/right



R.G.C - C.W

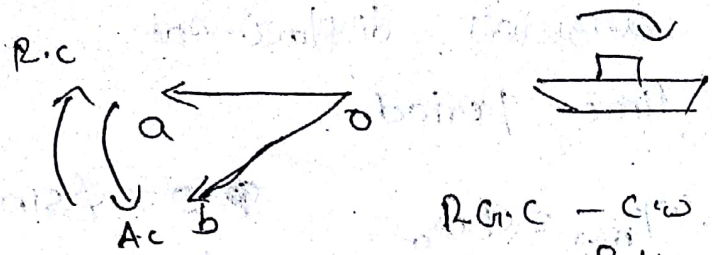
Bow - falls
stern raises

Seeing back left is port
Right is starboard



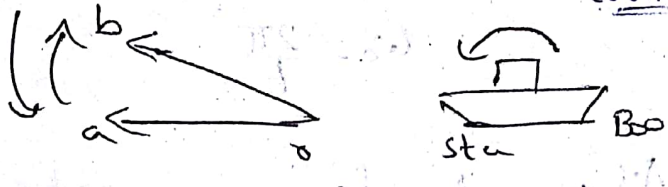
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Seeing from back anticlockwise direction turning right



R.G.C – C.W
Bow - falls
Stern - raises

Seeing from back anticlockwise direction turning right



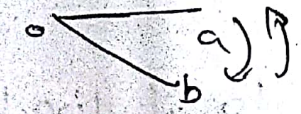
Direction R.G.C – A.C.W
Bow - raises
Stern - falls

Effect of Gyroscopic couple during pitching

The above to and fro motion of ship transverse axis is pitching

Seeing from back Engine in clockwise direction

right



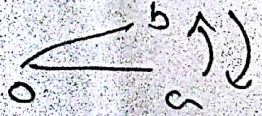
bow below



If bow lowers

ship moving towards Port side

Bow raises left



If bow raises

Ship moves towards starboard



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ω_0 = angular velocity of S.H.M

ϕ = Change of spin axis from normal position

θ = angular displacement

t = time Period

$\omega_{p_{max}} = \phi\omega_0$

$\omega_0 = \frac{2\pi}{t}$

$\theta = \phi \sin\omega_0 t$

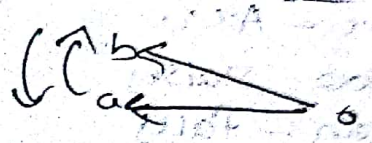
$\frac{d\theta}{dt} = \phi \cos\omega_0 t\,(\omega_0)$

$= \phi\,\omega_0 \cos\omega_0 t$

$\omega_p = \phi\omega_0$

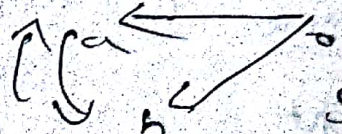
$\omega_0 = \frac{2\pi}{t}$

Engine A.C.W bow far left



move towards starboard

Engine A.C.W bow raise Right



Ship moves towards port side

Rolling:

During rolling longitudional axis and axis of spin remains parallel. so there is no gyroscopic effect.

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1 The rotor of turbine of a ship has 2500kg and rotates at a speed of 3,200rpm counter clockwise when seeing from stern side. The rotor has radius of gyration 4m. Determine the Gyroscopic couple and states its effect the ship steers to the left in a curve of radius 80m at speed of 15 knots

= 2.3

1 Not = 0.516m/s

15 = 7.74 m/s

Sol $m = 2500\ kg$

$N = 3,200\ rpm$

$k = 4m$

$I = mk^2$

$= 2500 \times 4^2$

$= 40000\ kgm^2$

1 ton = 1000 kg

$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 3200}{60} = 335.10\ rad/sec$

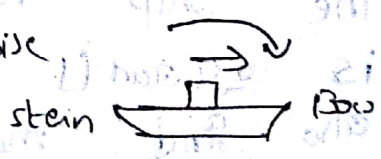
$V_p = 7.74 m/s$

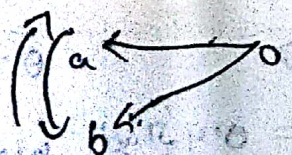
$\omega_p = \frac{V_p}{R} = \frac{7.74}{80} = 0.096\ rad/sec$

$C = 40000 \times 335.10 \times 0.096$

$C = 1286784\ N.m$

seeing back Anticlockwise





RG.C is C.W

so bow dip

stern raise

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2. The ship pitches 5° above and 5° below the normal position and bow is decending with its maximum velocity (pitching is SHM) with a time period of 40 sec

Sol: $\omega_p = \phi \omega_0$ $\quad \alpha_{max} = \phi \omega_0^2$

$\omega_\phi = \frac{2\pi}{T} = \frac{2\times\pi}{40} =$

$\omega_p = \frac{5\times\pi}{180} \times \frac{2\times\pi}{40}$

$= 0.013$ rad/sec

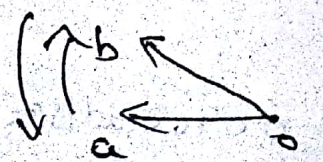
$C = I\omega_1\omega_p$

$= 401000 \times 335.10 \times 0.013$

$C = 1742522.1$ N-m







R.G.C is C.W
Ship moves towards stari board

3. The ship rolls, its angular velocity is 0.4 rad/s when viewed from stern also find maximum angular acceleration during pitching

ω_p = angular velocity = 0.4 rad/sec

$\theta = \phi \sin \omega_0 t$

$\omega_p = \frac{d\theta}{dt}$

$\alpha = \frac{d^2\theta}{dt^2} = \phi\omega_0^2$

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$\theta = \phi \sin \omega_0 t$

$= 5 \times \sin 0.4 \times 40$

$=$

$\alpha = \phi \omega_0^2$

$= 0.08 \times 0.157^2$

$\alpha = 0.013$

$\omega_0 = \frac{2\pi}{t}$

$= \frac{2\pi}{40}$

$\phi = 5 \times \frac{\pi}{180}$

Gyroscopic effect on 4-wheel drive car

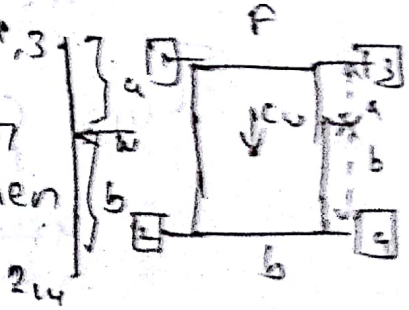
For safe drive of the car all the wheels should be in contact with the road. For this some force should act downwards, i.e the normal reaction by the road on the wheel should be upwards

Reactions on car

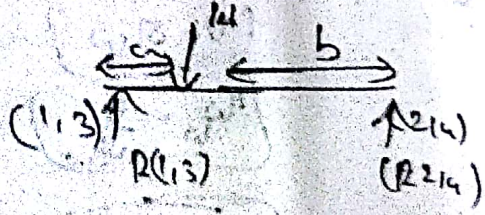


1) Reaction due to weight of car

a) If weight of the car lies at the centre then reaction on each wheel is $\frac{W}{4}$



If weight of the car do not lie at centre then reactions are.



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$\sum M_{13} = 0$

$-R_1(a+b) + R_{24}(a+b) = 0$

$R_{24} = \frac{w(a)}{a+b}$

$R_4 = R_2 = \frac{w(a)}{2(a+b)}$

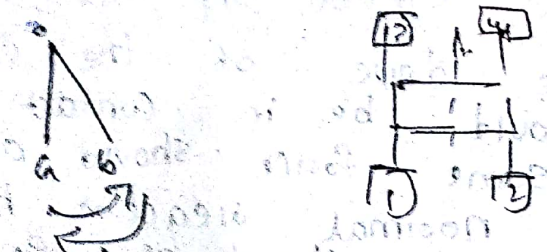
$R_{13} = \frac{w(b)}{a+b}$

$R_1 - R_3 = \frac{w(b)}{2(a+b)}$

$R_1 + R_3 = \frac{w \times b_1}{b_1 + b_2}$

$R_3 + R_4 = \frac{w \times b_2}{b_1 + b_2}$

2) Reactions due to gyroscopic couple



- Reactions due to G_T

$R_{1,2}$ → downwards

R_{3-4} → upwards

$R_{G_T} = \frac{C_G}{2w}$

$C_G = C_{wt} + C_e$

↓

Gyroscopic effect up

$\sqrt{I\omega_w \cdot \omega_p}$

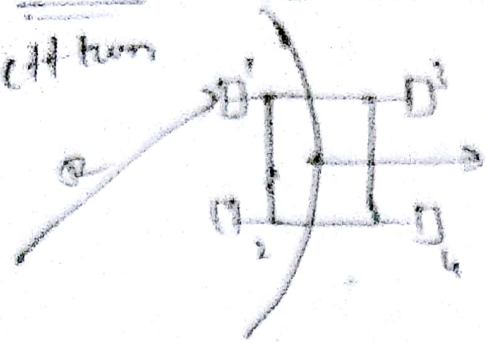
$I = mk^2$

$\omega_w = \frac{v}{r}$

$G = \frac{\omega_e}{\omega_w}$

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Reactions due to centrifugal force



Force on 3,4 is downward } left turn
$R_{3,4}$ is upward
$R_{1,2}$ is downward

1 Each wheel of a four wheeled rear Engine automobile has moment of inertia of 2.4 kgm² and effective diameter of 660mm. The rotating parts of Engine have a moment of inertia of 1.2 kgm². The Gear ratio of Engine to the back wheel is 3:1. The Engine axis is parallel to the rear axle and crank shaft rotates in the same sense as the road wheels. The mass of the vehicle is 2200 kg and the centre of mass is 550mm above the road level. The track width of vehicle is 1.5m. Determine the limiting speed of the vehicle around a curved with 80m radius so that all the four wheels maintain contact with the road surface.

Sol $I_w = 2.4$ kgm²
$r = \frac{d}{2} = 330$mm
wheel radius = 0.33m
$I_e = 1.2$ kgm²
$G = \frac{3}{1}$
$m = 2200$ kg
$h = 550$mm
$w = 1.5$m
$R = 80$m

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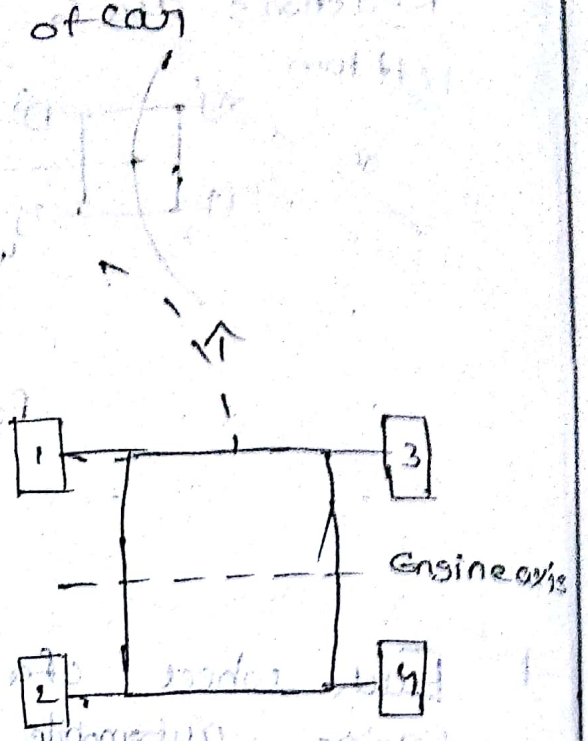
Reaction due to weight of car

$R_1 = R_2 = R_3 = R_4 = \frac{W}{4}$

$R_1 = \frac{mg}{4}$

$= \frac{2200 \times 9.81}{4}$

$= 5395.5\ N$



Reaction due to gyroscopic couple of wheels

force on 3 & 4 is ↓

Reaction on 3 & 4 is ↑

force on 1 & 2 is ↑

Reaction on 1 & 2 is ↓

$R_w = \frac{C\omega}{2G}$

$= \frac{4 \times I\omega \cdot \omega_s \cdot \omega_p}{2G}$

$= 4 \times 2.4 \times \frac{V}{r}$

$\omega_w = \frac{V}{r}$

$\omega_p = \frac{V}{R}$

$G = \frac{\omega_p}{\omega_w}$

$V = r \cdot \omega$

$= 0.3 \times 0.8$

$= 0.45$

$\omega_e = G \cdot \omega_p$

$\omega_p \omega_p = \frac{V}{R} = \frac{4 \times 2.4 \times \frac{V}{0.3} \times \frac{V}{80}}{2 \times 81.5}$

$R_w = 0.121 V^2$

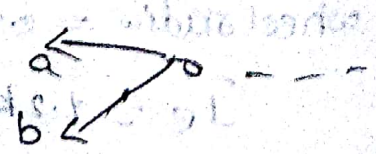
Reaction due to gyroscopic effect on Engine

For 3,4 is ↓

R on 3,4 is ↑

F on 1,2 is ↑

R on 1,2 is ↓



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$$R_e = \frac{C_e}{2 \times \omega}$$

$$= \frac{f_e \cdot \omega_e \cdot \omega_p}{2\omega} \qquad G = \frac{\omega_e}{\omega_w}$$

$$\omega_e = G \cdot \omega_w$$

$$= \frac{1.2 \times 3 \times \frac{V}{0.33} \times \frac{V}{80}}{2 \times 1.5}$$

$$R_e = 0.045 V^2$$

Reaction force due to centrifugal couple

For 3,4 ↓

$R_{3,4}$ is ↑

R on 1,2 ↑

R on 1,2 ↓

$$R_c = \frac{C_c}{2\omega}$$

$$= \frac{f_c \cdot h}{2\omega}$$

$$= \frac{\frac{1200 \times v^2}{80} \times 0.55}{2 \times 1.5}$$

$$f_c = \frac{mv^2}{R}$$

$$= \frac{1200 \times v^2}{80} \times 0.55$$

$$R_c = \frac{18.75 v^2}{5.04}$$

To find limiting speed of vehicle around curved path to maintain contact of wheels with road, the net reaction on inner wheel should be equal to 0

$$-0.12v^2 - 0.045v^2 - 5.04v^2 + 5395.5 = 0$$

$$v^2(0.12 + 0.045 + 5.04) = 5395.5$$

$$5.206 v^2 = 5395.5$$

$$v^2 = 32.19 \text{ m/s}$$

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Nov 12 A 2.2 ton racing car has a wheel base of 2.4m and a track of 1.4m. The Centre of mass of the car lies at 0.6m above ground and 1.4m from the rear axle. The equivalent mass of Engine parts is 140 kg with radius of gyration of 150mm. The back axle ratio is 5. The Engine shaft and flywheel rotate clockwise when viewed from front each wheel has a diameter of 0.8m and moment of inertia of 0.7 kg-m². Determine load distribution on wheels when the car is rounding a curve of 100m radius with speed of 72 km/hr to left, also

Given

$m = 2.2$ ton

$b = 2.4$ m

$w = 1.4$ m

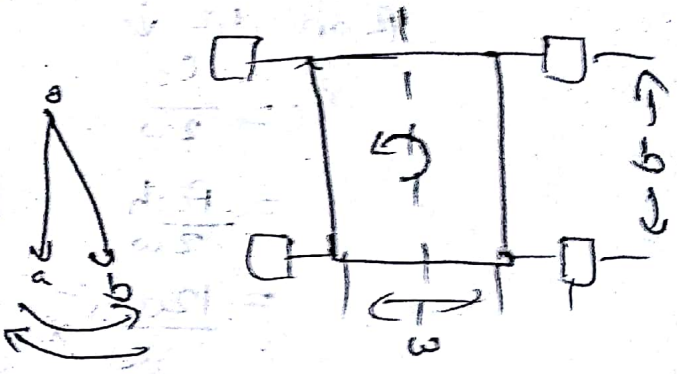
$h = 0.6$ m

$m_e = 140$ kg

$I_w = 0.7$ kg-m²

$D = 0.8$ m $r = 0.4$

$k = 150$ mm



$I_w = 0.7$ kg-m²

$R = 100$

$v = 72 \text{ km/hr} \times \frac{5}{18} = 20$ m/s

Reaction due to Gyroscopic Couple on Engine

For 1,3 is ↓

∴ R on 1,3 is ↑

For 2,4 is ↑

R on 2,4 is ↓

$R_e = \frac{C_e}{2b} = \frac{I_e \omega_e \omega_p \times G}{2b}$

$\omega_e = \frac{v}{r}$

$\omega_p = \frac{v}{R}$

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$I_e = mk^2$

$= 140 \times 150 \times (0.2)^2$

$\frac{21 \times 20 \times 4 \times 20}{60 \times 7\omega}$

$\omega_e = n \cdot \omega$

$n = \frac{\omega_e}{\omega}$

2×2.4

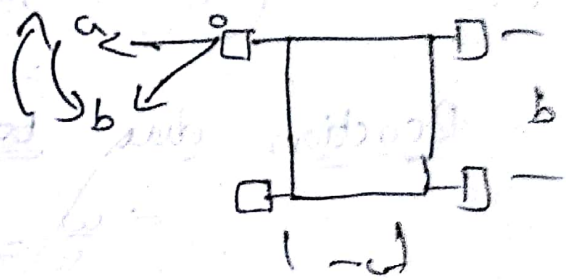
$R_c = 218.75$

$= 2 \times 3.15 \times \frac{20}{0.4} \times \frac{20}{100} \times 5$

2×2.4

$R_c = 32.8 \text{ N}$

Reaction due to Gyroscopic couple on wheels



For 3,4 is ↓

Ron 3,4 is ↑

For 1,2 is ↑

Ron 1,2 is ↓

$R_\omega = \frac{C_\omega}{2 \times \omega}$

$= \frac{4 \times 0.7 \times \omega_\omega \times \omega_p}{2 \times \omega}$

$\omega_\omega = \frac{V}{R}$

$= \frac{4 \times 0.7 \times 50 \times 0.2}{2 \times 1.4}$ $I_\omega =$

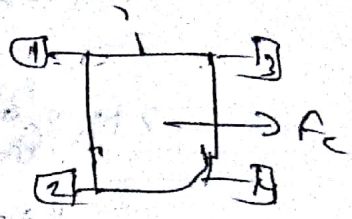
$\omega_\omega = \frac{V}{R}$

$\omega_p = \frac{V}{R}$

$R_\omega = 10 \text{ N}$

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Reaction due to Gyroscopic centrifugal couple



For 3,4 – ↓
Ron 3,4 ↑
For 1,2 ↑
Ron 1,2 ↓

1 ton = 100

$$R_c = \frac{C_c}{2w}$$

$$= \frac{F_c \cdot h}{2 \times w}$$

$$\frac{mv^2}{r} = \frac{2200 \times 20^2}{100}$$

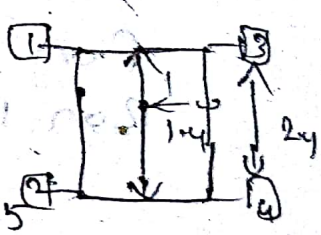
$$= \frac{8800 \times 0.6}{2 \times 1.4}$$

$$R_c = 1885.7 \text{ w}$$

Reaction due to weight of car

$$= \frac{w}{4} = \frac{mxg}{4}$$

$$= \frac{2200 \times 9.81}{4} = 5395.5$$



$$R_{13} = \frac{21582 \times 1.4}{2.4} = 12589.5 \quad (R_{1+3})$$

$$R_{24} = \frac{21582 \times 1}{2.4} = 8992.5 \quad (R_{2+4})$$

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Reactions on wheels	Engine	weight	Centrifugal Couple	net R
32.81	−10	$\frac{12589.5}{2}$	−18857	4431.8
− 11	+ 11	$\frac{8992.5}{2}$	+	6359.14
+ 11	− 11	$\frac{12589}{2}$	−	4430.8
− 11	+ 11	$\frac{8992.5}{2}$	+	6360.14

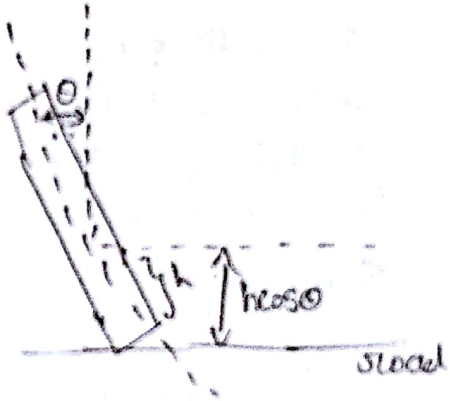
Gyroscopic effect on Motor cycle.

Gyroscopic Couple due to Engine and wheels along X

$C_{G1} = (C_\omega + C_e)\cos\theta$

$C_\omega = 2 I_\omega \cdot \omega_\omega \cdot \omega_p$

$C_e = I_e \cdot \omega_e \cdot \omega_p$ $\quad G_1 \cdot \frac{\omega_e}{\omega_\omega}$





$\cos\theta = \frac{h}{h}$

$h = h\cos\theta$

Couple due to Centrifugal force

$C_c = F_c \times h\cos\theta$

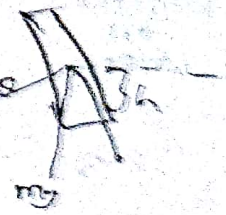
$= \frac{mv^2}{R} \cdot h\cos\theta$

Couple due to Gyroscopic and centrifugal force make couple vehicle to overturn towards outward

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$\Rightarrow$ overturn $= C_c + C_G$

Balancing couple $= mg\sin\theta \times h$



To avoid overturn calculate balancing couple with overturn couple.

1 Each wheel of motor cycle is 600mm D and has a moment of 1.2 kg/m² and the total mass of motor cycle and the rider is 180 kg and the combined centre of mass is 580mm above its ground level when the motor cycle is upright. The moment of inertia of the rotating parts of Engine is 0.2 kgm². The Engine speed is 5 times the speed of wheels and is in the same sense. Determine the angle necessary when the motor cycle takes a turn of 35m radius at a speed of 54 km/hr

Dia of wheel $D = 600mm$

$R = 300mm = 0.3m$

M.I of $I_w = 1.2 kg/m^2$

mass $m = 180 kg$

MI of $I_e = 0.2 kg/m^2$

$V_m = 15 \times \frac{N}{\omega}$ $\quad \omega_e = 5\omega_w$

$R = 35m$

$V = 54 km/hr = 54 \times \frac{5}{18} = 15 m/s$

$h = 580mm = 0.58m$

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Gyroscopic couple

$$= \left(2 I_\omega \omega_\omega \cdot \omega_p \cos\theta\right) + I_e \omega_e \cdot \omega_p \cos\theta$$

$$= \left(2 \times 1.2 \times \frac{1.5}{0.3} \times \frac{15}{35} \times \cos\theta\right) + \left(0.2 \times 5 \times \frac{15}{0.3} \times \frac{15}{35} \cos\theta\right)$$

$$(51.4 \cos\theta) + (21.42 \cdot \cos\theta)$$

$$= 72.82 \cos\theta$$

$\omega_\omega = \left(\frac{V}{r}\right)$

Centrifugal couple

$$C_c = F_c \times h \cos\theta \qquad g = \frac{\omega_e}{\omega_\omega}$$

$$= \frac{mv^2}{R} \times h \cos\theta$$

$$= \frac{180 \times 15^2}{35} \times 0.58 \cos\theta$$

$$= 55\ 671.14 \cos\theta$$

Balancing couple

$$C_B = mg \sin\theta \times h$$

$$= 180 \times 9.81 \times \sin\theta \times 0.58$$

$$= 1024.16 \sin\theta$$

overturning couple = Balancing couple

$$72.82 \cos\theta + 671.14 \cos\theta = 1024.16 \sin\theta$$

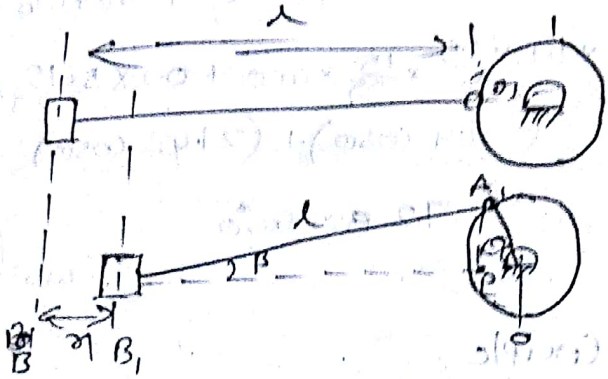
$$743.96 \cos\theta = 1024.16 \sin\theta$$

$$\tan\theta = \frac{743.96}{1024.16}$$

$$\theta = 35.9^\circ$$

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Dynamic analysis of slider crank mechanism



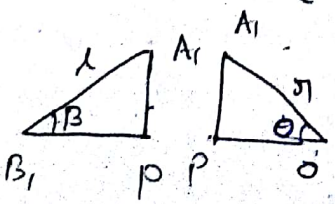
displacement = x

$x = BB_1$

$= Bo - B_1o$

$= Bo - (B_1P + Po)$

$x = Bo - [n r \cos\beta + r\cos\theta]$



$\sin\beta = \frac{A_1P}{l}$

$A_1P = l\sin\beta$

$\sin\theta = \frac{A_1P}{r}$

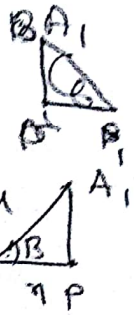
$A_1P = r\sin\theta$

$A_1P = l\sin\beta = r\sin\theta$

$\sin\beta = \frac{r\sin\theta}{l}$

$= \frac{r\sin\theta}{nr}$

$\sin\beta = \frac{\sin\theta}{n}$

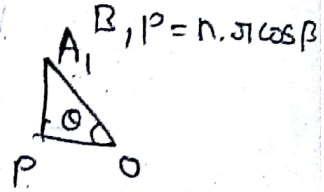


$\cos\theta = \frac{x}{l}$

$x = l\cos\beta$

$\frac{l}{r} = n$

$x = nr\cos\beta$



$B_1P = n.r\cos\beta$

$\cos\theta = \frac{OP}{OA_1}$

$OP = A_1O\cos\theta$

$OP = r\cos\theta$

$\cos\theta = \frac{Adj}{Hyp}$

$\sin\theta = \frac{OP}{Hyp}$

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$\cos\beta = \sqrt{1-\sin^2\beta}$

$= \sqrt{1 - \frac{\sin^2\theta}{n^2}}$

$= \sqrt{\frac{n^2 - \sin^2\theta}{n^2}} = \frac{1}{n}\sqrt{n^2-\sin^2\theta}$

$= (l+r) - [nr\cos\beta + r\cos\theta]$

$= l + r - nr\frac{1}{n}\sqrt{n^2-\sin^2\theta} - r\cos\theta$

$= l + r(1-\cos\theta) - r\sqrt{n^2-\sin^2\theta}$ $\quad \frac{l}{r} = n$, $l = nr$

$= nr + r - r\sqrt{n^2-\sin^2\theta} \cdot r\cos\theta$

$x = r[n+1-\sqrt{n^2-\sin^2\theta} - \cos\theta]$ — If $n \gg 0$, $\sin^2\theta \approx 0$

$= r[n+1-n-\cos\theta] = r(1-\cos\theta)$

This equation represents displacement of simple harmonic motion i.e if $n \gg 0$ then $l \gg r$ motion of piston is SHM

$v = \frac{dx}{dt} = \frac{dx}{d\theta} \cdot \frac{d\theta}{dt} - \omega$

$= r(n+1) - r\left(\sqrt{n^2-\sin^2\theta}\right) \cdot r\cos\theta$ $\quad \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

$= -r\frac{1}{2\sqrt{n^2-\sin^2\theta}} \frac{d}{d\theta}(n^2-\sin^2\theta) - r(-\sin\theta)$

$= -r\frac{1}{2\sqrt{n^2-\sin^2\theta}}\left[0 - 2\sin\theta \cdot \frac{d}{d\theta}(\sin\theta)\right] + r\sin\theta\omega$

$= -r\frac{1}{2\sqrt{n^2-\sin^2\theta}}\left[-2\sin\theta\cdot\cos\theta\right] + r\sin\theta\omega$

$V = \left[\frac{r}{2}\frac{\sin 2\theta}{\sqrt{n^2-\sin^2\theta}} + r\sin\theta\right]\cdot\omega$

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$$v = \left[\frac{r}{2}\frac{\sin 2\theta}{\sqrt{n^2-\sin^2\theta}} + r\sin\theta\right]\omega$$

if $n >> \theta$ then $\sin\theta \simeq \theta$

$$v = \left[\frac{r}{2}\frac{\sin 2\theta}{n} + r\sin\theta\right]\omega$$

$$v = r\cdot\omega\left[\frac{\sin 2\theta}{2n} + \sin\theta\right]$$

Piston Acceleration $a = \frac{dv}{dt} = \frac{dv}{d\theta}\cdot\frac{d\theta}{dt}$

$$a = r\cdot\omega\left[\frac{\cos 2\theta(2)}{2n} + \cos\theta\right]\cdot\omega$$

$$a = r\cdot\omega^2\left[\frac{\cos 2\theta}{n} + \cos\theta\right]$$

Angular displacement of CR

$\rightarrow \beta$



angular velocity

$\omega_c = \frac{d\beta}{dt}$ $\sin\beta = \frac{1}{n}\sin\theta$

Diff w.r.t

$$\cos\beta\cdot\frac{d\beta}{dt} = \frac{1}{n}\cdot\cos\theta\cdot\frac{d\theta}{dt}$$

$$\cos\beta\cdot\omega_c = \frac{1}{n}\cdot\cos\theta\cdot\omega$$

$$\omega_c = \frac{\omega}{n}\frac{\cos\theta}{\cos\beta}$$

$\sin\beta = \frac{\sin\theta}{n}$

$\cos\beta = \sqrt{1-\sin^2\beta}$

$= \sqrt{1-\frac{\sin^2\theta}{n^2}}$

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$$\sqrt{\frac{n^2-\sin^2\theta}{n^2}}$$

$$= \frac{1}{n}\sqrt{n^2-\sin^2\theta}$$

$$\omega_c = \frac{\omega}{n}\,\frac{\cos\theta}{\frac{1}{n}\sqrt{n^2-\sin^2\theta}}$$

$$\omega_c = \frac{\omega\cdot\cos\theta}{\sqrt{n^2-\sin^2\theta}}$$

Angular acceleration of connecting rod.

$$\alpha_c = \frac{d\omega_c}{dt}$$

$$\omega_c = \left[\omega\cdot\cos\theta\cdot(n^2-\sin^2\theta)^{-1/2}\right]\omega$$

$$= \left(\frac{d\omega_c}{d\theta}\times\frac{d\theta}{dt}\right)\omega$$

$$\frac{d}{d\theta}(n^2-\sin^2\theta)^{-1/2}$$

$$= -\tfrac{1}{2}(n^2-\sin^2\theta)^{-1/2-1}$$

$$(0-2\sin\theta\cos\theta)$$

$$= \frac{(n^2-\sin^2\theta)}{2}\cdot\sin 2\theta$$

$$= \left[\omega\cos\theta\,\frac{(n^2-\sin^2\theta)}{2}\sin 2\theta + (n^2-\sin^2\theta)^{-1/2}(-\sin\theta)\right]\frac{d\theta}{dt}\omega$$

$$= \frac{d\omega_c}{d\theta}\times\frac{d\theta}{dt}$$

$$= \omega\left[\cos\theta\cdot -\frac{1}{2}(n^2-\sin^2\theta)^{-3/2}(-\sin 2\theta) + (n^2-\sin^2\theta)^{-1/2}\cdot(-\sin\theta)\right]\omega$$

$$= \omega^2\sin\theta\left[(n^2-\sin^2\theta)^{-1/2}\left(\cos^2\theta\,(n^2-\sin^2\theta)^{-1}-1\right)\right]$$

$$-\frac{3}{2} = \frac{1}{2}+n$$

$$-\frac{3}{2}+\frac{1}{2} = -\frac{2}{2} = -1$$

$$\frac{1}{2}+n = \frac{3}{2}$$

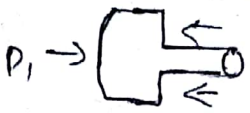
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$$\omega^2 \cdot \sin\theta \cdot (n^2 - \sin^2\theta)^{-\frac{1}{2}} \left[\frac{\cos^2\theta}{n^2 - \sin^2\theta} - 1\right]$$

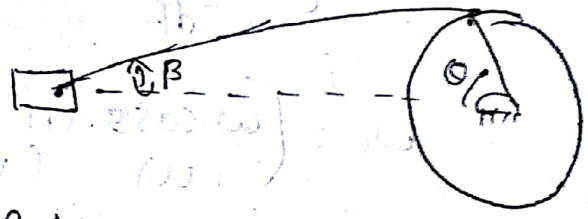
$$\omega^2 \sin\theta (n^2 - \sin^2\theta) \left[\frac{\cos^2\theta - n^2 + \sin^2\theta}{n^2 - \sin^2\theta}\right]$$

$$a_c = -\frac{\omega^2 \sin\theta \left[1 - n^2\right]}{(n^2 - \sin^2\theta)^{3/2}}$$

Force analysis on slider crank mechanism



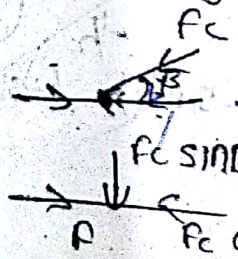
1) Piston effort (P)



$F_{gas} = F_p = P_1A_1 - P_2A_2$ $\quad A = \frac{\pi}{4}D_1^2$

$F = F_p - F_f - F_i + mg$ $\quad A_2 = \frac{\pi}{4}(D^2 - d^2)$

F_c = Force along connecting rod

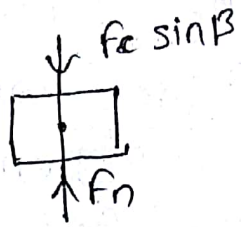


$$F = F_c \cos\beta$$

$$F_c = \frac{F}{\cos\beta}$$

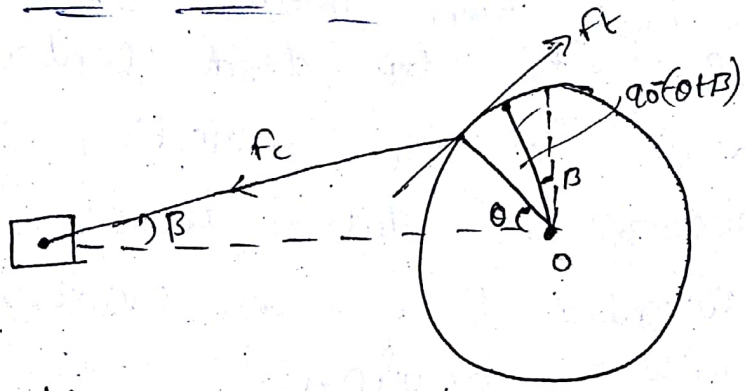
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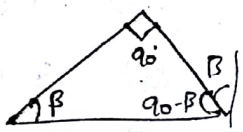
Normal force on wall of cylinder



$F_n = F_c \cdot \sin\beta$

Force on crank pin



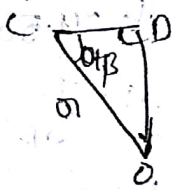


Equating couples on crankwheel about O

$F_c \times OD = F_t \times OC$

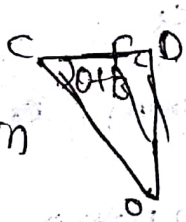
$F_c \times OJ \cdot \sin(\theta+\beta) = F_t \times OJ$

$F_t = F_c \cdot \sin(\theta+\beta)$



$\sin(\theta+\beta) = \frac{OD}{OJ}$

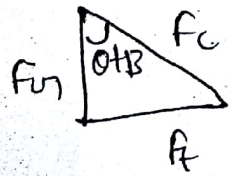
$OD = OJ \sin(\theta+\beta)$



F_c acting on crankpin



tangential radial



$\cos(\theta+\beta) = \frac{F_n}{F_c}$

$F_n = F_c \cdot \cos(\theta+\beta)$

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In a vertical double acting steam Engine, the connecting rod is 4.5 times of crank. The weight of reciprocating parts is 120kg, and stroke of piston is 440mm. The Engine runs at 250 rpm if the net load on the piston due to steam pressure is 25kN. when the crank has turn through an angle of 120° from the top dead centre. Determin,

a) Thrust in the connecting rod
b) Pressure on slide bars
c) Tangential force on crankpin
d) Thrust on bearings
e) Turning moment of crank shaft

Given

$l_c = 4.5r$

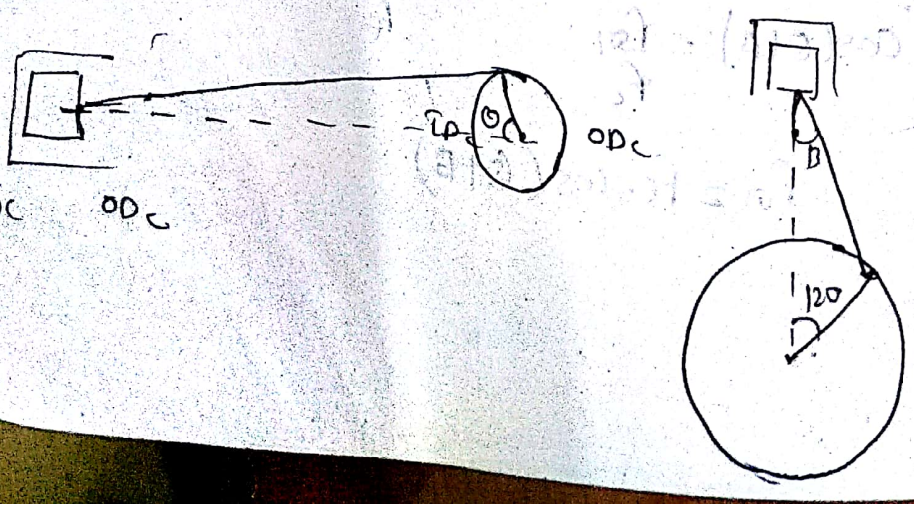
$n = \frac{l_c}{r}$ $\boxed{n = 4.5}$

mass $m = 120kg$

Stroke $L = 440mm$ $\frac{L}{r} = 2$ $r = \frac{440}{2} = 220mm$
$= 0.22m$

Speed $N = 250 rpm$

$F_p = 25kN$



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$$F_c = \frac{F}{\cos\beta}$$

$$F = F_p - F_i + mg \qquad \omega = \frac{2\pi N}{60} = 26.17$$

$$F_i = ma$$

$$= m \times r\omega^2 \left[\frac{\cos 2\theta}{n} + \cos\theta\right]$$

$$= 120 \times 0.22 \times 26.17^2 \left[\frac{\cos 2\times(120)}{4.5} + \cos(120)\right]$$

$$F_i = -1104.9 \text{ N}$$

$$F = 25\times10^3 + 1104.92 + 120 \times 9.81$$

$$= 37586.4 \text{ N}$$

$$\sin\beta = \frac{\sin\theta}{n}$$

$$\sin\beta \quad \frac{\sin 120}{4.5}$$

$$\beta = 11.09^\circ$$

$$F_c = \frac{37586.4}{\cos 11.09}$$

$$F_c = 38\,301.6 \text{ N}$$

ii) Normal force

$$F_n = F_c \sin\beta \quad 38301.6 \times \sin 11.09$$

$$= 7367.3 \text{ N}$$

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iii) $F_t = F_c \sin(\theta + \beta)$

$= 38301.6 \sin(120 + 11.09)$

$= 28867.07$ N

$F_n = F_c \cos(\theta + \beta)$

$= 37226.9 \cos(120 + 11.09)$

$= -24466.8$ N

Turning Moment on crankshaft



$T.M = F_t \times r$

$= 28867.07 \times 0.22$

$T.M = 6350.7$ N-m

5) Static force analysis:

Whenever there are moving masses in a mechanism, inertia forces are produced

While analysing a mechanism if the inertia forces are neglected such analysis is known as static force analysis

ex: Force analysis of lifting cranes.

Dynamic force analysis

While analysing a mechanism if the effect of inertia forces are considered such a analysis is known as Dynamic force analysis ex: Force

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Fly wheels

Introduction

Torque on crank wheel

$T = F_t \times r$

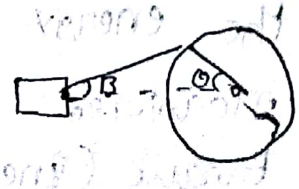
$T = F_t \times \sin(\theta+\beta) \times r$

$T = I \cdot \alpha$

I → M.I, α → angular acceleration $= f(\theta, \beta)$

$\omega_f = \omega_i + \alpha t$

ω_f → fluctuating speed



Here, α is angular acceleration of crank wheel. During rotation of crank wheel, AS (β, θ) are changing so α is not constant. Then

$\omega_f = \omega_i + \alpha t$

ω_f → fluctuating speed

In this equation ω_f, ω_i are angular speeds which are not same.

Conclusion: In a cycle engine rotates at different speeds, to control the fluctuation of speeds a device is name as fly wheel.

→ fly wheel is used to control the variations in speed during each cycle of an Engine

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Chp-1

1) What is gyroscopic couple? State its expression?

A) Whenever a rotating body changes its axis of rotation, a couple is applied on the rotating body. The couple is known as gyroscopic couple.

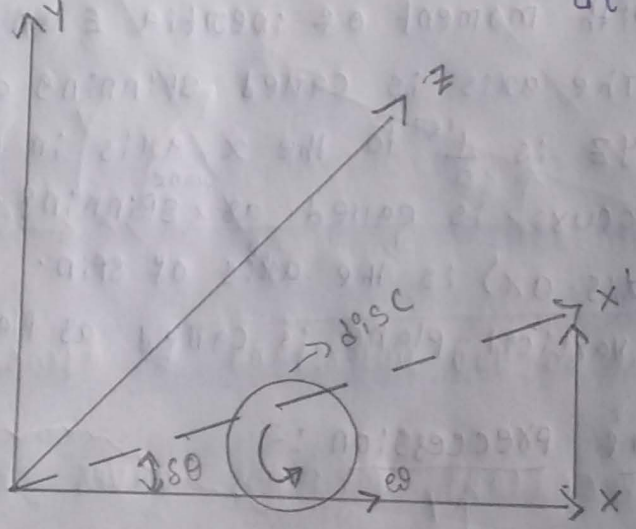
Gyroscopic couple, $C = I.\omega.\omega_p$ N-m

I = moment of inertia ($kg-m^2$)

ω = Angular velocity of spin (rad/sec)

ω_p = Angular speed of precession (rad/sec)

$\omega = \frac{2\pi N}{60}$; $I = mk^2$; $\omega_p = \frac{d\phi}{dt}$.



2) Define

A) a) Angular velocity of precession :-

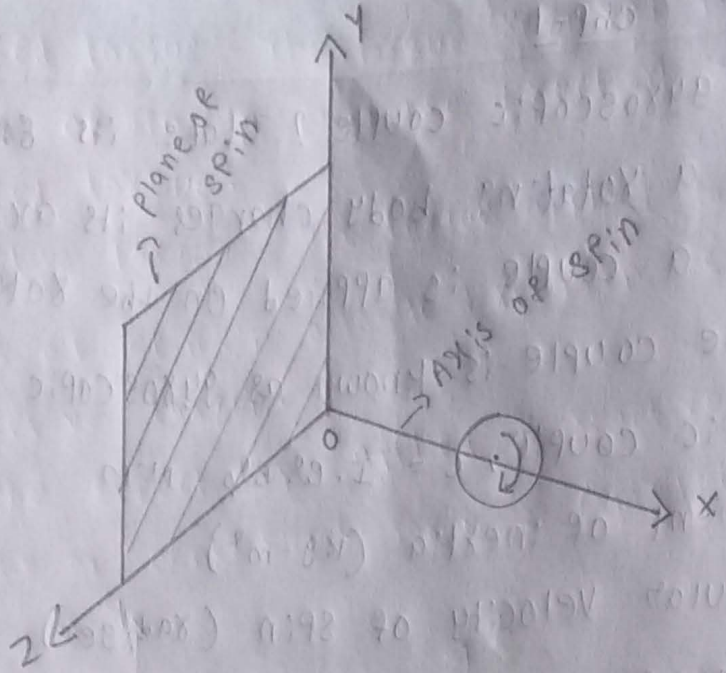
The angular velocity of the axis of spin, about which a disc or body revolves is known as angular velocity of pression.

It is given by, $\omega_p = \frac{d\theta}{dt}$.

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b) Axis of spin :-

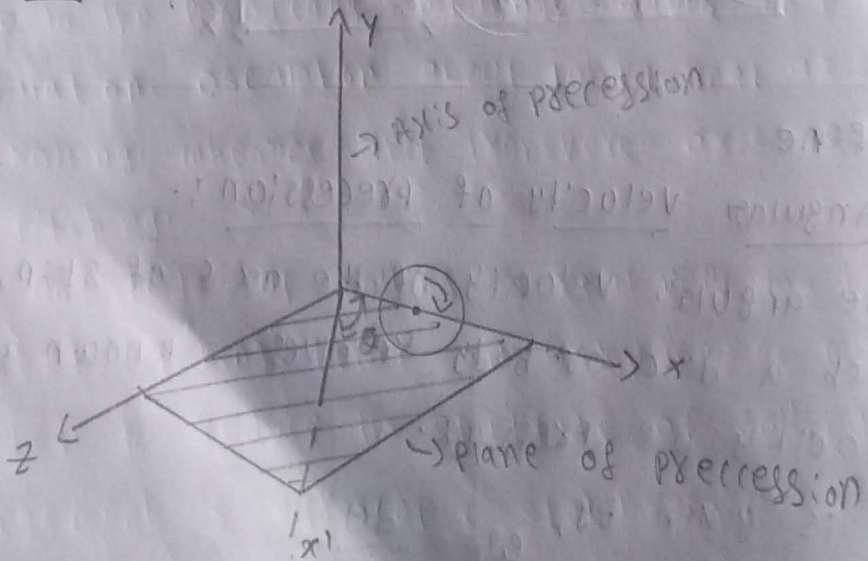


Spining is defined as a angular motion of a body about the axis of symmetry. The rotation of disc with moment of inertia I an angular velocity. The axis is called spinning of a disc. The plane yz is $\perp^{er}$ to the x-axis in which the rotation occurs is called as plane of spinning.

x-axis (axis ox) is the axis of spin.

yoz is a vertical plane is called as plane of spinning.

c) Axis of Precession :-

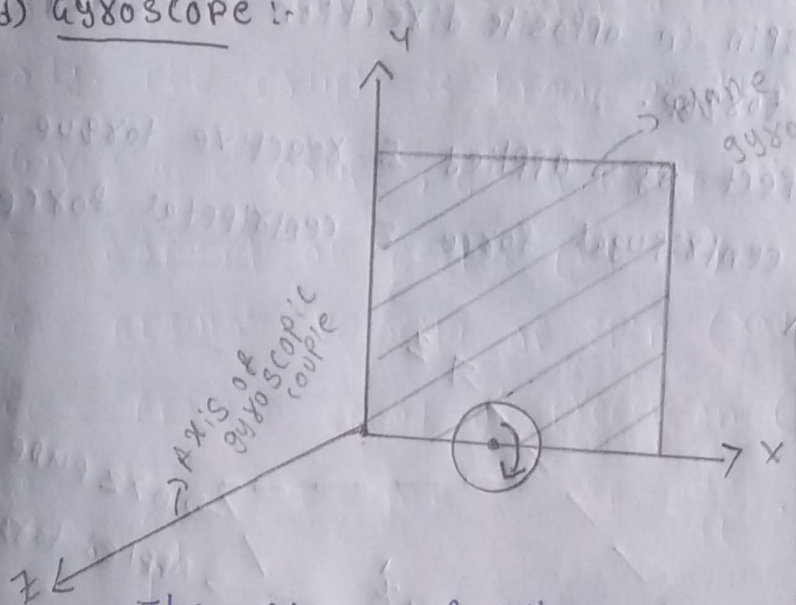


The plane of precession is defined as the plane perpendicular to the axis of precession. The axis of

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precession is the rotation of spining axis from ox to ox' with an angle $\delta\theta$ along the y-axis. The horizontal plane xoz $\perp^{ley}$ to the axis of precession (y-axis) is known as plane of precession.

d) Gyroscope :-



The plane of gyroscopic couple is defined as the plane $\perp^{ler}$ to the axis of reactive gyroscopic couple when an active couple is applied on the rotor. The axis of gyroscopic couple about z-axis & the axis of active gyrocouple which is in vertical plane (x-y) perpendicular to z-axis is known as the plane of gyrocouple.

z-axis (~~axis oy~~) is the axis of gyroscopic couple xoy is a vertical plane called as the plane of gyroscopic couple.

e) Precessional Angular motion :-

When the axis of spin moves in angular direction about the axis of precession, it is called as precessional angular motion.

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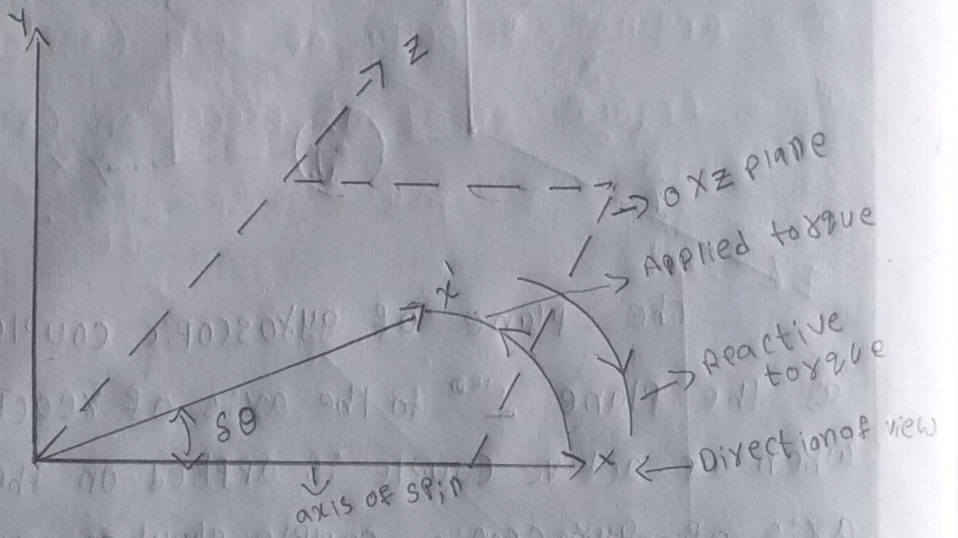
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3) What is meant by applied torque & reactive torque?

A) Applied torque :- The torque required to rotate the axis of spin in clockwise direction is called an applied torque.

Reactive torque :- The torque applied to rotate the axis of spin in opposite direction is called a Reactive torque.

The effect of applied & reactive torque is same as centrifugal force & centripetal force.



4) Explain what do you understand by gyroscopic stabilization?

A) Due to the periodically impinings waves, ship undergoes oscillation. oscillation about longitudinal axis is known as rolling, whearas, abot transverse axis is known as pitching.

But amplitude of rolling is relatively too large than that of pitching. Hence, an arrangement must be provided to reduce the amplitude of rolling. This arrangement is gyroscopic stabilization.

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5) Explain the naval ship movements :-

A) **Bow** :- The front end of the navel ship is known as Bow.

Stern & **aft** :- The rear end of the navel ship is known as stern or aft.

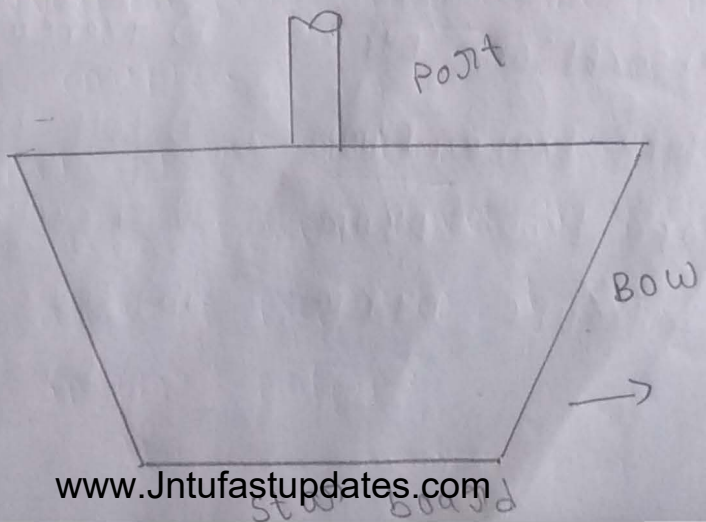
Port :- It is the left side, when looking from the stern.

star board :- It is the Right side, when looking from the stern.

steering :- It is the turning of a complete ship in a curve towards left or Right, while it moves forward.

Pitching :- It is the limited angular motion of the ship about the transverse axis ie., it is the movement of a complete ship up & down in vertical Plane about the axis of precession.

Rolling :- It is the limited angular motion of the ship about the longitudinal axis. In this case the axes of ~~spin~~ precession is parallel to the axis of spin. Hence, no effect of gyroscopic couple acts on the body of a ship.



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