UNIT-1

ECOMONIC OPERATION OF POWER SYSTEMS

Introduction:-

→ The main aim of the economic duration of the power systems is to minimize the total cost of generating real power at various stations.

→ The first problem in power system is called unit commitment problem (UC) and the 2nd is load scheduling 'or' economic dispatch problem (LS)

unit comittment problem:-

→ It is required to select optimally out of the availab-le generating sources to operate to be expected load. over a specified period of time

Economic dispatch problem:- It is also called as online economic dispatch problem where it is required to distribute the load among the generating units paralle with the system in such a manner is as to minimize total cost of the generation minite to minite requirment of the system.

→ The cost of generation is not fixed fo a particular load demand but depends upon the operating conftrains

System conftrains (or) limitations:-

They are two types of system conftrains.

1. Equality conftrains
2. Inequality conftrains.

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Equality constraints :- These constraints arise out of the necenity for the system to balance the load & generation

$$P_i = P_D + P_L$$

Inequality constraints :- These constraints arise due physical and operation limitations of the respective units and components.

→ It is divided in to different types of constraints.

↓ Generator constraints :- For proper operation each generator should have a minimum & minimum units to ensure that

1. real power.

$$P_{imin} \le P_i \le P_{imax} \quad i = 1,2,3 \dots n_p$$

2. Reactive power $Q_{imin} \le Q_i \le Q_{imax} \quad i = 1,2,3 \dots n_q$

→ where 'np' & 'nq' are the total no. of real and reactive power sources.

voltage constraints :- It is essential that the voltage magnitude & phase angles at various loads should vary within limits.

$$V_{imin} \le V_i \le V_{imax} \quad i = 1,2,3 \dots n_r.$$

phase angle :- $\delta_{imin} \le \delta_i \le \delta_{imax} \quad i = 1,2,3 \dots n_s$

Transformer Tap settings :-

1. If an autotransformer is used the minimum tap setting could be zero & maximum is one

$$0 \le t \le 1 \qquad t = \text{tap setting transformer.}$$

2. For two winding transformer if tapings are provided on the secondary side $0 \le t \le n$.

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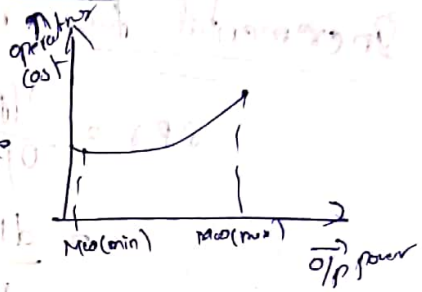
3) For phase shifting transformer the limitations are

$$\theta_{i\min} \le \theta_i \le \theta_{i\max}$$

4) Transmission line constraint :- The flow of active and Reactive power through the transmission line ckt is limited by the thermal capability of the system and it is expressed as $P_{ij} \le P_{ij\max}$ where $P_{ij\max}$ = maximum load in capacity of the line.

optimal operation of generators in thermal power generation :-

→ The total generator operating cost includes fuel, labour & maintance cost & simplicity fuel cost is the only one consider to be variable.

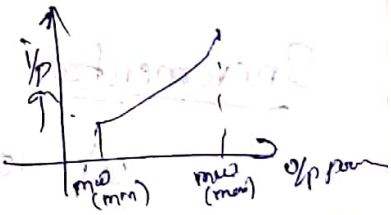


→ The fuel cost mining fuel in case of thermal power energy stored is operated, the operating maximum

i/p o/p characteristics of thermal station :-

i/p o/p characteristics of thermal station

Let us consider generating unit perform curve as shown in fig the generating unit consist of a boiler, turbine, generator only, the fig represent the i/p o/p curve in a power plant for each individual thermal unit

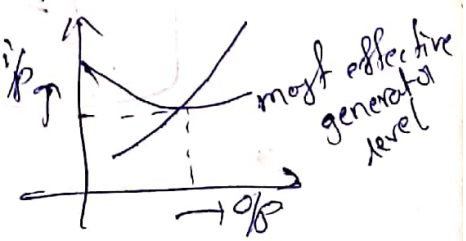


→ o/p unit power (MW) the i/p unit

Incremental rate :- It defined as ratio of i/p to o/p fuel curve

i.e, heat rate curve = $\frac{i/p}{o/p}$

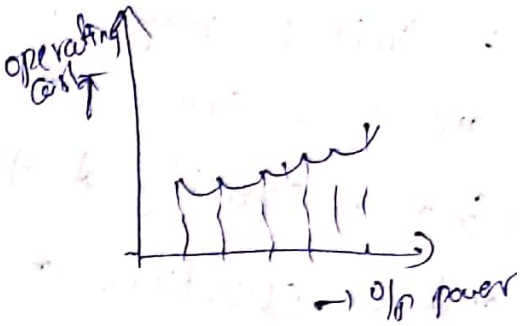
where both heat rate curve and IR are same curve.



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Cost curve :- The day continuity operater on the two situation when it becomes easy to o/p with the help of boiler are steam condenyor.

→ when the cost represent operation in entire power station.



Incremental fuel rate curve :-

$$IFR = \frac{\delta i/p}{\delta o/p}$$

$$= \frac{df(i/p)}{dP(o/p)}$$

$$\boxed{IFR = \frac{dF}{dP}}$$

For cost curve $ICR = \frac{dC}{dP}$

Incremental protection curve :-

It consist of increment fuel cost, labour, water transmitation

efficiency :- The overall efficiency of plant is determined by the

$$\boxed{\eta = \frac{o/p}{i/p}}$$

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optimum Generation allocation with line losses aleglected :-

F_t → it is the total input to the system.

F_n → Fuel input to the n^{th} planet

P_D → Total load demand

P_n → Total generation of n^{th} unit

→ The economic dispatch problem is define as

$$\min F_t = \sum_{n=1}^{N} F_n \longrightarrow (1)$$

It is subjected to equilate constrain.

[Input = output + loss-Neglected]

$$\sum_{n=1}^{N} P_n = P_D \longrightarrow (2)$$

eye of legrangial multiplyer the accel. auxiliary function is expressed as,

$$F = F_t + \lambda \left(P_D - \sum_{n=1}^{N} P_n \right)$$

differentiat w.r.to P_n

$$\frac{dF}{dP_n} = \frac{dF_t}{dP_n} + \lambda \frac{dP_D}{dP_n} - \lambda \frac{d\left(\sum_{n=1}^{N} P_n\right)}{dP_n}$$

when the $\frac{dF}{dP_n} = 0 \quad \lambda \frac{dP_D}{dP_n} = 0.$

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$$0 = \frac{dF_t}{dP_n} + 0 - \lambda(1)$$

$$0 = \frac{dF_t}{dP_n} - \lambda$$

$$\boxed{\lambda = \frac{dF_t}{dP_n}} \quad (\text{or}) \quad \boxed{\lambda = \frac{dC_t}{dP_n}}$$

Similarly for cost function. $\lambda = \frac{dC_t}{dP_n}$

unit :- rupees / mega watt hr (or) Rs/mwhr

→ The condition of optimum operation is,

$$\boxed{\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \cdots = \frac{dF_n}{dP_n} = \lambda}$$

Similarly for cost function

$$\boxed{\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \cdots = \frac{dC_n}{dP_n} = \lambda}$$

λ = legranigeal multiplyer.

$\frac{dF_n}{dP_n}$ = Incremental production cost of n^{th} unit

∴ The above eqn is called as "co-ordination

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problems:-

method ① :-

① The fuel cost of two units are given by

$$C_1 = 1.5 + 20P_{G_1} + 0.1P_{G_1}^2 \text{ Rs/hr}$$

$$C_2 = 1.9 + 30P_{G_2} + 0.1P_{G_2}^2 \text{ Rs/hr}.$$

if the total demand of the generators is 200mwatts. Find the economic load scheduling of the two units.

Sol?:-

Given data,

$$C_1 = 1.5 + 20P_{G_1} + 0.1P_{G_1}^2 \text{ Rs/hr}$$

$$C_2 = 1.9 + 30P_{G_2} + 0.1P_{G_2}^2 \text{ Rs/hr}.$$

$P_{G_1} + P_{G_2} = 200\text{mw}$. $P_{G_1} = ?$ $P_{G_2} = ?$ $\lambda = ?$

formula,

$$\frac{dC_1}{dP_{G_1}} = \frac{dC_2}{dP_{G_2}} = \lambda$$

$$\frac{dC_1}{dP_{G_1}} = 0 + 20 + 0.2P_{G_1}$$

$$\frac{dC_1}{dP_{G_1}} = 20 + 0.2P_{G_1} \rightarrow ①$$

$$\frac{dC_2}{dP_{G_2}} = 30 + 0.2P_{G_2} \rightarrow ②$$

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① = ②

$$\frac{dC_1}{dP_{G_1}} = \frac{dC_2}{dP_{G_2}}$$

$$20 + 0.2 P_{G_1} = 30 + 0.2 P_{G_2}$$

$$0.2 P_{G_1} - 0.2 P_{G_2} = 30 - 20$$

$$(P_{G_1} - P_{G_2})\, 0.2 = 10$$

$$P_{G_1} - P_{G_2} = \frac{100}{2}$$

$$P_{G_1} - P_{G_2} = 50 \rightarrow (3)$$

$$P_{G_1} - P_{G_2} = 50 \rightarrow (5)$$

$$P_{G_1} + P_{G_2} = 200 \rightarrow (4)$$

$$P_{G_1} = 125 \text{ MW}$$

$$P_{G_2} = 75 \text{ MW}$$

$$\frac{dC_1}{dP_{G_1}} = \lambda \; ; \; \frac{dC_2}{dP_{G_2}} = \lambda$$

$$\frac{dC_1}{dP_{G_1}} = 20 + 0.2 P_{G_1}$$

$$= 20 + 0.2 (125)$$

$$\frac{dC_1}{dP_{G_1}} = 45$$

$$\lambda = 45$$

$$\frac{dC_2}{dP_{G_2}} = 30 + 0.2 (75)$$

$$\lambda = 45$$

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⑨ The fuel input per hour of plant ① & ② are given by $C_1 = 0.2 P_{G_1}^2 + 40 P_{G_1} + 120$ Rs/hr.

$C_2 = 0.25 P_{G_2}^2 + 30 P_{G_2} + 150$ Rs/hr. If the total demand on the generators is 180MW. Find the economic load schedule of the units

Soln:- Given data,

$C_1 = 0.2 P_{G_1}^2 + 40 P_{G_1} + 120$ Rs/hr

$C_2 = 0.25 P_{G_2}^2 + 30 P_{G_2} + 150$ Rs/hr.

$P_{G_1} + P_{G_2} = 180$.

$$\frac{dC_1}{dP_{G_1}} = \frac{dC_2}{dP_{G_1}} = \lambda$$

$$\frac{dC_1}{dP_{G_1}} = 0.4 P_{G_1} + 40$$

$$\frac{dC_2}{dP_{G_2}} = 0.5 P_{G_2} + 30.$$

$$\frac{dC_1}{dP_{G_1}} = \frac{dC_2}{dP_{G_2}}$$

$$0.4 P_{G_1} + 40 = 0.5 P_{G_2} + 30.$$

$$0.4 P_{G_1} - 0.5 P_{G_2} = -10.$$

$$P_{G_1} + P_{G_2} = 180.$$

$P_{G_1} = 88.8$; $P_{G_2} = 91.1$

$$\frac{dC_1}{dP_{G_1}} = 0.4(88.8) + 40 = 75.52.$$

$\lambda = 75.52$

$$\frac{dC_2}{dP_{G_2}} = 0.5(91.1) + 30$$

$\lambda = 75.55$

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(3) A power system consisting of 200 MW units whose input cost data are represented by equations below

$C_1 = 0.04 P_1^2 + 22 P_1 + 800$ Rs/hr.

$C_2 = 0.045 P_2^2 + 15 P_2 + 1000$ Rs/hr.

If the total received power $P_R = 150$ MW. determine load sharing the two units for most economic operation.

Sol:-

$C_1 = 0.04 P_1^2 + 22 P_1 + 800$ Rs/hr

$C_2 = 0.045 P_2^2 + 15 P_2 + 1000$ Rs/hr.

$P_1 + P_2 = 150$ MW.

$\frac{dC_1}{dP_1} = 0.08 P_1 + 22$.

$\frac{dC_2}{dP_2} = 0.09 P_2 + 15$.

$0.08 P_1 + 22 = 0.09 P_2 + 15$

$0.08 P_1 - 0.09 P_2 = -7$

$P_1 + P_2 = 150$.

$P_1 = 38.23$ MW $\quad P_2 = 111.76$ MW.

$\frac{dC_1}{dP_1} = 0.08(38.23) + 22$

$\lambda = 25.05$ Rs/MWhr

$\frac{dC_2}{dP_2} = 0.09(111.76) + 15$

$\lambda = 25.05$ Rs/MWhr

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④ $C_1 = 1.6 + 25P_1 + 0.1P_1^2$ & $C_2 = 2.1 + 32P_2 + 0.1P_2^2$

If the load demand is = 250 MW

Soln:-

$$\frac{dC_1}{dP_1} = 25 + 0.2P_1$$

$$\frac{dC_2}{dP_2} = 32 + 0.2P_2$$

$$25 + 0.2P_1 = 32 + 0.2P_2$$

$$0.2P_1 - 0.2P_2 = 7$$

$$P_1 + P_2 = 250.$$

$$P_1 = 142.5 \quad P_2 = 107.5$$

$$\frac{dC_1}{dP_1} = 25 + 0.2(142.5)$$

$$\lambda = 53.5 \text{ Rs/hr}$$

$$\frac{dC_2}{dP_2} = 32 + 0.2(107.5)$$

$$\lambda = 53.5 \text{ Rs/hr}$$

⑤ Incremental fuel cost in Rs/hr for 2 units are given by. $\frac{dF_1}{dP_1} = 0.075P_1 + 15$

$\frac{dF_2}{dP_2} = 0.085P_2 + 12$. If the total load of 200 MW. Find out P_1 & P_2 & λ.

Soln:- Given data

$$\frac{dF_1}{dP_1} = 0.075P_1 + 15$$

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$$\frac{dP_2}{dP_2} = 0.085 P_2 + 12.$$

$$\frac{dP_1}{dP_1} = \frac{dP_2}{dP_2}$$

$$0.075 P_1 + 15 = 0.085 P_2 + 12$$

$$0.075 P_1 - 0.085 P_2 = -3.$$

$$P_1 + P_2 = 200.$$

$$\boxed{P_1 = 87.5 \text{ MW}} \quad \boxed{P_2 = 112.5 \text{ MW}}$$

$$\frac{dP_1}{dP_1} = 0.075(87.5) + 15$$

$$\boxed{\lambda = 21.56 \text{ Rs/MWhr}}$$

$$\frac{dP_2}{dP_2} = 0.085(112.5) + 12$$

$$\boxed{\lambda = 21.56 \text{ Rs/MWhr}}$$

Method (2)

(1) A generating station has units having the following fuel cost rupees per MWhr

$$\frac{dC_1}{dP_{G1}} = 0.24 P_{G1} + 60$$

$$\frac{dC_2}{dP_{G2}} = 0.20 P_{G2} + 45$$

$$\frac{dC_3}{dP_{G3}} = 0.30 P_{G3} + 75$$: The total load of 500 MW for station. determine.

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① The real power to be generated by each unit ② The station. ① For most economic operation.

Soln:-

$$\frac{dC_1}{dP_{G_1}} = 0.24 P_{G_1} + 60 \rightarrow ①$$

$$\frac{dC_2}{dP_{G_2}} = 0.20 P_{G_2} + 45 \rightarrow ②$$

$$\frac{dC_3}{dP_{G_3}} = 0.30 P_{G_3} + 75 \rightarrow ③$$

$P_{G_1} + P_{G_2} + P_{G_3} = 500 \rightarrow ⑥$

$P_{G_1} = ?$

$P_{G_2} = ?$

$P_{G_3} = ?$

$$\frac{dC_1}{dP_{G_1}} = \frac{dC_2}{dP_{G_2}} = \frac{dC_3}{dP_{G_3}} = \lambda$$

①

① & ② $\frac{dC_1}{dP_{G_1}} = \frac{dC_2}{dP_{G_2}}$

$$0.24 P_{G_1} + 60 = 0.20 P_{G_2} + 45$$

$$0.24 P_{G_1} - 0.20 P_{G_2} + 15 \rightarrow ④$$

④ = ③

$$\frac{dC_1}{dP_{G_1}} = \frac{dC_3}{dP_{G_3}}$$

$$0.24 P_{G_1} + 60 = 0.30 P_{G_3} + 75$$

$$0.24 P_{G_1} - 0.30 P_{G_3} = 15 \rightarrow ⑤$$

⑥ $P_{G_3} = 500 - (P_{G_1} + P_{G_2}) \rightarrow ⑦$

Sub ⑦ in ⑤

$$0.24 P_{G_1} - 0.30 (500 - P_{G_1} - P_{G_2}) = 15$$

$$0.24 P_{G_1} - 150 + 0.30 P_{G_1} + 0.30 P_{G_2} = 15$$

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$0.54 P_{G_1} + 0.30 P_{G_2} = 165 \rightarrow (3)$

$0.24 P_{G_1} + 0.20 P_{G_2} = 115 \rightarrow (4)$

$\boxed{P_{G_1} = 158.3 \text{ MW}}$ $\boxed{P_{G_2} = 265 \text{ MW}}$

$P_{G_3} = 500 - P_{G_1} - P_{G_2}$

$\boxed{P_{G_3} = 76.7 \text{ MW}}$

$\lambda = \frac{dC_1}{dP_{G_1}} = 0.24 P_{G_1} + 60$

$= 0.24 (158.3) + 60$

$\boxed{\lambda = 97.9 \text{ Rs/MWhr}}$

$\frac{dC_2}{dP_{G_2}} = 0.20 P_{G_2} + 45$

$= 0.20 (265) + 45$

$\boxed{\lambda = 98.8 \text{ Rs/MWhr}}$

$\frac{dC_3}{dP_{G_3}} = 0.30 P_{G_3} + 75$

$= 0.30 (76.7) + 75$

$\boxed{\lambda = 98.0 \text{ Rs/MWhr}}$

(7) $F_1 = 350 + 7.2 P_1 + 0.004 P_1^2$

$F_2 = 500 + 7.3 P_2 + 0.0025 P_2^2$

$F_3 = 600 + 6.74 P_3 + 0.003 P_3^2$

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(i) If the total load demand $P_D = 800$ MW

(ii) If the " " " $P_D = 450$ MW.

Soln:-

$$\frac{dF_1}{dP_1} = 7.2 + 0.008 P_1 \rightarrow (1)$$

$$\frac{dF_2}{dP_2} = 7.3 + 0.005 P_2 \rightarrow (2)$$

$$\frac{dF_3}{dP_3} = 6.74 + 0.006 P_3 \rightarrow (3)$$

(i) $P_1 + P_2 + P_3 = 800 \rightarrow (4)$

(1) & (2)

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

$$7.2 + 0.008 P_1 = 7.3 + 0.005 P_2$$

$$0.008 P_1 - 0.005 P_2 = 0.1 \rightarrow (3)$$

(4) & (3).

$$\frac{dF_1}{dP_1} = \frac{dF_3}{dP_3}$$

$$7.2 + 0.008 P_1 = 6.74 + 0.006 P_3$$

$$0.008 P_1 - 0.006 P_3 = -0.46 \rightarrow (4)$$

Eqn (4) $\Rightarrow$ $P_3 = 800 - (P_1 + P_2) \rightarrow (5)$

Sub (5) in (4)

$$0.008 P_1 - 0.006 (800 - P_1 - P_2) = -0.46$$

$$0.008 P_1 - 4.8 + 0.006 P_1 + 0.006 P_2 = -0.46$$

$$0.014 P_1 + 0.006 P_2 = 4.34 \rightarrow (6)$$

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③ & ⑥

$$0.008P_1 - 0.005P_2 = 0.1$$

$$0.014P_1 + 0.006P_2 = 4.34$$

$$\boxed{P_1 = 188.9\ MW} \quad \boxed{P_2 = 282.3\ MW}$$

$$\boxed{P_3 = 328.8\ MW}$$

$$\frac{dF_1}{dP_1} = 7.2 + 0.008P_1$$

$$= 7.2 + 0.008(188.9)$$

$$\boxed{\lambda = 8.7\ Rs/MWhr}$$

$$\frac{dF_2}{dP_2} = 7.3 + 0.005P_2$$

$$= 7.3 + 0.005(282.3)$$

$$\boxed{\lambda = 8.7\ Rs/MWhr}$$

$$\frac{dF_3}{dP_3} = 6.74 + 0.006(328.8)$$

$$\boxed{\lambda = 8.7\ Rs/MWhr}$$

⑤

ii) $\frac{dC_1}{dP_1} = 0.1P_1 + 20$

$\frac{dC_2}{dP_2} = 0.1P_2 + 16$

$P_1 + P_2 + P_3 = 450$

$P_3 = 450 - P_1 - P_2$

$$0.008P_1 - 0.006(450 - P_1 - P_2) = -0.46$$

$$0.008P_1 - 2.7 + 0.006P_1 + 0.006P_2 = -0.46$$

$$0.014P_1 + 0.006P_2 = 2.24 \quad \rightarrow (7)$$

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(5) 8 (4)

$0.008P_1 - 0.005P_2 = 0.1$

$0.014P_1 + 0.006P_2 = 2.24$

$P_1 = 100\,MW$ $\quad P_2 = 140\,MW$

$P_3 = 210\,MW$

$\frac{dF_1}{dP_1} = 7.2 + 0.008(100)$

$\lambda = 8\,Rs/MWhr$

$\frac{dF_2}{dP_2} = 7.3 + 0.005(140)$

$\lambda = 8\,Rs/MWh$

$\frac{dF_3}{dP_3} = 6.74 + 0.006(210)$

$\lambda = 8\,Rs/MWhr$

H.W

$\frac{dC_2}{}$

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Method ③:-

① Three power plants of total capacity 425 MW are scheduled for operation to supply total system load of 300 MW. Find the optimum load schedule if the plants have the following incremental cost characteristic

$$\frac{dC_1}{dP_1} = 30 + 0.15P_1 \; ; \; 25 \le P_1 \le 175$$

$$\frac{dC_2}{dP_2} = 40 + 0.2P_2 \; ; \; 30 \le P_2 \le 100$$

$$\frac{dC_3}{dP_3} = 15 + 0.18P_3 \; ; \; 50 \le P_3 \le 200$$

where, $\frac{dC_i}{dP_i}$ is in Rs/mwhr for $i = 1,2,3$.

Soln:-

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \frac{dC_3}{dP_3} = \lambda$$

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2}$$

$$30 + 0.15P_1 = 40 + 0.2P_2$$

$$0.15P_1 - 0.2P_2 = 10. \longrightarrow ①$$

$$\frac{dC_1}{dP_1} = \frac{dC_3}{dP_3}.$$

$$30 + 0.15P_1 = 15 + 0.18P_3.$$

$$0.15P_1 - 0.18P_3 = -15 \longrightarrow ②$$

$$P_1 + P_2 + P_3 = 300. \Rightarrow P_3 = 300 - P_1 - P_2$$

$$0.15P_1 - 0.18(300 - P_1 - P_2) = -15.$$

$$0.15P_1 - 54 + 0.18P_1 + 0.18P_2 = -15.$$

$$0.33P_1 + 0.18P_2 = 39. \longrightarrow ③$$

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$0.15P_1 - 0.2P_2 = 10$

$0.33P_1 + 0.18P_2 = 39$

$P_1 = 103.9$ MW $P_2 = 27.$ MW

$P_3 = 169.39$ MW

Since the value of P_2 obtaine is lower then the permisable lower limit of 30 MW

→ Therefore, fix the value of P_2 at 30 MW.

$P_2 = 30$ MW. $P_1 + P_2 + P_3 = 300.$

eq (2) $\therefore P_1 + P_3 = 300 - 30$

$P_1 + P_3 = 270.$ MW.

$0.15P_1 - 0.18P_3 = -15$

$P_1 = 101.8$ MW $P_3 = 168.1$ MW

so, The ecanamic load scheduling of the three units are

$P_1 = 101.8$ MW

$P_2 = 30$ MW

$P_3 = 168.1$ MW

(2) 3 Plants of total capacity 500 MW are scheduled for operation to supply a total system load of 310 MW evaluate the optimum load schaduling if the plants have the following Incremental fuel characteristics and the limitation

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$\frac{dC_1}{dP_1} = 30 + 0.12P_1 \quad ; \quad 30 \le P_1 \le 150$ Rs/Mwhr.

$\frac{dC_2}{dP_2} = 40 + 0.20P_2 \quad ; \quad 20 \le P_2 \le 100$ Rs/Mwhr

$\frac{dC_3}{dP_3} = 10 + 0.15P_3 \quad ; \quad 50 \le P_3 \le 250$ Rs/Mwhr

Soln :-

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2}$$

$$30 + 0.12P_1 = 40 + 0.20P_2$$

$$0.12P_1 - 0.20P_2 = 10. \rightarrow (1)$$

$$\frac{dC_1}{dP_1} = \frac{dC_3}{dP_3}$$

$$30 + 0.12P_1 = 10 + 0.15P_3$$

$$0.12P_1 - 0.15P_3 = -20. \rightarrow (2)$$

$$P_1 + P_2 + P_3 = 310 \text{ MW}.$$

$$P_3 = 310 - P_1 - P_2 \rightarrow (3)$$

Sub (2) in (3).

$$0.12P_1 - 0.15(310 - P_1 - P_2) = -20$$

$$0.12P_1 - 46.5 + 0.15P_1 + 0.15P_2 = -20.$$

$$0.27P_1 + 0.15P_2 = 26.5 \rightarrow (4)$$

$$0.12P_1 - 0.20P_2 = 10.$$

$$\boxed{P_1 = 94.9 \text{ MW}} \quad \boxed{P_2 = 6.66 \text{ MW}}$$

$$\boxed{P_3 = 208.9 \text{ MW}}$$

Since the value of P_2 obtained is lower than the permissible lower limit of 20MW.

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P_2 is 20 MW.

$$P_1 + P_2 + P_3 = 310$$

$$P_1 + P_3 = 290 \text{ MW}$$

$$0.12P_1 + 0.15P_3 = 40$$

$P_1 = 177.5$ MW $P_3 = 112.5$ MW.

(3) $C_1 = 800 + 45P_{G_1} + 0.01P_{G_1}^2$ $50 \text{ MW} \le P_{G_1} \le 200$ MW.

$C_2 = 200 + 43P_{G_2} + 0.003P_{G_2}^2$ $50 \text{ MW} \le P_{G_2} \le 600$ MW.

Total load is 700 MW find the optimal with & without considering generator limits have been expressed as,

$50 \text{ MW} \le P_{G_1} \le 200 \text{ MW}$.

$50 \text{ MW} \le P_{G_2} \le 600 \text{ MW}$.

Soln:

$$\frac{dC_1}{dP_{G_1}} = 45 + 0.02P_{G_1}$$

$$\frac{dC_2}{dP_{G_2}} = 43 + 0.006P_{G_2}$$

$$\frac{dC_1}{dP_{G_1}} = \frac{dC_2}{dP_{G_2}}$$

$$45 + 0.02P_{G_1} = 43 + 0.006P_{G_2}$$

$$0.02P_{G_1} - 0.006P_{G_2} = -2$$

$$P_{G_1} + P_{G_2} = 700 \text{ MW}.$$

$P_{G_1} = 84.6$ MW $P_{G_2} = 615.3$ MW

Taken limit $P_{G_2} = 600$.

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$P_{G_1} + P_{G_2} = 700$.

$P_{G_1} + 600 = 700$

$\boxed{P_{G_1} = 100 \text{ MW}}$ $\boxed{P_{G_2} = 600 \text{ MW}}$

~~Problem~~

(4) The cost function of a 50MW generator is obtained and is given below ~~if~~

$C_i = 225 + 53 P_{G_i} + 0.02 P_{G_i}^2$

where P_{G_i} is the generator loading. Find.

(i) Incremental fuel cost IFC

(ii) IFC when 100% load applying

(iii) IFC when 50% load is apply.

Sol:- $C_i = 225 + 53 P_{G_i} + 0.02 P_{G_i}^2$

$\frac{dC_i}{P_{G_i}} = 53 + 0.04 P_{G_i}$

(ii) 100% load is IFC becomes

$P_{G_i} = 50$.

$\frac{-dC_i}{dP_{G_1}} = 53 + 0.04(50)$

$\boxed{\frac{dC_i}{dP_{G_1}} = 55 \text{ Rs/MWhr}}$

(iii) 50% $P_{G_i} = 25$.

$\frac{dC_i}{dP_{G_1}} = 53 + 0.04(25)$

$\boxed{\frac{dC_1}{dP_{G_1}} = 54 \text{ Rs/MWhr}}$

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Method (II) :-

① The incremental fuel inputs for a unit steam plant are given by

$$\frac{dC_1}{dP_1} = 0.08P_1 + 15 \quad Rs/MWhr$$

$$\frac{dC_2}{dP_2} = 0.08P_2 + 20 \quad Rs/MWhr$$

Find the loss in operating economy if the two units share a load of 300 MW equally instead of operating in an optimum day.

Sol:- 6/7/19

Given that

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2}$$

$$0.08P_1 + 15 = 0.08P_2 + 20$$

$$0.08P_1 - 0.08P_2 = 5$$

$$P_1 + P_2 = 300$$

$$\boxed{P_1 = 181.25 MW} \quad \boxed{P_2 = 118.75 MW}$$

If the total load of 300 MW equally divided because of equally injected operating units that's why

$P_1 = 150 MW$; $P_2 = 150 MW$.

For unit (1) :- $\int_{\text{economic}}^{\text{equal}} \frac{dC_1}{dP_1} dP_1 = \int_{}^{181.2}$

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For economic $P_1 = 181.2$ MW

For equal $P_1 = 150$ MW.

$$= \int_{181.2}^{150} (0.08P_1 + 15)\, dP_1$$

$$= \left[\frac{0.08P_1^2}{2} + 15P_1\right]_{181.2}^{150}$$

$$= (0.04P_1^2 + 15P_1)_{181.2}^{150}$$

$$= \left[0.04(150)^2 + 15(150)\right] - \left[0.04(181.2)^2 + 15(181.2)\right]$$

$$= 3150 - 4031.3$$

$$= -881.3 \text{ Rs/hr}$$

∴ The '−' indicates that there is a reduction in cost.

For unit ② :- $\int_{\text{economic}}^{\text{equal}} \frac{dC_2}{dP_2} \times dP_2$ = economic $P_2 = 118.7$, equal $P_2 = 150$.

$$= \int_{118.7}^{150} (0.08P_2 + 20)\, dP_2$$

$$= \left[\frac{0.08P_2^2}{2} + 20P_2\right]_{118.7}^{150}$$

$$= \left[0.04P_2^2 + 20P_2\right]_{118.7}^{150}$$

$$= \left[0.04(150)^2 + 20(150)\right] - \left[0.04(118.7)^2 + 20(118.7)\right]$$

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$= 3900 - 2937.5$

$= 962.4$ Rs/hr.

The net loss in operating economy due to equal load sharing instead of operating unit in an optimum way is

$= 962.4 - 881.3$

$= 81.1$ Rs/hr.

(2) A constant load of 300 MW is supplied two 200 MW generators (1) & (2) for which the respective incremental fuel cost are.

$$\frac{dC_1}{dP_{G_1}} = 0.10 P_{G_1} + 20$$

$$\frac{dC_2}{dP_{G_2}} = 0.12 P_{G_2} + 15$$

with the power P_G in MW and cost in Rs/hr. determine

(i) the most economical division of load b/w the two generators.

(ii) The saving in Rs/day their by obtained compare. to equal load sharing b/w two generators.

Sol:-

$$\frac{dC_1}{dP_{G_1}} = 0.10 P_{G_1} + 20$$

$$\frac{dC_2}{dP_{G_2}} = 0.12 P_{G_2} + 15$$

$P_{G_1} + P_{G_2} = 300$ MW.

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$$\frac{dC_1}{dP_{G_1}} = \frac{dC_2}{dP_{G_2}}$$

$$0.01 P_{G_1} + 20 = 0.12 P_{G_2} + 15$$

$$0.01 P_{G_1} - 0.12 P_{G_2} = -5$$

$$P_{G_1} + P_{G_2} = 300 MW$$

$P_{G_1} = 140 MW$ $P_{G_2} = 160 MW$

(i) unit ① :- $P_{G_1} = 150 MW$ $P_{G_1} = 150 MW$.

equal

$$\int_{140}^{150} \frac{dC_1}{dP_{G_1}} dP_{G_1} = \int_{140}^{150} (0.10 P_{G_1} + 20) dP_{G_1}$$

economic

$$= \left[\frac{0.10 P_{G_1}^2}{2} + 20 P_{G_1} \right]_{140}^{150}$$

$$= \left[0.05 P_{G_1}^2 + 20 P_{G_1} \right]_{140}^{150}$$

$$= \left[0.05 (150)^2 + 20(150)\right] - \left[0.05 (140)^2 + 20(140)\right]$$

$$= 4125 - 3780.$$

$$= 345 \text{ Rs/hr.}$$

unit ②

$$\int_{160}^{150} \frac{dC_2}{dP_{G_2}} dP_{G_2} = \int_{160}^{150} (0.12 P_{G_2} + 15) dP_{G_2}$$

$$= \left[\frac{0.12}{2} P_{G_2}^2 + 15 P_{G_2} \right]_{160}^{150}$$

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Method-5 :-

① Each of two units of 200 mw in a thermal power plant on operating all the time throughout the year the max & min load on each unit is 200 mw & 50 mw respectively. The incremental fuel cost characteristics for the two units are

$$\frac{dC_1}{dP_{G_1}} = 15 + 0.08 P_{G_1} \text{ Rs/mwhr}$$

$$\frac{dC_2}{dP_{G_2}} = 13 + 0.1 P_{G_2} \text{ Rs/mwhr}$$

If the total load varies b/w 100 mw & 400 mw. Find the incremental fuel cost and allocation of load to two units for minimum total fuel cost for various total loads.

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Soln:-

$$\frac{dC_1}{dP_{G_1}} = \frac{dC_2}{dP_{G_2}} = \lambda$$

$$\frac{dC_1}{dP_{G_1}} = \lambda$$

$P_{G_1} \neq 0$

For min load $P_{G_1} = 50$ MW on unit 1

$$15 + 0.08 P_{G_1} = \lambda$$

$$15 + 0.08(50) = \lambda$$

$$\boxed{\lambda = 19 \text{ Rs/MWhr}}$$

$$\frac{dC_2}{dP_{G_2}} = \lambda$$

$$13 + 0.1 P_{G_2} = \lambda$$

$$13 + 0.1(50) = \lambda$$

$$\boxed{\lambda = 18 \text{ Rs/MWhr}}$$

→ $\lambda = 19$ Rs/MWhr is placed on unit ② then P_{G_2} value is,

$$13 + 0.1 P_{G_2} = \lambda$$

$$13 + 0.1 P_{G_2} = 19$$

$$P_{G_2} = \frac{6}{0.1}$$

$$\boxed{P_{G_2} = 60 \text{ MW}}$$

$$15 + 0.08 P_{G_1} = \lambda$$

$$15 + 0.08 P_{G_1} = 19$$

$$0.08 P_{G_1} = 4$$

$$P_{G_1} = \frac{4}{0.08}$$

$$\boxed{P_{G_1} = 50 \text{ MW}}$$

unit ① $\lambda = 19$ Rs/MWhr and delivers 50MW.

unit ② $\lambda = 19$ Rs/MWhr and delivers 60MW.

∴ The total load at equal incremental cost(λ) of 19 Rs/MWhr is 110 MW (60 + 50)

Now, we can assume various values and calculating the o/p for each unit.

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the process is continued until each time that the load on unit ① reaches the max. value 200MW.

λ (Rs/MWh)	P_{G_1} (MW)	P_{G_2} (MW)	$P_{G_1}+P_{G_2}$ (MW)	
18	37	50	87	uneconomical
19	50	60	110	economical operation
20	62.5	70	132.5	
22	87	90	177	
25	125	120	245	
28	162.5	150	312.5	
29	175	160	335	
30	187.5	170	357.5	
32	212.5	190	402.5	uneconomical

$\lambda = 20$

$15 + 0.08 P_{G_1} = 20$

$P_{G_1} = \frac{5}{0.08}$

$P_{G_1} = 62.5$ MW.

$13 + 0.1 P_{G_2} = 20$

$P_{G_2} = \frac{7}{0.1}$

$P_{G_2} = 70$ MW.

$\lambda = 25$

$15 + 0.08 P_{G_1} = 25$

$P_{G_1} = \frac{10}{0.08}$

$= 125$ MW

$13 + 0.1 P_{G_2} = 25$

$P_{G_2} = 120$ MW

$\lambda = 29$

$15 + 0.08 P_{G_1} = 29$

$P_{G_1} = 175$ MW

$13 + 0.1 P_{G_2} = 29$

$P_{G_2} = 160$ MW

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(2) Incremental fuel cost in Rs/MWhr for a plant consisting of two units are $\frac{dC_1}{dP_1} = 0.2P_1 + 40$ Rs/MWhr

$\frac{dC_2}{dP_2} = 0.25P_2 + 30$ Rs/MWhr

Assume that both the units are operating at all times and total load varies from 40 MW to 250 MW and the max loads on each unit are to be 125 and 200 MW respectively. How will the load be shared by the two units as a system load varies over the full range? what are the corresponding values of the plant incremental cost.

Sol:- $\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \lambda$

for min load.

$P_1 = 20$

$\frac{dC_1}{dP_1} = \lambda$

$0.2(20) + 40 = \lambda$

$\lambda = 44$ Rs/MWhr

$P_2 = 20$

$\frac{dC_2}{dP_2} = \lambda$

$0.25(20) + 30 = \lambda$

$\lambda = 35$ Rs/MWhr

$\lambda = 44$

$0.2P_1 + 40 = 44$

$0.2P_1 = 4$

$P_1 = 20$ MW

$0.25P_2 + 30 = 44$

$0.25P_2 = 14$

$P_2 = \frac{14}{0.25}$

$P_2 = 56$ MW

$P_1 + P_2 = 56$

$\lambda = 35$

$0.2P_1 + 40 = 35$

$P_1 = -25$ MW

$0.25P_2 + 30 = 35$

$P_2 = \frac{5}{0.25}$

$P_2 = 20$ MW

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λ	P_1 (MW)	P_2 (MW)	P_1+P_2	
35	25	20		uneconomical
44	20	56	76	
50	50	80	130	
55	75	100	175	economical condition
60	100	120	220	
61	105	124	229	
62	110	128	238	
65	125	140	265	uneconom.

$\lambda = 55$

$0.2P_1 + 40 = 55$; $0.25P_2 + 30 = 55$

$P_1 = 75$ MW

$P_2 = 100$ MW

$\lambda = 62$

$0.2P_1 + 40 = 62$; $0.25P_2 + 30 = 62$

$P_1 = 110$ MW

$P_2 = 128$ MW

$\lambda = 65$

$0.2P_1 + 40 = 65$; $0.25P_2 + 30 = 65$

$P_1 = 125$ MW

$P_2 = 140$ MW

$\lambda = 61$

$0.2P_1 + 40 = 61$; $0.25P_2 + 30 = 61$

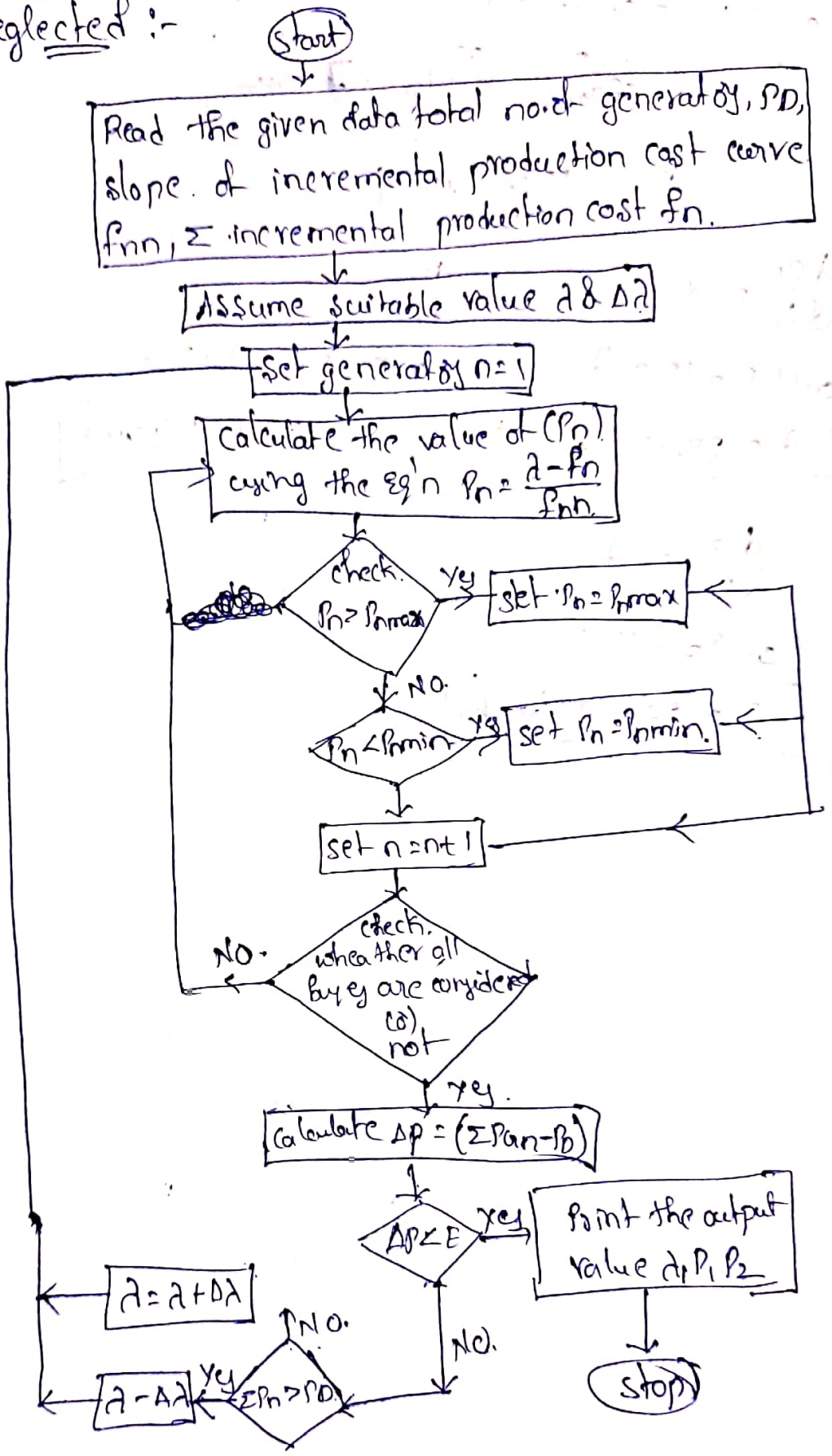
$P_1 = 105$ MW

$P_2 = 124$ MW

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10/7/19

optimum generation allocation with line losses neglected :-



optimum flowchart for obtaining optimal scheduling unit by Neglecting the Transmission losses.

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Inorder to draw the flowchart for obtaining optimum scheduling of generating units (Neglecting losses) we need to prepare the following algorithm.

step ① :- Assume a suitable value of lagrangeal multiplyer 'λ' This value should be more then the largest intercept of the incremental characteristics of the various generators such that $\lambda \geq F_i$ where F_i = highest incremental production cost of i^{th} unit

step ② :- Depending upon incremental production cost calculate the individual generators from the eqn

$$\frac{dF_n}{dP_n} = \lambda \text{ (or) } \frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \cdots = \frac{dF_n}{dP_n} = \lambda \longrightarrow (1)$$

$$\frac{dF_1}{dP_1} = \lambda \; ; \; \frac{dF_2}{dP_2} = \lambda$$

The above eqn is called as Co-ordination eqn.

step ③ :- with the help of eqn check the equality $\sum_{i=1}^{N} P_{gn} = P_D \longrightarrow (2)$ is satisfied (input = output)

step ④ :- If the eqn (2) in step ③ is not satisfy then assuming 2nd case for the legrazial multiplayer $\lambda = \lambda'$ and repeat the above step's.

step ⑤ :- Now, step ③ is satisfy then the individual generations are calculated using co-ordination eqn in step ② which give the optimum operating strategi.

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$\frac{dC_3}{dP_3} = 15$

11/7/19

optimum generation allocation including T.L losses

~~neglecting~~

$$P_T = \sum_{i=1}^{n} P_i \rightarrow (1)$$

P_T = Total fuel i/p to the system

P_i = Fuel i/p to the i^{th} plant

P_{Gi} = Total generation for i^{th} plant

P_D = Total load demand

P_L = Total power loss

The mathematical formulation is Now expressed

as,

$$\min P_T = \sum_{i=1}^{n} P_i \rightarrow (1)$$

with, i/p = o/p + losses

then,

$$\sum_{i=1}^{n} P_{Gi} = P_D + P_L$$

$$P_D + P_L - \sum_{i=1}^{n} P_{Gi} = 0 \rightarrow (2)$$

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where, P_L =

The auxiliary function are legrangial multiplyer the function is now written as,

$$L = F_T + \lambda\left(P_D + P_L - \sum_{i=1}^{n} P_{Gi}\right)$$

Take the partial differention w.r.to P_{Gi}

$$\therefore \frac{\partial L}{\partial P_{Gi}} = \frac{dF_T}{\partial P_{Gi}} + \lambda \frac{\partial P_D}{\partial P_{Gi}} + \lambda \frac{\partial P_L}{\partial P_{Gi}} - \lambda \frac{\partial\left(\sum_{i=1}^{n} P_{Gi}\right)}{\partial P_{Gi}} \rightarrow (1)$$

$\frac{\partial L}{\partial P_{Gi}} \rightarrow 0$; $\frac{\partial P_D}{\partial P_{Gi}} \rightarrow 0$

$$\therefore 0 = \frac{\partial F_T}{\partial P_{Gi}} + \lambda \frac{\partial P_L}{\partial P_{Gi}} - \lambda$$

$$\boxed{\frac{\partial F_T}{\partial P_{Gi}} + \lambda \frac{\partial P_L}{\partial P_{Gi}} = \lambda}$$

$$\frac{\partial F_T}{\partial P_{Gi}} = \frac{\partial P_L}{\partial P_{Gi}}$$

$$\boxed{\frac{\partial F_T}{\partial P_{Gi}} + \lambda \frac{\partial P_L}{\partial P_{Gi}} = \lambda} \rightarrow (3)$$

∴ for problem analysis

$$\boxed{\frac{\partial C_T}{\partial P_{Gi}} + \lambda \frac{\partial P_L}{\partial P_{Gi}} = \lambda}$$

for cost, the above eqn is called as

'co-ordination eqn'

→ The eqn (3) is the condition for optimum allocation of total load (including T.L loss) where the losses are neglected.

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$$\frac{dF_T}{dP_{Gi}} = \lambda$$

divide with $\left(1 - \frac{\partial P_L}{\partial P_{Gi}}\right)$ then,

$$\frac{\frac{\partial P_T}{\partial P_{Gi}}}{\left(1 - \frac{\partial P_L}{\partial P_{Gi}}\right)} = \lambda$$

$$\frac{dF_T}{dP_{Gi}} \times L_T = \lambda \qquad \left(\because L_T = \frac{1}{1 - \frac{dP_L}{dP_{Gi}}}\right)$$

$L_T = \frac{1}{1 - \frac{dP_L}{dP_{Gi}}}$ is called as penality factor of i^{th} plant.

→ Therefore, for two plants the above eqn is

$$\frac{dF_1}{dP_1} \times L_1 = \frac{dF_2}{dP_2} \times L_2 = \lambda$$

Note :-

A general expression for the loss P_L is given by, $P_L = \sum_m \sum_n P_m P_n B_{mn}$

For two plants P_L is

$$P_L = B_{11}P_1^2 + B_{22}P_2^2 + 2P_1P_2B_{12}$$

where B = loss coefficient units :- $\frac{1}{MW}$ (or) MW^{-1}

Matrix form $B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$

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P① A two bus system is shown in figure. If 100MW is transmitted from plant ① to the load a transmission loss of 10MW. Find the required generation for each plant the power received by the load. when the system 'λ' is 25 Rs/MWhr. The incremental fuel cost of the two plants are given below.

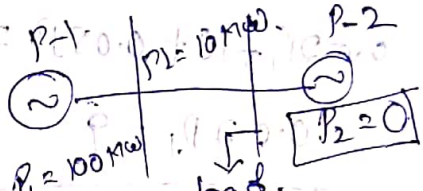
$$\frac{dC_1}{dP_1} = 0.02P_1 + 16 \text{ Rs/MWhr}$$

$$\frac{dC_2}{dP_2} = 0.04P_2 + 20 \text{ Rs/MWhr}$$

Sol:- Given data,

$$\frac{dC_1}{dP_1} = 0.02P_1 + 16$$

$$\frac{dC_2}{dP_2} = 0.04P_2 + 20$$



Find :- $P_1 = ?$ $P_2 = ?$ $P_D = ?$

$\lambda = 25$ Rs/MWhr

For two plants co-ordination eqn is

$$\frac{\partial C_1}{\partial P_1} + \lambda \frac{\partial P_L}{\partial P_1} = \frac{\partial C_2}{\partial P_2} + \lambda \frac{\partial P_L}{\partial P_2} = \lambda$$

$$P_L = B_{11}P_1^2 + B_{22}P_2^2 + 2B_{12}P_1P_2$$

(0) (0)

$B_{12} = 0$

$$P_L = B_{11}P_1^2$$

$$10 = B_{11}(100)^2$$

$$\boxed{B_{11} = 0.001 \text{ MW}^{-1}}$$

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$$\frac{\partial P_L}{\partial P_1} = 2 B_{11} P_1$$

$$\frac{\partial P_L}{\partial P_1} = 2(0.001) P_1$$

$$= 0.002 P_1$$

$$\frac{\partial P_L}{\partial P_2} = 0$$

Unit -1

$$\frac{dC_1}{dP_1} + \lambda \frac{\partial P_L}{\partial P_1} = \lambda$$

$$(0.02 P_1 + 16) + 25(0.002 P_1) = 25$$

$$0.02 P_1 + 16 + 0.05 P_1 = 25$$

$$0.07 P_1 = 9$$

~~$P_1 = 0.007700$~~

$$P_1 = 128.57 \text{ MW}$$

Unit 2 :-

$$\frac{dC_2}{dP_2} + \lambda \frac{\partial P_L}{\partial P_2} = \lambda$$

$$(0.04 P_2 + 20) + (0)25 = 25$$

$$0.04 P_2 = 25 - 20$$

$$P_2 = \frac{5}{0.04}$$

$$P_2 = 125 \text{ MW}$$

$$P_1 + P_2 = P_D + P_L$$

$$P_D = P_1 + P_2 - P_L$$

$$= 128.57 + 125 - 10$$

$$P_D = 243.57 \text{ MW}$$

(2) **Penality factor method :-**

$$\frac{dC_1}{dP_1} \times L_1 = \frac{dC_2}{dP_2} \times L_2 = \lambda$$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}}$$

$$L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_2}}$$

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$L_1 = \frac{1}{1-0.0028P_1}$ | $L_2 = \frac{1}{1-0.}$

$\frac{dC_1}{dP_1} \times L_1 = \lambda$ | $\frac{dC_2}{dP_2} \times L_2 = \lambda$

$(0.02P_1 + 16)\frac{1}{1-0.0028P_1} = 25$ | $(0.04P_2 + 20)1 = 25$

$0.02P_1 + 16 = 25 - 0.05P_1$ | $P_2 = 125$ MW.

$0.07P_1 = 9.$

$P_1 = 128.5$ MW

$P_1 + P_2 = P_D + P_L$

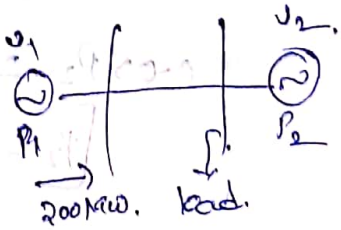
$P_D = 128 + 125 - 10$

$P_D = 243$ MW

② A system consists of two plants connected by a T.L and a load is at power plant ② is shown in fig. Data for the loss equation coefficient of the information that 200 MW transmitted from plant ① to the load results in a transmission loss of 20 MW. Find the optimum generation scheduled

$\frac{dC_1}{dP_1} = 0.025P_1 + 14$ Rs/MWhr

$\frac{dC_2}{dP_2} = 0.05P_2 + 16$ Rs/MWhr



Soln:- given,

$\frac{dC_1}{dP_1} = 0.025P_1 + 14$

$\frac{dC_2}{dP_2} = 0.05P_2 + 16$

$\lambda = 30$ Rs/MWhr.

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$$\frac{dC_1}{dP_1} + \lambda \frac{\partial P_2}{\partial P_1} = \frac{dC_2}{dP_2} + \lambda \frac{\partial P_2}{\partial P_2} = \lambda$$

$$\therefore P_2 = B_{11}P_1^2 + B_{22}P_2^2 + 2B_{12}P_1P_2$$

$B_{12} = 0$

$$P_2 = B_{11}P_1^2$$

$$20 = B_{11}(200)^2$$

$$\boxed{B_{11} = 0.0005 \text{ mho}^{-1}} \qquad \frac{\partial P_2}{\partial P_2} = 0$$

$$\frac{\partial P_2}{\partial P_1} = 2B_{11}P_1$$

$$= 2(0.0005)P_1$$

$$= 0.001P_1$$

$$\frac{dC_1}{dP_1} + \lambda \frac{\partial P_2}{\partial P_1} = \lambda$$

$$(0.025P_1 + 14) + 30(0.001P_1) = 30$$

$$0.025P_1 + 14 + 0.03P_1 = 30$$

$$0.055P_1 = 16$$

$$\boxed{P_1 = 290.9 \text{ MW}}$$

$$\frac{dC_2}{dP_2} + \lambda \frac{\partial P_2}{\partial P_2} = \lambda$$

$$(0.05P_2 + 16) + 0 = 30$$

$$0.05P_2 = 14$$

$$\boxed{P_2 = 280}$$

$$P_1 + P_2 = P_D + P_L$$

$$P_D = 290 + 280 - 20$$

$$\boxed{P_D = 550 \text{ MW}}$$

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penality factor :-

$$\frac{dC_1}{dP_1} \times L_1 = \frac{dC_2}{dP_2} \times L_2 = \lambda$$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} \qquad L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_2}}$$

$$= \frac{1}{1 - 0.001 P_1} \qquad L_2 = \frac{1}{1-0} = 1$$

$$(0.025P_1 + 12)\left(\frac{1}{1 - 0.001P_1}\right) = 30$$

$$0.025P_1 + 14 = 30 - 0.03P_1$$

$$0.055P_1 = 16$$

$$\boxed{P_1 = 290.9 \text{ MW}}$$

$$P_1 + P_2 = P_D + P_L$$

$$P_D = 290 + 280 - 20$$

$$\boxed{P_D = 550 \text{ MW}}$$

$$(-0.05P_2 + 16) \times 1 = 30$$

$$\boxed{P_2 = 280}$$

(gate ~~Qest~~)

(3) Determine the incremental cost of recivied power and penality factor of the plant shown in fig if the incremental cost of production is

$$\frac{dC_1}{dP_{G_1}} = 0.1 P_{G_1} + 30 \text{ Rs/MW-hr}$$

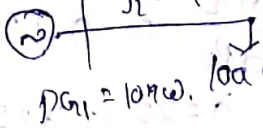
Soln :

$$\frac{dC_1}{dP_1} \times L_1 = \lambda \qquad L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}}$$

$$(0.1(100) + 30)(1.25) = \lambda \qquad = \frac{1}{1 - \frac{2}{10}}$$

$$(31)(1.25) = \lambda \qquad = \frac{1}{1 - \frac{1}{5}} = 1.25$$

$$\boxed{\lambda = 38.75 \text{ Rs/MWh}}$$



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④ A simple two plant system have the incremental cost are $\frac{dC_1}{dP_{G_1}} = 0.01 P_{G_1} + 20$

$$\frac{dC_2}{dP_{G_2}} = 0.01 P_{G_2} + 15$$

and the total load on the system is distributed optimally b/w two stations as $P_{G_1} = 60$ MW ; $P_{G_2} = 110$ MW and the loss coefficient of the system are given

P	Q	B_{PQ}
1	1	0.0015
1	2	-0.0015
2	2	0.0025

determine the transmission loss

Sol^n:- $P_L = B_{11} P_{G_1}^2 + B_{22} P_{G_2}^2 + 2 B_{12} P_{G_1} P_{G_2}$

$= (0.0015)(60)^2 + (0.0025)(110)^2 + 2(-0.0015)(60)(110)$

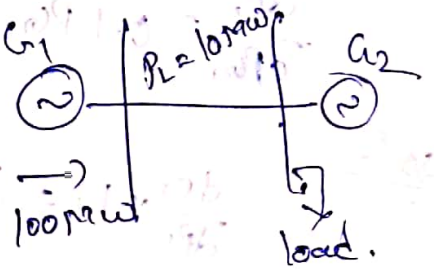
$= 5.4 + 30.25 - 19.8$

$P_L = 15.85$ MW

⑤ $\frac{dC_1}{dP_1} = 0.1 P_1 + 13$ Rs/MWhr

$\frac{dC_2}{dP_2} = 0.12 P_2 + 12$ Rs/MWhr.

① $\lambda = 22$ ② $\lambda = 25$ ③ $\lambda = 30$



Sol^n:- ~~0.0222~~

$$\frac{dC_1}{dP_1} + \lambda \frac{dP_L}{dP_1} = \frac{dC_2}{dP_2} + \lambda \frac{dP_L}{dP_2} = \lambda$$

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$$P_L = B_{11}P_1^2 + B_{22}P_2^2 + 2B_{12}P_1P_2$$

(0) (0)

$$P_L = B_{11}P_1^2$$

$$10 = B_{11}(100)^2$$

$$\boxed{B_{11} = 0.001 \text{ mho}}$$

$$\frac{dP_L}{dP_2} = 0$$

$$\frac{dP_L}{dP_1} = 2B_{11}P_1$$

$$= 2(0.001)P_1$$

$$= 0.002P_1$$

(i) $\lambda = 22$

$$\frac{dC_1}{dP_1} + \lambda\frac{dP_L}{dP_1} = \lambda$$

$$(0.1P_1 + 13) + 22(0.002P_1) = 22$$

$$0.1P_1 + 13 + 0.044P_1 = 22$$

$$0.144P_1 = 9$$

$$\boxed{P_1 = 62.5 \text{ MW}}$$

$$\frac{dC_2}{dP_2} + \lambda\frac{dP_L}{dP_2} = \lambda$$

$$(0.12P_2 + 12) + 22(0) = 22$$

$$0.12P_2 + 12 = 22$$

$$\boxed{P_2 = 83.33 \text{ MW}}$$

$$P_1 + P_2 = P_D + P_L$$

$$P_D = P_1 + P_2 - P_L$$

$$= 62.5 + 83.3 - 10$$

$$\boxed{P_D = 135.8 \text{ MW}}$$

(ii) $\lambda = 25$

$$\frac{dC_1}{dP_1} + \lambda\frac{dP_L}{dP_1} = \lambda$$

$$(0.1P_1 + 13) + 25(0.002P_1) = 25$$

$$0.1P_1 + 13 + 0.05P_1 = 25$$

$$0.15P_1 = 12$$

$$\boxed{P_1 = 80 \text{ MW}}$$

$$\frac{dC_2}{dP_2} + \lambda\frac{dP_L}{dP_2} = \lambda$$

$$(0.12P_2 + 12) + 25(0) = 25$$

$$0.12P_2 = 13$$

$$\boxed{P_2 = 108.3 \text{ MW}}$$

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$P_1 + P_2 = P_D + P_L$

$P_D = P_1 + P_2 - P_L$

$= 108.3 + 150 - 10$

$\boxed{P_D = 248.3 \text{ MW}}$

(iii) $\lambda = 30$.

$\frac{dC_1}{dP_1} + \lambda \frac{dP_L}{dP_1} = \lambda$

$(0.1P_1 + 13) + 30(0.002P_1) = 30$

$0.1P_1 + 13 + 0.06P_1 = 30$

$0.16P_1 = 17$

$\boxed{P_1 = 106.25 \text{ MW}}$

$\frac{dC_2}{dP_2} + \lambda \frac{dP_L}{dP_2} = \lambda$

$(0.12P_2 + 12) + 30(0) = 30$

$0.12P_2 = 18$

$\boxed{P_2 = 150 \text{ MW}}$

$P_1 + P_2 = P_D + P_L$

$P_D = P_1 + P_2 - P_L$

$= 106.25 + 150 - 10$

$\boxed{P_D = 246.25 \text{ MW}}$

12/7/19

Choose an initial set of values for P_1, P_2, P_3 such that they add upto total loads $P_1 = 400$ MW, $P_2 = 300$ MW, $P_3 = 150$ MW calculate incremental transmission losses and the total loss. Given the transmission loss $P_L = 0.00003P_1^2 + 0.00009P_2^2 + 0.00012P_3^2$ and $\frac{dC_1}{dP_1} = 0.00312P_1 + 7.92$

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$\frac{dC_1}{dP_1} = 0.00388 P_2 + 7.85$

$\frac{dC_2}{dP_2} = 0.00924 P_2 + 7.97$

$P_1 = 0.00003 P_1^2 + 0.00009 P_2^2 + 0.00012 P_3^2$

$\frac{\partial P_1}{\partial P_1} = 0.00006 P_1$; $\frac{\partial P_1}{\partial P_2} = 0.00018 P_2$

$\frac{\partial P_1}{\partial P_3} = 0.00024 P_3$.

$\frac{\partial P_1}{\partial P_1} = (0.00006)(1100)$ $\frac{\partial P_1}{\partial P_2} = 0.00018 \times 300$

$= 0.054$

$= 0.024$.

$\frac{\partial P_1}{\partial P_3} = 0.00024 \times 150$

$= 0.036$

$P_1 = 0.00003(1100)^2 + 0.00009(300)^2 + 0.00012(150)^2$

$P_1 = 15.6$ MW.

$\frac{dC_1}{dP_1} + \lambda \frac{dP_1}{dP_1} = \lambda$

$0.00312(1100) + \lambda(0.024) + 7.92 = \lambda$

$9.768 = \lambda(1 - 0.024)$

$\lambda = 9.37$ Rs/MWhr

0.976

$\frac{dC_2}{dP_2} + \lambda \frac{dP_2}{dP_2} = \lambda$

$0.00388(300) + \lambda(0.054) + 7.85 = \lambda$

$1.164 = \lambda(1 - 0.054)$

$9.014 = \lambda(0.946)$

$\lambda = 9.52$ Rs/MWhr

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$$\frac{dC_3}{dP_3} + \lambda \frac{dP_L}{dP_3} = \lambda$$

$$0.00964(150) + 9.97 + \lambda(0.036) = \lambda$$

$$9.416 = \lambda(1 - 0.036)$$

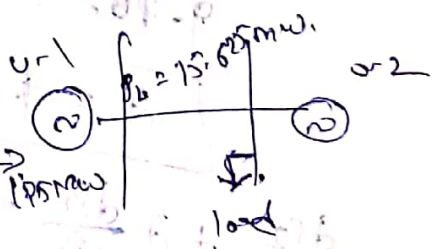
$$\boxed{\lambda = 9.76 \text{ Rs/MWhr}}$$

④ A two bar system shown in fig if a load of 120 MW is transmitted from plant ① to the load a loss of 15.625 MW. determine the generation schedule and load demand, if the cost of received power is 24 Rs/MWhr. Solve the problem using Co-ordination eqn & penality factor method. The incremental production cost of the plants are

$$\frac{dC_1}{dP_1} = 0.025P_1 + 15$$

$$\frac{dC_2}{dP_2} = 0.05P_2 + 20$$

Soln ⇒ $\lambda = 24$ Rs/MWhr



$$\frac{dC_1}{dP_1} + \lambda \frac{dP_L}{dP_1} = \frac{dC_2}{dP_2} + \lambda \frac{dP_L}{dP_2} = \lambda$$

$$P_L = B_{11}P_1^2 + B_{22}P_2^2 + 2B_{12}P_1P_2$$

$$P_L = B_{11}P_1^2 \quad (\because B_{22} = 0,\ B_{12} = 0)$$

$$15.625 = B_{11}(125)^2$$

$$\boxed{B_{11} = 0.001 \text{ MW}^{-1}}$$

$$\frac{\partial P_L}{\partial P_1} = 2B_{11}P_1$$

$$= 2 \times 0.001 P_1$$

$$\frac{\partial P_L}{\partial P_1} = 0.002 P_1$$

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$\lambda = 24$ Rs/MWhr

$$\frac{dC_1}{dP_1} + \lambda \frac{dP_2}{dP_1} = \lambda$$

$$(0.025P_1 + 15) + 24(0.002P_1) = 24$$

$$0.025P_1 + 15 + 0.048P_1 = 24$$

$$0.073P_1 = 9$$

$$\boxed{P_1 = 123.28 \text{ MW}}$$

$$\frac{dC_2}{dP_2} + \lambda \frac{dP_2}{dP_2} = \lambda$$

$$(0.05P_2 + 20) + 24(0) = 24$$

$$0.05P_2 = 4$$

$$\boxed{P_2 = 80 \text{ MW}}$$

$$P_1 + P_2 = P_D + P_L$$

$$P_D = P_1 + P_2 - P_L$$

$$= 123.28 + 80 - 15.625$$

$$\boxed{P_D = 187.65 \text{ MW}}$$

(B) The incremental fuel cost of two bary for

$$\frac{dC_1}{dP_1} = 0.08P_1 + 16 \text{ Rs/MWhr}$$

$$\frac{dC_2}{dP_2} = 0.08P_2 + 12 \text{ Rs/MWhr}$$

Given that $B_{11} = 0.001/\text{MW}$; $B_{22} = 0.0024/\text{MW}$; $B_{12} = -0.0005/\text{MW}$

and $\lambda = 20$ Rs/MWhr. Find P_1, P_2, P_D and P_L

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$\lambda = 20$

$$\frac{dC_1}{dP_1} + \lambda \frac{dP_L}{dP_1} = \lambda$$

$$(0.08P_1 + 16) + 20(0.002P_1) = 20$$

$$0.08P_1 + 16 + 0.04P_1 = 20$$

~~$0.12P_1 =$~~

$$P_L = B_{11}P_1^2 + B_{22}P_2^2 + 2B_{12}P_1P_2$$

$$\frac{\partial P_L}{\partial P_1} = 2B_{11}P_1 + 2B_{12}P_2 = 0.002P_1 + 0.001P_2$$

$$\frac{\partial P_L}{\partial P_2} = 2B_{22}P_2 + 2B_{12}P_1 = 0.0048P_2 + 0.001P_1$$

$$\frac{dC_1}{dP_1} + \lambda \frac{dP_L}{\partial P_1} = \lambda$$

$$(0.08P_1 + 16) + 20(0.002P_1 + 0.001P_2) = 20.$$

$$0.08P_1 + 16 + 0.04P_1 + 0.02P_2 = 20$$

$$0.12P_1 + 0.02P_2 = 4 \rightarrow (1)$$

$$\frac{dC_2}{dP_2} + \lambda \frac{dP_L}{\partial P_2} = \lambda$$

$$(0.08P_2 + 12) + 20(0.0048P_2 + 0.001P_1) = 20$$

$$0.08P_2 + 12 + 0.096P_2 + 0.02P_1 = 20$$

$$-0.02P_1 + 0.176P_2 = 8 \rightarrow (2)$$

from (1) & (2)

$P_1 = 41.69$ MW $P_2 = 50.19$ MW

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$$P_L = (0.001)(41.69)^2 + (0.0024)(50.19)^2 + 2(-0.0005)(41.69)(50.19)$$

$$\boxed{P_L = 5.69 \text{ MW}}$$

$$P_1 + P_2 = P_D + P_L$$

$$P_D = P_1 + P_2 - P_L$$

$$P_D = 41.69 + 50.19 + 5.69$$

$$\boxed{P_D = 86.19 \text{ MW}}$$

(9) The fuel i/p to the plants are given by

$$C_1 = 0.015P_1^2 + 16P_1 + 50$$

$$C_2 = 0.025P_2^2 + 12P_2 + 30$$

The loss coefficient of the system are given by $B_{12} = -0.0012$; $B_{11} = 0.003$, $B_{22} = 0.002$. the load to be met to be 200 MW. determine the economic operation schedule and the corresponding cost of generation if the transmission line losses are coordinated

Soln:- $\lambda = ?$ $P_1 = ?$

$$P_D = 200$$

$$P_1 + P_2 = 200$$

$$\frac{\partial P_L}{\partial P_1} = 2 \times B_{12} P_1$$

$$= 2 \times -0.0012 \times P_1$$

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \lambda$$

$$\frac{dC_1}{dP_1} = 0.03P_1 + 16$$

$$\frac{dC_2}{dP_2} = 0.05P_2 + 30$$

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$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2}$$

$$0.03P_1 + 16 = 0.05P_2 + 30$$

$$0.03P_1 - 0.05P_2 = 14.$$

$$P_1 + P_2 = 200$$

$$P_1 = 300 \text{ MW} \qquad P_2 = -100 \text{ MW}$$

$$\frac{dC_1}{dP_1} = \lambda$$

$$0.03P_1 + 16 = \lambda$$

$$0.03(300) + 16 = \lambda$$

$$\lambda = 25 \text{ Rs/MWhr}$$

$$P_L = B_{11}P_1^2 + B_{22}P_2^2 + 2B_{12}P_1P_2$$

$$= (0.005)(300)^2 + (0.002)(-100)^2 + 2 \times -0.0012 \times 300 \times -100.$$

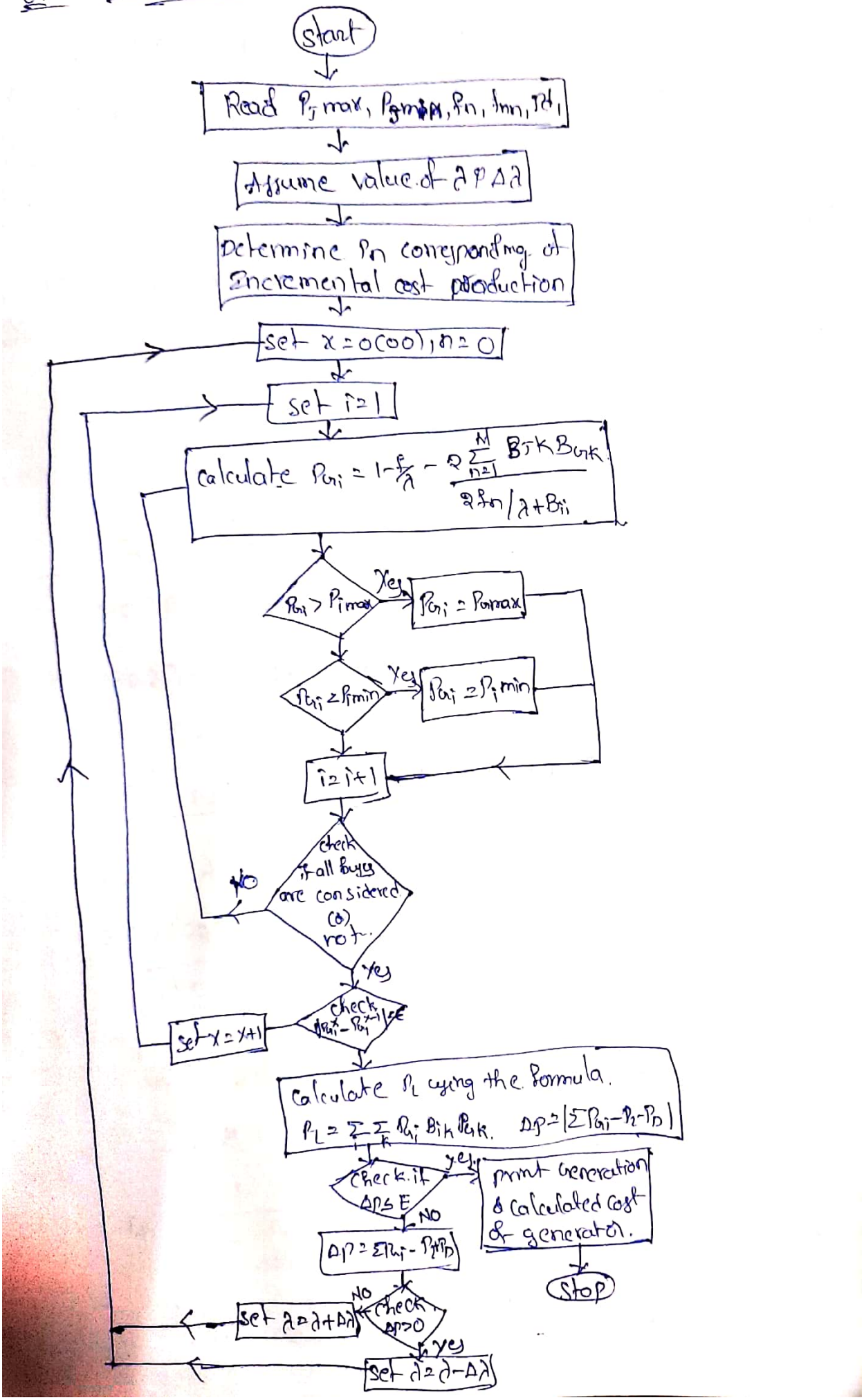
$$= 450 + 20 + 72$$

$$P_L = 542 \text{ MW}$$

10/7/19

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13/7/17: optimum generation allocation including effect of transmission line :-



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(16) $\frac{dC_1}{dP_1} = 0.02P_1 + 16$; $\frac{dC_2}{dP_2} = 0.04P_2 + 20$ consider a system with a load of 243.57 MW of buy (2) Find the optimum distribution b/w the two plants for

i) when losses are included but not co-ordinated

ii) when losses are co-ordinated

iii) And also find the savings in $/hr when losses are co-ordinated.

Soln:- given,

$$\frac{dC_1}{dP_1} = 0.02P_1 + 16$$

$$\frac{dC_2}{dP_2} = 0.04P_2 + 20$$

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2}$$

(i) $0.02P_1 + 16 = 0.04P_2 + 20$

$0.02P_1 - 0.04P_2 = 4$

$P_1 + P_2 = 243.57$

$P_1 = 229.04$ MW. $\quad P_2 = 14.52$ MW.

$0.02P_1 + 16 = \lambda$

$0.02(229.04) + 16 = \lambda$

$\lambda = 20.58$ Rs/MWhr

(ii) $\frac{dC_1}{dP_1} + \lambda \frac{\partial P_L}{\partial P_1} = \frac{dC_2}{dP_2} + \lambda \frac{\partial P_L}{\partial P_2} = \lambda$.

$\frac{dC_1}{dP_1} * L_1 = \frac{dC_2}{dP_2} \times L_2 = \lambda$

$\frac{\partial P_L}{\partial P_1} = 2B_{11}P_1$

$= 0.002P_1$.

$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}}$

$= \frac{1}{1 - 0.002P_1}$

$L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_2}}$

$= \frac{1}{1-0}$

$= 1$

$P_1 = 128.5$ MW

$P_2 = 125$ MW

$P_D = 243$ MW

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$P_2 = B_{11} P_1^2$

$\frac{\partial P_2}{\partial P_2} = 0$

$0 = B_{11} (229.024)^2$

$\boxed{B_{11} = 0.000019 \text{ MW}^{-1}}$

$\frac{\partial P_2}{\partial P_1} = 2 B_{11} P_1$

$= 2(0.000019) P_1$

$= 0.00038 P_1$

$\frac{\partial C_1}{\partial P_1} * \lambda_1 = \lambda$

$(0.02 P_1 + 16)\left(\frac{1}{1 - 0.00038 P_1}\right) = 20.5$

$0.02 P_1 + 16 = 20.5 - 0.00779 P_1$

$0.02779 P_1 = 4.5$

$\boxed{P_1 = 161.92 \text{ MW}}$

$\frac{dC_2}{dP_2} \times \lambda_1 = \lambda$

$(0.04 P_2 + 20)(1) = 20.5$

$0.04 P_2 = 0.5$

$\boxed{P_2 = 12.5 \text{ MW}}$

$P_1 + P_2 = P_D + P_L$

$P_D = P_1 + P_2 - P_L$

$P_D = 161.92 + 12.5 - 10$

$\boxed{P_D = 164.42 \text{ MW}}$

(iii)

$P_1 = 229.024 \text{ MW}$ $P_2 = 14.029 \text{ MW}$

Req. $P = 243.57 \approx 244$

$P_{G1} = 122$; $P_{G2} = 122$

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unit ① :-

$$\int_{\text{cost}}^{\text{equal}} \frac{dC_1}{dP_1} dP_1 = \int_{229.04}^{122} (0.02P_1 + 16)\, dP_1$$

$$= \left[\frac{0.02P_1^2}{2} + 16P_1\right]_{229.04}^{122}$$

$$= \left(0.01P_1^2 + 16P_1\right)_{229.04}^{122}$$

$$= \left[0.01(122)^2 + 16(122)\right] - \left[0.01(229.04)^2 + 16(229.04)\right]$$

$$= 2100.84 - 4189.23$$

$$= -2088.39 \text{ MW}$$

unit ② :-

$$\int_{\text{cost}}^{\text{equal}} \frac{dC_2}{dP_2} dP_2 = \int_{14.52}^{122} (0.04P_2 + 20)$$

$$= \left[\frac{0.04P_2^2}{2} + 20P_1\right]_{14.52}^{122}$$

$$= \left[0.02(122)^2 + 20(122)\right] - \left[0.02(14.52)^2 + 20(14.52)\right]$$

$$= 2737.68 - 294.6$$

$$= 2443.06 \text{ MW}$$

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15/7/19

Transmission line loss formulae:-

→ The 1st approach expresses transmission power loss P_L interms of loss coefficient for the N/w.

→ Here the system transmission loss are written as function of time and is called as "B-coefficient".

→ According to the equation,

$$P_L = \sum_{i=1}^{n} \sum_{k=1}^{n} P_{Gi} B_{ik} P_{Gk} \longrightarrow (1)$$

→ where, P_{Gi} & P_{Gk} are the power o/p units i,k. respectively and 'n' is the total no. of plants

→ In matrix form eq (1) can be written as

$$P_L = [P_G]^T [B] [P_G] \longrightarrow (2)$$

→ $P_G^T = [P_{G1}\ P_{G2} \cdots P_{Gn}]$

$$\therefore [B] = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1n} \\ B_{21} & B_{22} & \cdots & B_{2n} \\ \vdots & & & \\ B_{n1} & B_{n2} & \cdots & B_{nn} \end{bmatrix}$$

where, 'P_G' are in 'MW', 'B' are in 'MW'

→ For two plant system the loss formula is,

$$P_L = B_{11} P_{G1}^2 + B_{22} P_{G2}^2 + 2 B_{12} P_{G1} P_{G2}$$

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→ For a three plant system the loss formula is,

$$P_L = B_{11}P_{G_1}^2 + B_{22}P_{G_2}^2 + B_{33}P_{G_3}^2 + 2B_{12}P_{G_1}P_{G_2} + 2B_{23}P_{G_2}P_{G_3} + 2B_{31}P_{G_3}P_{G_1}$$

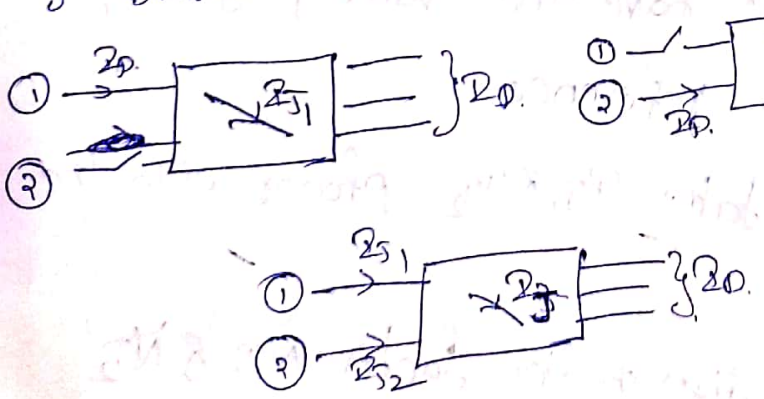
where $P = \begin{bmatrix} P_{G_1} \\ P_{G_2} \end{bmatrix}$

$$B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

→ The method for obtaining these B coefficient will be developed here based on several assumptions for system having any no. of nodes and plants:

→ Let us consider two generating plants connected to an arbitary no. of nodes through a transmission N/w as shown in fig.

→ one line of the N/w may be considered as j^{th} branch



→ From the fig, the total load of $[I_D]$ supplied by plant ① and ct is j^{th} branch is I_j, then ct distribution factor is,

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$$N_{J1} = \frac{Z_{J1}}{Z_D} \quad \rightarrow (3)$$

→ From the fig, the total load (or ckt Z_D) supplied by plant ② and ckt in J^{th} branch is Z_{J1}, then ckt distribution factor is

$$N_{J2} = \frac{Z_{J2}}{Z_D} \rightarrow (4)$$

→ when both plants ① & ② are supplied in the load with sharing of ckt Z_{G1} & Z_{G2} the current I_J in the J^{th} branch is expressed as,

$$\boxed{I_J = N_{J1} I_{G1} + N_{J2} I_{G2}} \rightarrow (5)$$

→ Based on supper position principle let us make two assumptions

① for all branches of the N/w the ratio XY are is same

② all load currents have the same phase angle w.r.to a common reference.

→ Therefore, let us take I_{J1} & I_{J2} phasor with the 'D'.

→ This makes ckt distribution factor N_{J1}' & N_{J2}' real rather then complex.

→ Therefore, $I_{G1} = I_{G1}\cos\alpha_1 + jI_{G1}\sin\alpha_1$

$I_{G2} = I_{G2}\cos\alpha_2 + jI_{G2}\sin\alpha_2$

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→ where, α_1 & α_2 are the phase angles with common reference

→ using 5th eqn.

$$Z_T = NI_1 (ZG_1 \cos\alpha_1 + jZG_1 \sin\alpha_1) + NI_2 [ZG_2 \cos\alpha_2 + jZG_2 \sin\alpha_2]$$

$$Z_T = \left[NI_1 (ZG_1 \cos\alpha_1 + jZG_1 \sin\alpha_1)\right] + \left[NI_2 (ZG_2 \cos\alpha_2 + jZG_2 \sin\alpha_2)\right]$$

$$= NI_1 ZG_1 \cos\alpha_1 + jNI_1 ZG_1 \sin\alpha_1 + NI_2 ZG_2 \cos\alpha_2 + jNI_2 ZG_2 \sin\alpha_2$$

$$Z_T = NI_1 ZG_1 \cos\alpha_1 + NI_2 ZG_2 \cos\alpha_2 + j(NI_1 ZG_1 \sin\alpha_1 + NI_2 ZG_2 \sin\alpha_2)$$

$$|Z_T| = \left[(NI_1 ZG_1 \cos\alpha_1 + NI_2 ZG_2 \cos\alpha_2) + j(NI_1 ZG_1 \sin\alpha_1 + NI_2 ZG_2 \sin\alpha_2)\right]$$

$$\left[\because Z = a + jb \quad |Z| = \sqrt{a^2+b^2} \quad Z^2 = a^2+b^2\right]$$

$$\left[\because Z^2 = a+jb \quad Z^2 = \sqrt{a^2+b^2}\right]$$

$$|Z_T|^2 = (NI_1 ZG_1 \cos\alpha_1 + NI_2 ZG_2 \cos\alpha_2)^2 + (NI_1 ZG_1 \sin\alpha_1 + NI_2 ZG_2 \sin\alpha_2)^2$$

$$|Z_T|^2 = NI_1^2 ZG_1^2 \cos^2\alpha_1 + NI_2^2 ZG_2^2 \cos^2\alpha_2 + 2NI_1 NI_2 ZG_1 ZG_2 \cos\alpha_1 \cos\alpha_2 + NI_1^2 ZG_1^2 \sin^2\alpha_1 + NI_2^2 ZG_2^2 \sin^2\alpha_2 + 2NI_1 ZG_1 NI_2 ZG_2 \sin\alpha_1 \sin\alpha_2$$

$$= NI_1^2 ZG_1^2 [\cos^2\alpha_1 + \sin^2\alpha_1] + NI_2^2 ZG_2^2 (\cos^2\alpha_2 + \sin^2\alpha_2) + 2NI_1 NI_2 ZG_1 ZG_2 (\cos\alpha_1 \cos\alpha_2 + \sin\alpha_1 \sin\alpha_2)$$

$$\boxed{|Z_T|^2 = NI_1^2 ZG_1^2 + NI_2^2 ZG_2^2 + 2NI_1 NI_2 ZG_1 ZG_2 [\cos(\alpha_1 - \alpha_2)]} \rightarrow (6)$$

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→ Now, interms of power generation P_{G1} & P_{G2} of currents may be expressed as,

$$P = \sqrt{3} V I \cos\theta$$

$$I_{G1} = \frac{P_{G1}}{\sqrt{3} V_1 \cos\theta_1} \rightarrow (7)$$

$$I_{G2} = \frac{P_{G2}}{\sqrt{3} V_2 \cos\theta_2} \rightarrow (8)$$

where, V_1 & V_2 are the busvtges at the plants and $\cos\theta_1$ & $\cos\theta_2$ are the power factors of the plants.

→ Consider, transmission loads loss for j^{th} branch

$$P_L = 3 I_j^2 R_j$$

$$\Rightarrow P_L = \sum_j 3 I_j^2 R_j$$

$$P_L = \sum_j 3 \left[N_{j1}^2 I_{G1}^2 + N_{j2}^2 I_{G2}^2 + 2 N_{j1} N_{j2} I_{G1} I_{G2} \cos(\alpha_1 - \alpha_2) \right] R_j$$

$$P_L = \sum_j 3 \left[N_{j1}^2 \frac{P_{G1}^2}{3 V_1^2 \cos^2\theta_1} + N_{j2}^2 \frac{P_{G2}^2}{3 V_2^2 \cos^2\theta_2} + 2 N_{j1} N_{j2} \frac{P_{G1} P_{G2}}{3 V_1 V_2 \cos\theta_1 \cos\theta_2} \cos(\alpha_1 - \alpha_2) \right] R_j$$

$$= \sum_j \frac{N_{j1}^2 P_{G1}^2 R_j}{V_1^2 \cos^2\theta_1} + \sum_j \frac{N_{j2}^2 P_{G2}^2 R_j}{V_2^2 \cos^2\theta_2} + \frac{2 N_{j1} N_{j2} P_{G1} P_{G2} R_j}{V_1 V_2 \cos\theta_1 \cos\theta_2} \cos(\alpha_1 - \alpha_2)$$

$$\boxed{P_L = B_{11} P_{G1}^2 + B_{22} P_{G2}^2 + 2 B_{12} P_{G1} P_{G2}}$$

$$B_{11} = \sum_j \frac{N_{j1}^2 R_j}{V_1^2 \cos^2\theta_1} \; ; \; B_{22} = \sum_j \frac{N_{j2}^2 R_j}{V_2^2 \cos^2\theta_2} \; ; \; B_{12} = \sum_j \frac{N_{j1} N_{j2} R_j}{V_1 V_2 \cos\theta_1 \cos\theta_2} \cos(\alpha_1 - \alpha_2)$$

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