

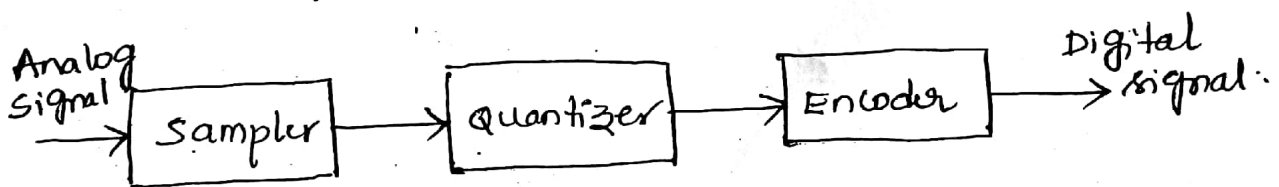
Sampling

①

Introduction:- Now a days, everything is digitalization. So we need to have a digital signal, but unfortunately there is no digital signal in the real world.

The solution for this, we have to produce a digital signal from the available signal i.e. analog signal. Therefore, the process of producing a digital signal based on analog signal is called as A/D conversion and is done by a physical circuit called A/D converter.

Block diagram of A/D converter:-



The block diagram of A/D converter includes sampler, quantizer and encoder, the detailed description of every block is given below

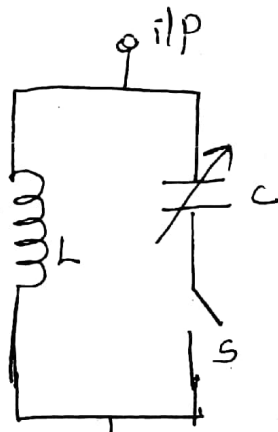
Sampling:- Sampler is a physical circuit that performs sampling operation and may be defined as the process of converting 'continuous amplitude, continuous time' to 'continuous amplitude and discrete time'.

→ sampling may be done by S/H (Sampling and Hold) circuit.

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SLH circuit :-

→ Parallel combination of Inductor (L) and capacitor (C) is called tank tuned circuit and is used for frequency selection.

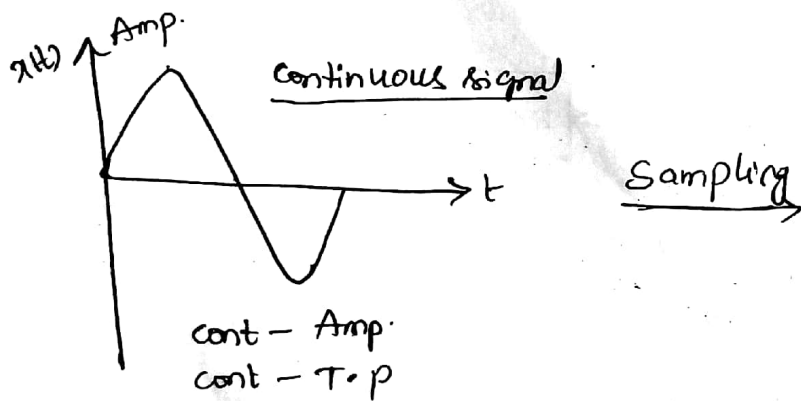


On a no circuit.
L||C - Tank/Tuned ckt
S - switch.

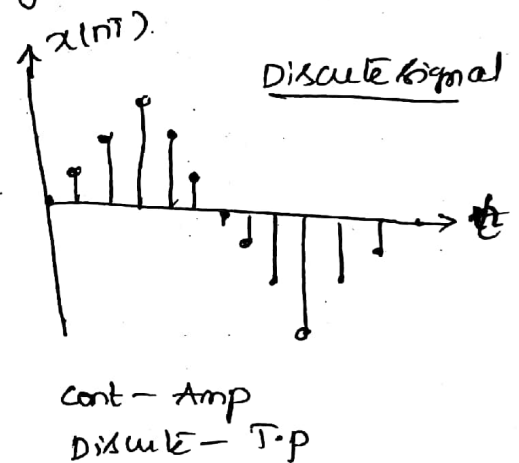
→ At the ilp of SLH circuit we may have frequency spectrum, among those all frequencies to have a single frequency, we use tuned ckt.

→ After this, signal which was selected is passed through a switch, which may be randomly ON/OFF.

→ Due to ON/OFF action, the signal may get sampled.



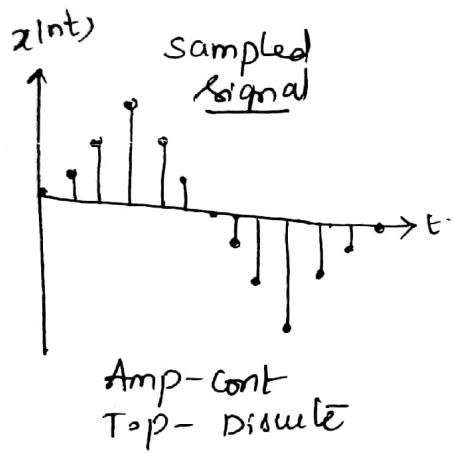
Sampling



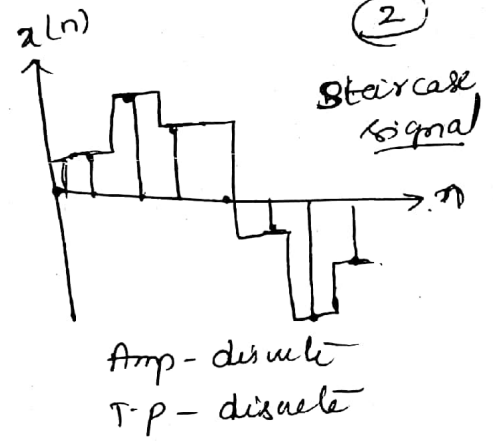
Quantization:- The output of sampler, i.e; continuous amplitude discrete time period, signal is given to the quantizer. Here the quantizer output is discrete amplitude and discrete time period.

→ Quantization includes 2 steps.

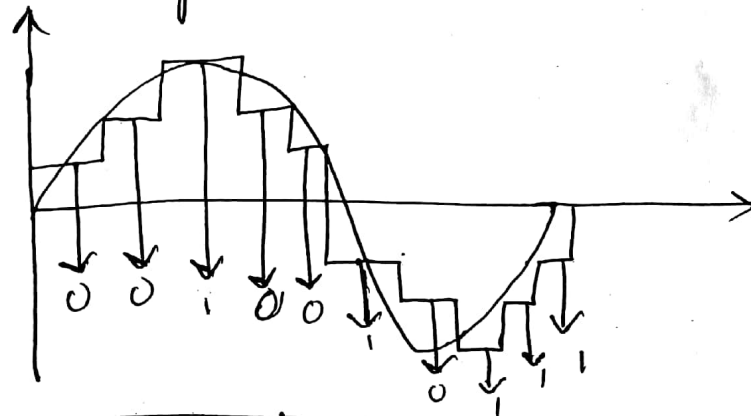
- 1) Elimination of redundant samples. (Samples which carry same information).
- 2) Approximation of sample amplitudes to its nearest positive integer values.



Quantizer



Encoder:— The purpose of encoder is to produce a digital signal, by comparing ilp signal (analog) with staircase signal.



	Amplitude	Time period.
Analog	Continuous	Continuous
Sampled	Continuous	Discrete
Quantized	Discrete	Discrete

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Sampling theorem: - Sampling theorem states that for a band limited signal, the original signal can be recovered back from its samples if and only if it satisfies the condition.

$$f_s \geq 2f_m \quad \text{where}$$

f_s = Sampling frequency
 f_m = Maximum signal frequency / message signal freq

Nyquist rate: - The minimum sample rate at which signal can be recovered from its samples, called Nyquist rate or Nyquist interval.

$$f_s = 2f_m$$

Types of sampling: - There are 3 classifications of sampling

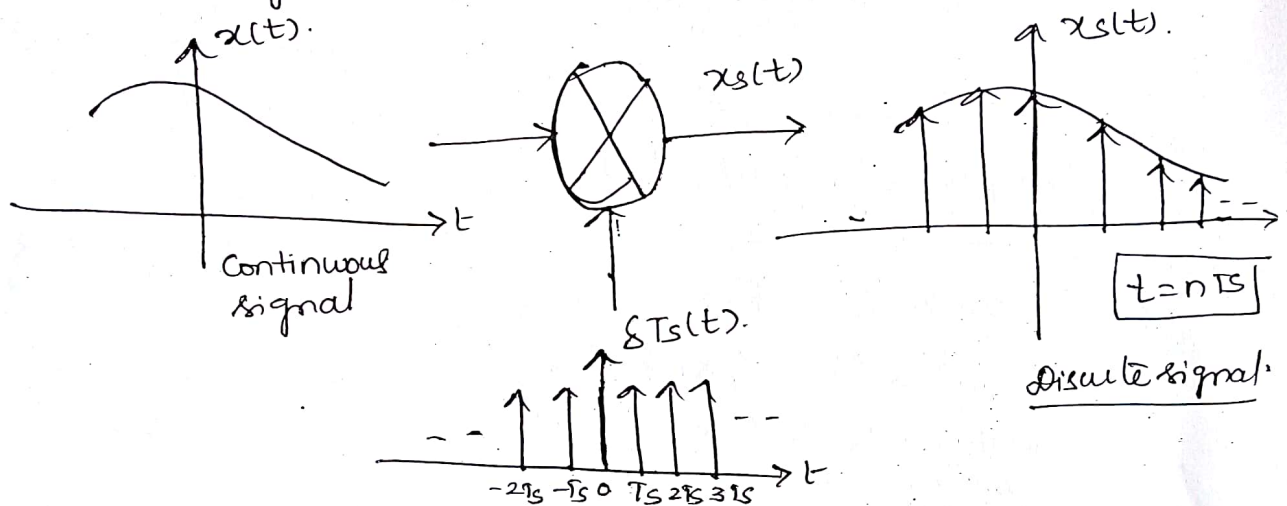
- 1) Oversampling $\rightarrow f_s > 2f_m$ condition
- 2) Critical sampling $\rightarrow f_s = 2f_m$ condition
- 3) under sampling $\rightarrow f_s < 2f_m$ condition.

Instantaneous sampling (or)

Representation of sampling by mathematical equations:-

To get a sampled signal, the analog signal is to be multiplied by a periodic impulse train using a product multiplier, thereby producing a sampled signal.

The arrangement can be shown as like below.



From the above figure.

$$x_s(t) = x(t) * \delta_{Ts}(t)$$

$$x_s(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(t) \delta(t - nT_s) \rightarrow \textcircled{1}$$

Wkf

$$\int x(t) \delta(t - t_0) dt = \int x(t - t_0) dt$$

In freq domain,

$$X_s(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(f - n f_s)$$

$$X_s(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(f - n f_s)$$

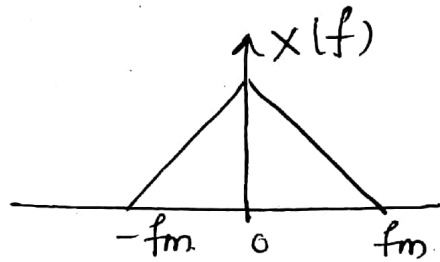
$$X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - n f_s) \rightarrow \textcircled{2}$$

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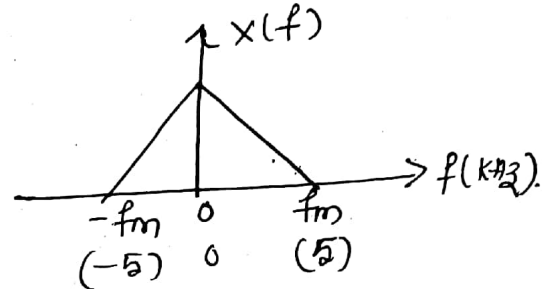
Effects of sampling:-

1) Over sampling $f_s > 2f_m$:-

Let a signal $x(t)$ which is band limited can be shown as



Let $f_m = 5 \text{ kHz}$.



WKT

$$X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - n f_s)$$

Let $n=0$, $X_s(f) = f_s X(f)$.

$n=1$, $X_s(f) = f_s X(f - f_s)$

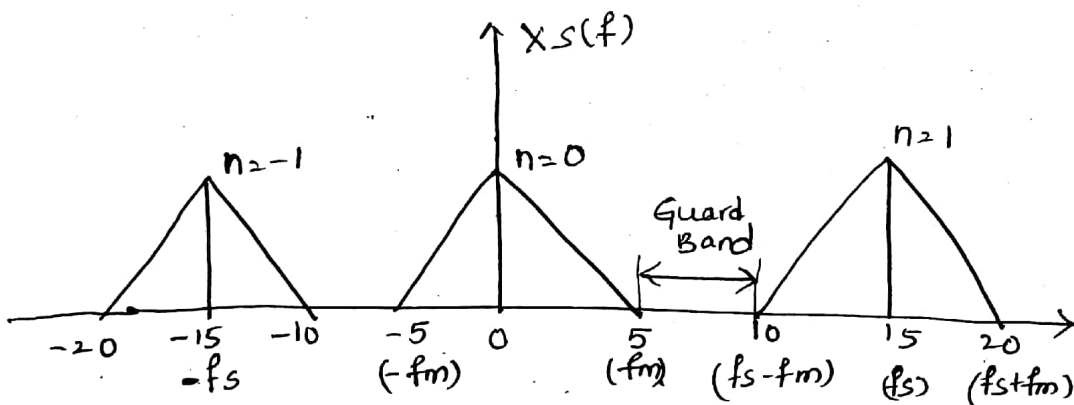
$n=-1$, $X_s(f) = f_s X(f + f_s)$

Let $f_m = 5 \text{ kHz}$

$f_s = 15 \text{ kHz}$

$f_s > 2f_m$

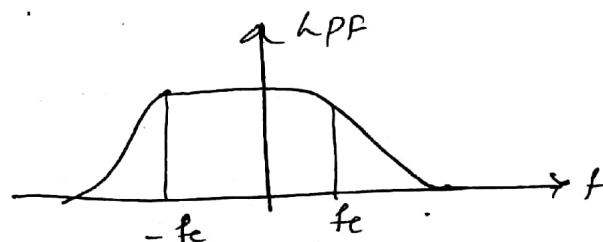
$15 > 10 \text{ kHz}$



$$\text{Guard Band} = (f_s - f_m) - f_m$$

$$GB = f_s - 2f_m$$

→ use practical LPP for signal reconstruction because of guard band is there.



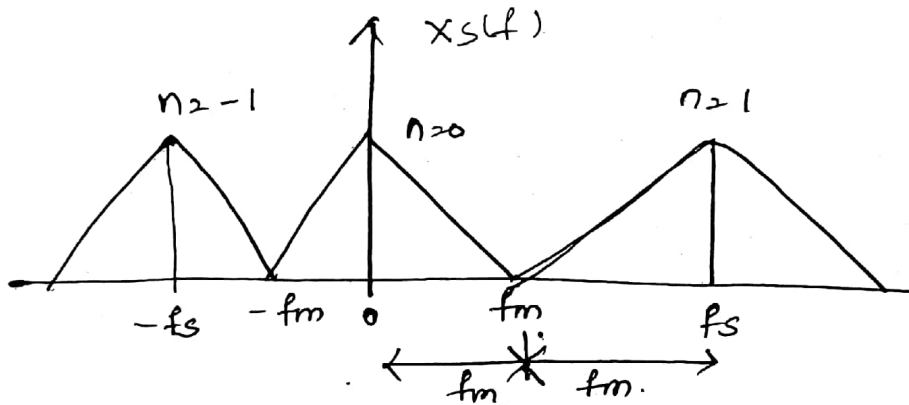
2) Perfect sampling or critical sampling ($f_s = 2f_m$)

(4)

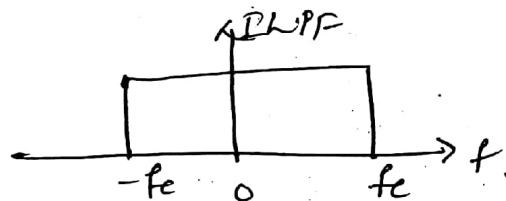
$$X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

$$f_s = 2f_m$$

$$10 = 2(5) \text{ kHz}$$



$f_s = 2f_m$, use Ideal LPP, for signal reconstruction.

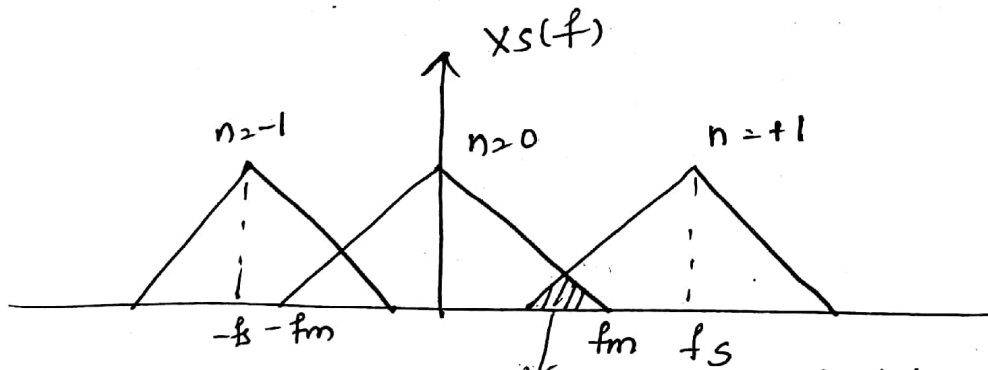


3) under sampling: $f_s < 2f_m$:-

$$f_s < 2f_m$$

$$f_s < 2(5) \text{ kHz}$$

$$f_s < 10 \text{ kHz}$$



Aliasing effect or folded over error.

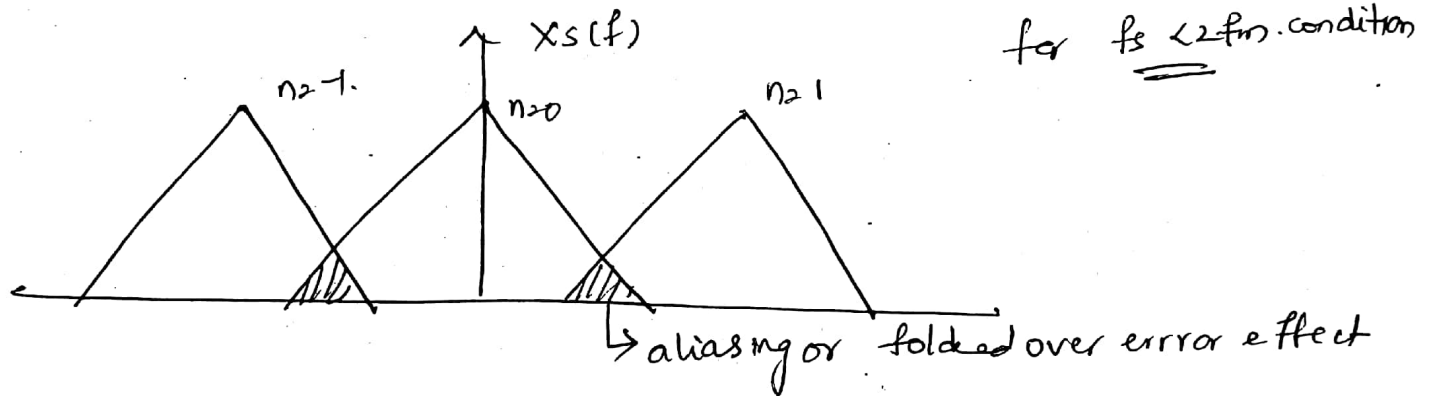
⇒ use Antialiasing filter for signal reconstruction, generally a LPP.

Aliasing effect and remedies of aliasing effect (or) Effect of under sampling and how to overcome it:-

When the signal $x(t)$ is sampled by a periodic impulse train for a condition $f_s < 2f_m$ i.e; under sampling then the spectrum of $X_s(f)$, one region is overlapped with other region. Hence this type of phenomenon is called aliasing effect or folded over error effect.

WKT

$$X_s(f) = f_s \sum_{n=-\infty}^{\infty} x(f - f_s)$$



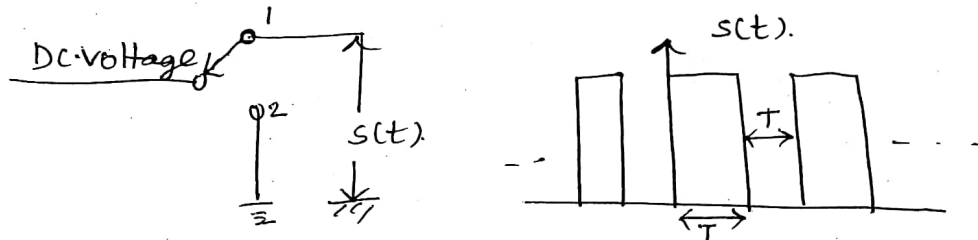
Note:- for $f_s < 2f_m$, aliasing occurs, To avoid this, before sampling, the signal should be passed through, a pre-alias filter (i.e; LPP). to avoid aliasing effect. i.e; Antialiasing filter

Remedies for aliasing effect:-

- 1) using a pre-alias filter, before sampling
- 2) sampling should be done at $f_s \geq 2f_m$.

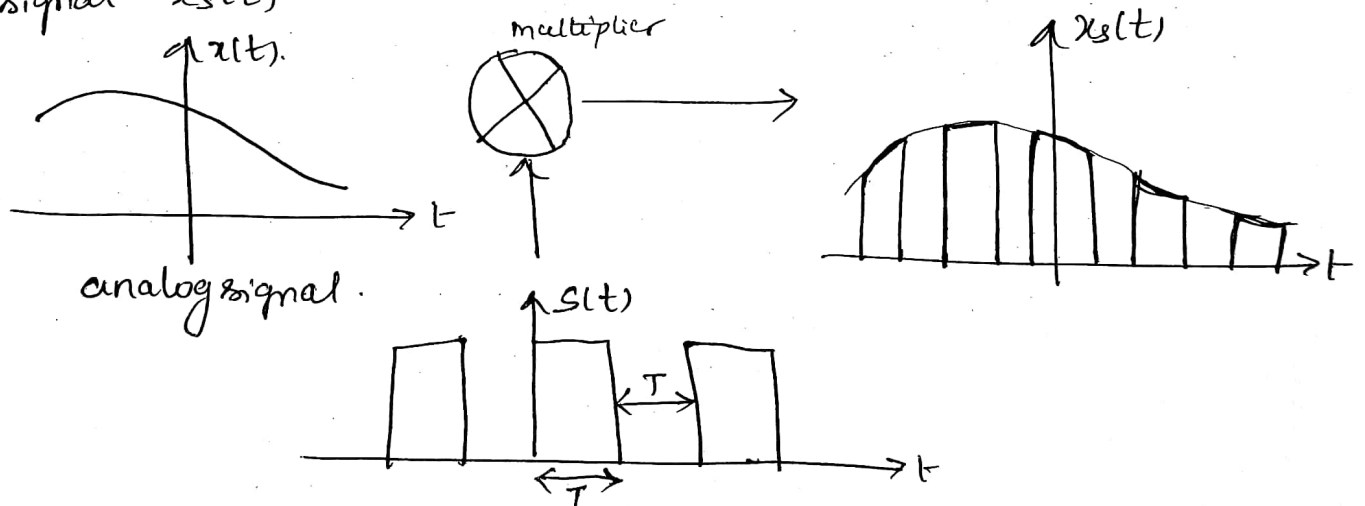
Natural sampling or pulse sampling:-

→ Generation of pulse train:- The arrangement of generating periodic pulse train depicted as



A. D.C voltage is connected to a switch of 2 possible positions 1 & 2. When it was connected to position 1, we get a pulse of duration ' T ', after this the switch was connected to position 2 for a duration of ' T ', and it continues to generate a pulse train $s(t)$.

The analog signal $x(t)$ was convolved using a product multiplier with $s(t)$, i.e. periodic pulse train to get a sampled signal $x_s(t)$.



Mathematically, it can be written as.

$$x_s(t) = x(t) \cdot s(t), \text{ where.}$$

$s(t)$ is a periodic signal-

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Since $x(t)$ is periodic impulse train, it can be represented in exponential Fourier series form as.

$$s(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_s t} \quad \text{--- (1)}$$

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} s(t) e^{-jn\omega_s t} dt \quad \text{--- (2)}$$

periodic impulse train

$$s(t) = \sum_{k=-\infty}^{\infty} p(t - kT) \quad \text{--- (3)}$$

eq (3) in eq (2).

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} s(t) e^{-jn\omega_s t} dt$$

$$C_n \Rightarrow \frac{1}{T} \int_{-T/2}^{T/2} p(t) e^{-jn\omega_s t} dt$$

$$C_n \Rightarrow \frac{1}{T} p(n\omega_s) \big|_{\omega = \omega_s} \quad \text{--- (4)}$$

eq (4) in (1).

$$s(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_s t}$$

$$s(t) = \sum_{n=-\infty}^{\infty} \left[\frac{1}{T} p(n\omega_s) \right] e^{jn\omega_s t}$$

$X_s(t) = x(t) \cdot s(t)$. w.k.t; then $X_s(j\omega) = FT[X_s(t)]$

$$X_s(j\omega) = FT \left[\frac{1}{T} \sum_{n=-\infty}^{\infty} p(n\omega_s) e^{jn\omega_s t} \cdot x(t) \right]$$

$$X_s(j\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} p(n\omega_s) \cdot X(j\omega) \cdot FT[e^{jn\omega_s t}]$$

$$X_s(j\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} p(n\omega_s) X(j\omega) \cdot \delta(\omega - n\omega_s)$$

$$\therefore X_s(j\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} p(n\omega_s) X(j[\omega - n\omega_s])$$

Flat top sampling:- (or) Practical sampling

(6)

→ Why Flat top (or) Need of flat top sampling:-

During transmission, noise is introduced at the top of transmission pulse which can be easily removed, if pulse is in the form of flat top.

Top flat, i.e.; Having constant amplitude. Hence it is called a flat top or practical sampling.

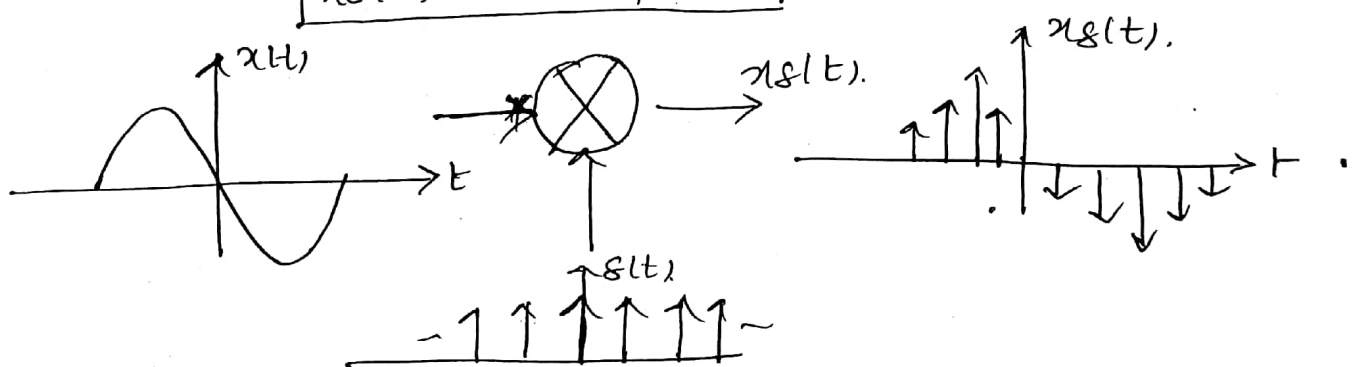
→ In order to get a, flat top sampling, the pulse $p(t)$ is to be convolved (multiplied) with a sampled signal.

Mathematically, it may be given as

$$x_s(t) = p(t) * x_s(t) \quad \begin{array}{l} \downarrow \\ \text{pulse} \end{array} \quad \begin{array}{l} \rightarrow \\ \text{Ideal sampled signal} \end{array}$$

→ Generation of $x_s(t)$ includes the following

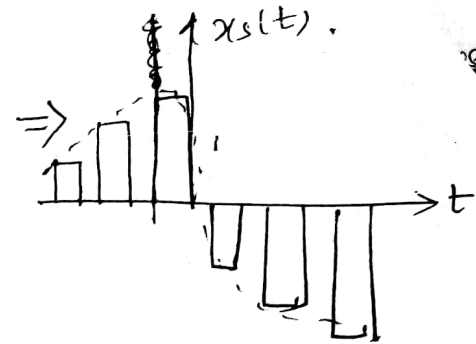
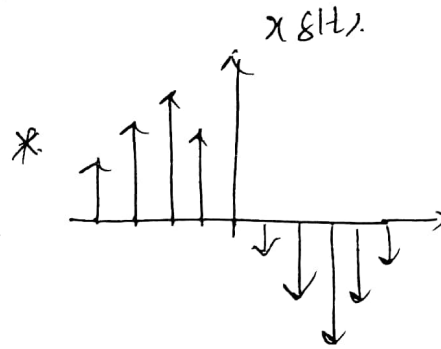
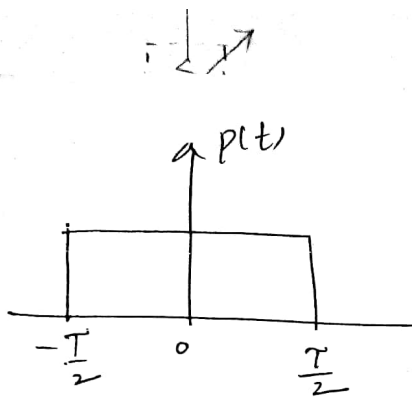
$$x_s(t) = x(t) * \delta(t)$$



→ The flat top sampling can be obtained as

$$x_s(t) = x_s(t) * p(t)$$

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Mathematically,

$$x_s(t) = p(t) * x_s(t)$$

$$x_s(t) \Rightarrow p(t) * \left[\sum_{k=-\infty}^{\infty} x(kT_s) \delta(t - kT_s) \right]$$

FT on both sides.

$$X_s(\omega) = p(\omega) \cdot \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$$

wkT

$$x_s(t) = \sum_{k=-\infty}^{\infty} x(t) \delta(t - kT_s)$$

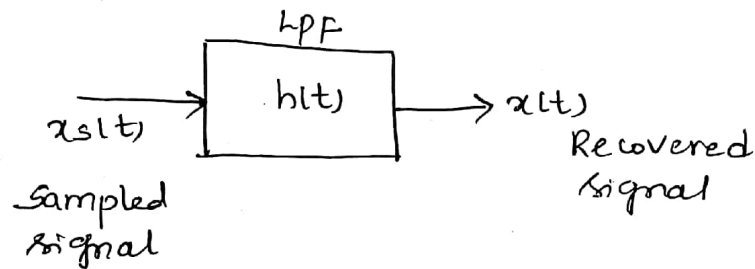
$$x_s(t) = \sum_{k=-\infty}^{\infty} x(kT_s) \delta(t - kT_s)$$

$$\therefore X_s(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} p(\omega) X(\omega - n\omega_s)$$

Signal Reconstruction:-

(7)

The original signal $x(t)$ can be recovered from the sampled signal $x_s(t)$, when it is passed through a LPF with impulse response $h(t)$.



From the above figure, we can write

$$x(t) = x_s(t) * h(t). \quad \text{--- (1)}$$

$$\text{WKT } x_s(t) = \sum_{n=-\infty}^{\infty} x(nTs) \delta(t - nTs)$$

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nTs) \delta(t - nTs) \quad \text{--- (2)}$$

sub eq (2) in eq (1), we get

$$x(t) = \left[\sum_{n=-\infty}^{\infty} x(nTs) \delta(t - nTs) \right] * h(t).$$

$$x(t) \Rightarrow \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nTs) \delta(t - nTs) h(t - \tau) d\tau.$$

$$\Rightarrow \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nTs) \delta(\tau - nTs) h(t - \tau) d\tau.$$

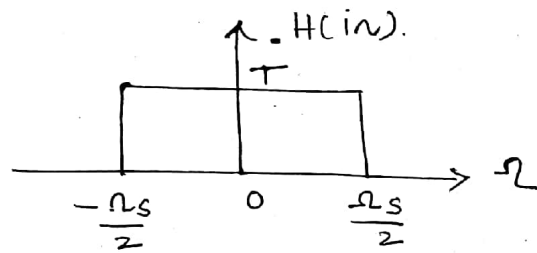
$$\Rightarrow \sum_{n=-\infty}^{\infty} x(nTs) \int_{-\infty}^{\infty} h(t - \tau) \delta(\tau - nTs) d\tau$$

$$x(t) \Rightarrow \sum_{n=-\infty}^{\infty} x(nTs) h(t - nTs). \quad \text{--- (3)}$$

The frequency response of LPF can be expressed as

$$H(\omega) = T \text{ for } |\omega| \leq \frac{\omega_s}{2}$$

$$= 0, \text{ else.}$$



The impulse response of LPF is

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{j\omega t} d\omega$$

$$\Rightarrow \frac{1}{2\pi} \int_{-\omega_s/2}^{\omega_s/2} T e^{j\omega t} d\omega$$

$$= \frac{T}{2\pi} \int_{-\omega_s/2}^{\omega_s/2} e^{j\omega t} d\omega \Rightarrow \frac{T}{2\pi} \left\{ \frac{e^{j\omega t}}{jt} \right\}_{-\omega_s/2}^{\omega_s/2}$$

$$\Rightarrow \frac{T}{2\pi jt} \left[\frac{e^{j\omega_s t/2} - e^{-j\omega_s t/2}}{2j} \right]$$

$$\Rightarrow \frac{T}{\pi t} \sin \frac{\omega_s t}{2}$$

$$\Rightarrow \frac{T}{\pi t} \sin \left(\frac{\omega_s t}{2} \right)$$

$$h(t) \Rightarrow \frac{T}{\pi t} \sin \left(\frac{\pi t}{T} \right)$$

$$h(t) = \frac{T}{\pi t} \sin \left(\frac{\pi t}{T} \right)$$

$$\Rightarrow \frac{T}{\pi} \frac{\sin \left(\frac{\pi t}{T} \right)}{t}$$

$$h(t - nT_s) = \frac{T}{\pi} \frac{\sin \frac{\pi}{T} (t - nT_s)}{(t - nT_s)} \quad \text{--- (4)} \Rightarrow \frac{T}{\pi} \text{sinc} \left(\frac{t - nT_s}{T} \right)$$

Sub (4) in (3)

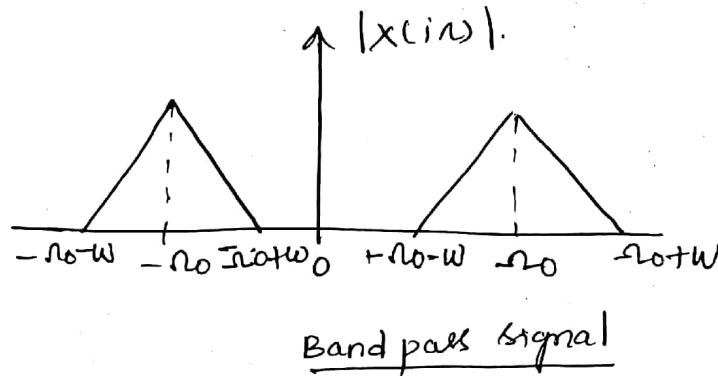
$$\therefore x(t) = \frac{T}{\pi} \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \text{sinc} \frac{\pi}{T} (t - nT_s)$$

Band pass signal:-

(8)

A signal $x(t)$ is a band pass signal whose frequency response $|X(\omega)| = 0$ for $|\omega - \omega_0| \geq W$. where ' ω_0 ' is referred as central frequency of band pass signal.

A band pass signal can be depicted as follows.



→ Sampling of bandpass signals:-

Let us consider a signal $x(t)$ whose highest frequency component is ' f_m ' and low frequency component ' $f_L = 0$ '. Then the sampling frequency must be greater than or equal to $2f_m$.

$$\text{ie; } f_s \geq 2f_m.$$

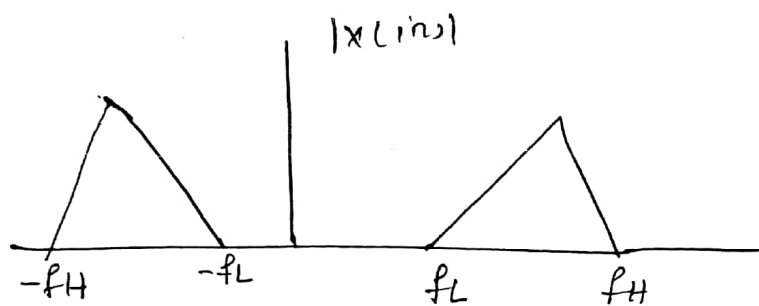
$$\therefore f_s \geq 2(f_H - f_L), \text{ if } f_L = 0.$$

$$\boxed{f_s \geq 2f_H} \rightarrow \text{No aliasing.}$$

$$f_s \geq 2(f_H - f_L), \text{ considering minimum condition}$$

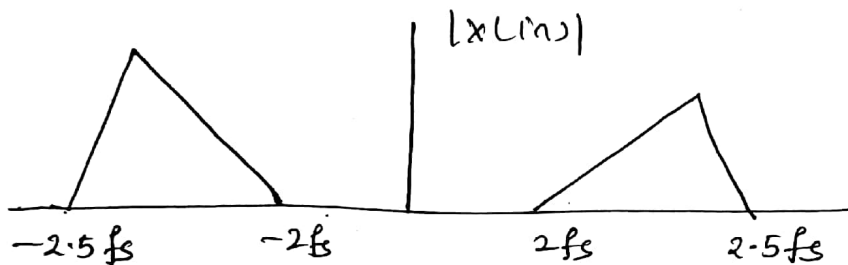
$$\text{ie; } f_s \geq 2(f_H - f_L), f_L \neq 0, \text{ No aliasing.}$$

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Band pass signal with f_H and f_L

Let us consider a sampling frequency $f_s = 2(f_H - f_L)$.
 For our convenience f_L is integral multiple of f_s - i.e; $f_L = n f_s$.
 then the above spectra is shifted to right side and also left side by an amount f_s .



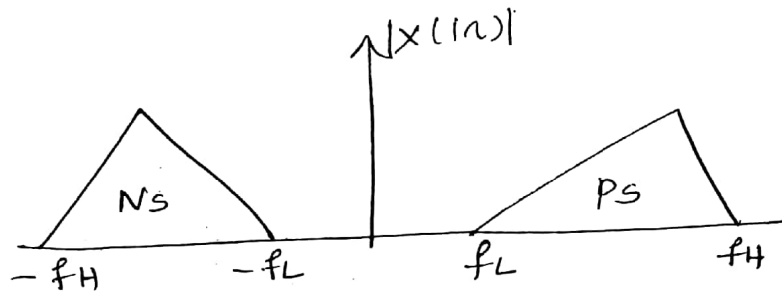
The above spectrum extends between ~~2f_s~~ $2f_s$ and $2.5f_s$.
 there is no overlap. Similarly if the spectrum of $x(t)$ extends between $2.5f_s$ and $3f_s$, there will be no overlap.

However, if $x(t)$ were not confined either to first half or second half of the interval b/w sampling frequency, there would be overlap b/w the spectra pattern and signal recovery would not possible

Hence minimum sampling frequency is $f_s \geq 2(f_H - f_L)$

(9)

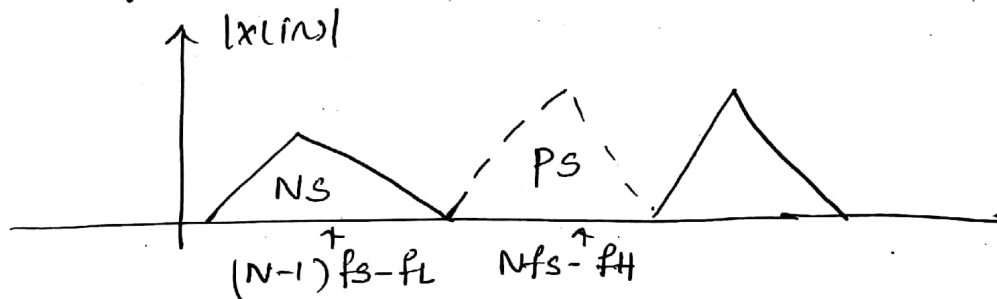
The above spectrum can be represented as positive spectra (PS) and negative spectra (NS) as.



If we maintain $f_s = 2(f_H - f_L)$ then the patterns ^{of PS} will not overlap with PS. The shifted pattern of NS to the left of NS will not overlap with PS.

However right shifted patterns of NS cause an overlap.

The right shift pattern of NS due to $(N-1)^{th}$ and N^{th} harmonic of sampling waveform.



To avoid, overlaps

$$(N-1)f_s - f_L \leq f_L$$

$$Nf_s - f_H > f_L$$

$$\text{Let } B = f_H - f_L$$

$$(N-1)f_s \leq 2(f_H - B) \text{ and}$$

$$Nf_s \geq 2f_H$$

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Let $K = \frac{f_H}{B}$, then $f_s \leq 2B \left(\frac{K-1}{N-1} \right)$ and
 $f_s \geq 2B \left(\frac{K}{N} \right)$ in which $K \geq N$ since
 $f_s \geq 2B$

From the above discussion, a band pass signal with highest frequency f_H and bandwidth B , can be recovered from its samples through bandpass filtering by sampling it with high frequency $f_s = \frac{2f_H}{K}$, where K is largest integer not exceeding $\frac{f_H}{B}$.