

18-1. Introduction

A hydraulic turbine is a *prime mover* (a machine which uses the raw energy of a substance and converts into mechanical energy) that uses the energy of flowing water and converts it into the mechanical energy (in the form of rotation of the runner). This mechanical energy is used in running an electric generator which is directly coupled to the shaft of the hydraulic turbine; from this electric generator, we get electric power which can be transmitted over long distances by means of transmission lines and transmission towers. The hydraulic turbines are also known as 'water turbines' since the fluid medium used in them is water.

First hydroelectric station was probably started in America in 1882 and thereafter development took place very rapidly. In India, the first major hydroelectric development of 4.5 MW capacity name as Sivasamudram Scheme in Mysore was commissioned in 1902.

Hydro (water) power is a conventional renewable source of energy which is clean, free from pollution and generally has a good environment effect. However, the following factors are major obstacles in the utilisation of hydropower resources:

- (i) Large investments,
- (ii) Long gestation period, and
- (iii) Increased cost of power transmission.

Fig. 18-1, shows the flow sheet of hydroelectric power plant.

18-2. Classification of Hydraulic Turbines

The hydraulic turbines are classified as follows :

1. According to the head and quantity of water available.
2. According to the name of the originator.
3. According to the action of water on moving blades.

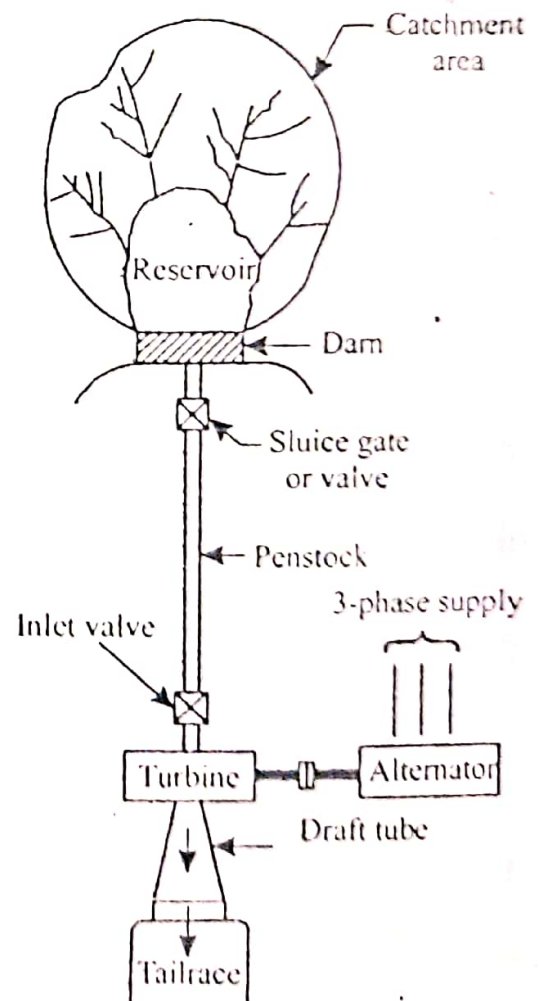


Fig. 18.1. Flow sheet of hydroelectric power plant.

4. According to the direction of flow of water in the runner.
5. According to the disposition of the turbine shaft.
6. According to the specific speed N_s .

1. According to the head and quantity of water available :

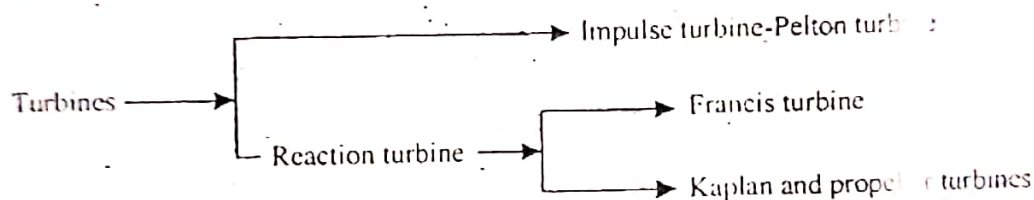
- (i) *Impulse turbine ... requires high head and small quantity of flow.*
- (ii) *Reaction turbine ... requires low head and high rate of flow.*

Actually there are two types of reaction turbines, one for medium head and medium flow and the other for low head and large flow.

2. According to the name of the originator :

- (i) *Pelton turbine ... named after Lester Allen Pelton of California (U.S.A.). It is an impulse type of turbine and is used for high head and low discharge*
- (ii) *Francis turbine ... named after James Bichens Francis. It is a reaction type of turbine from medium high to medium low heads and medium small to medium large quantities of water.*
- (iii) *Kalpan turbine ... named after Dr. Victor Kaplan. It is a reaction type of turbine for low heads and large quantities of flow.*

3. According to action of water on the moving blades :



4. According to direction of flow of water in the runner :

- (i) Tangential flow turbines (Pelton turbine)
- (ii) Radial flow turbine (no more used)
- (iii) Axial flow turbine (Kaplan turbine)
- (iv) Mixed (radial and axial) flow turbine (Francis turbine).

In *tangential flow* turbine of Pelton type the water strikes the runner tangential to the path of rotation.

In *axial flow* turbine water flows parallel to the axis of the turbine shaft. Kaplan turbine is an axial flow turbine. In Kaplan turbine the runner blades are *adjustable and can be rotated* about pivot, fixed to the boss of the runner. If the runner blades of the axial flow turbines are *fixed*, these are called "*propeller turbines*".

In *mixed flow* turbines the water enters the blades radially and comes out axially, parallel to the turbine shaft. *Modern Francis turbines have mixed flow runners.*

5. According to the disposition of the turbine shaft :

Turbine shaft may be either vertical or horizontal. In modern practice, Pelton turbines usually have horizontal shafts whereas the rest, especially the large units, have vertical shafts.

6. According to specific speed :

The *specific speed* of a turbine is defined as the speed of a geometrically similar turbine that would develop *1 kW* under *1 m* head. All geometrically similar turbines (irrespective of the sizes) will have the same specific speeds when operating under the same head.

$$\text{Specific speed, } N_s = \frac{N \sqrt{P}}{H^{5/4}}$$

where, N = The normal working speed,
 P = Power output of the turbine, and
 H = The net or effective head in metres.

Turbines with low specific speeds work under high head and low discharge conditions, while high specific speed turbines work under low head and high discharge conditions.

The following table gives the comparison between the impulse and reaction turbines with regard to their operation and application.

Table 18.1. Comparison between Impulse and Reaction Turbines

No.	Aspects	Impulse turbine	Reaction turbines
1.	Conversion of fluid energy	The available fluid energy is converted into K.E. by a nozzle.	The energy of the fluid is partly transformed into K.E. before it (fluid) enters the runner of the turbine.
2.	Changes in pressure and velocity	The pressure remains same (atmospheric) throughout the action of water on the runner.	After entering the runner with an excess pressure, water undergoes changes both in velocity and pressure while passing through the runner.
3.	Admittance of water over the wheel	Water may be allowed to enter a part or whole of the wheel circumference.	Water is admitted over the circumference of the wheel.
4.	Water-tight casing	Required	Not necessary.
5.	Extent to which the water fills the wheel/turbine	The wheel/turbine does not run full and air has a free access to the buckets.	Water completely fills all the passages between the blades and while flowing between inlet and outlet sections does work on the blades.
6.	Installation of unit	Always installed above the tail race. No draft tube is used.	Unit may be installed above or below the tail race-use of a draft tube is made.
7.	Relative velocity of water	Either remaining constant or reduces slightly due to friction.	Due to continuous drop in pressure during flow through the blade, the relative velocity increases.
8.	Flow regulation	— By means of a needle valve fitted into the nozzle. — Impossible without loss.	— By means of a guide-vane assembly. — Always accompanied by loss.

18.1. Impulse Turbines—Pelton Wheel

In an impulse turbine the *pressure energy of water is converted into kinetic energy* when passed through the nozzle and forms the high velocity jet of water. The formed water jet is used for driving the wheel.

Pelton wheel (named after the American engineer Lester Allen Pelton), among the various impulse turbines that have been designed and utilized, is by far the important. The Pelton wheel or Pelton turbine is a *tangential flow impulse turbine*.

Important Pelton turbine installations in India :

S. No.	Scheme/Project	Location (State)	Source of water
1.	Koyana hydroelectric project	Koyana (Maharashtra)	Koyana river
2.	Mahatma Gandhi hydroelectric works	Sharavathi (Karnataka)	Sharavathi river
3.	Mandi hydroelectric scheme	Joginder Nagar (Himachal Pradesh)	Uhl river
4.	Pallivasal power station	Pallivasal (Kerala)	Mudirapuzle river
5.	Pykara hydroelectric scheme	Pykara (Tamil Nadu)	Pykara river.

18.3.1. Construction and working of Pelton Wheel/Turbine

A Pelton wheel/turbine consists of a rotor, at the periphery of which are mounted equally spaced double hemispherical or double ellipsoidal buckets. Water is transferred from a high head source through penstock which is fitted with a nozzle, through which the water flows out at a high speed jet. A needle spear moving inside the nozzle controls the water flow through the nozzle and the same time, provides a smooth flow with negligible energy loss. All the available potential energy is thus converted into kinetic energy before the jet strikes the buckets of the runner. The pressure all over the wheel is constant and equal to atmosphere, so that energy transfer occurs due to purely impulse action.

The pelton turbine is provided with a casing, the function of which is to prevent the splashing of water and to discharge water to the tail race.

When the nozzle is completely closed by moving the spear in the forward direction the amount of water striking the runner is reduced to zero but the runner due to inertia continues revolving for a long time. In order to bring the runner to rest in a short time, a nozzle (brake) is provided which directs the jet of water on the back of buckets; this jet of water is called braking jet.

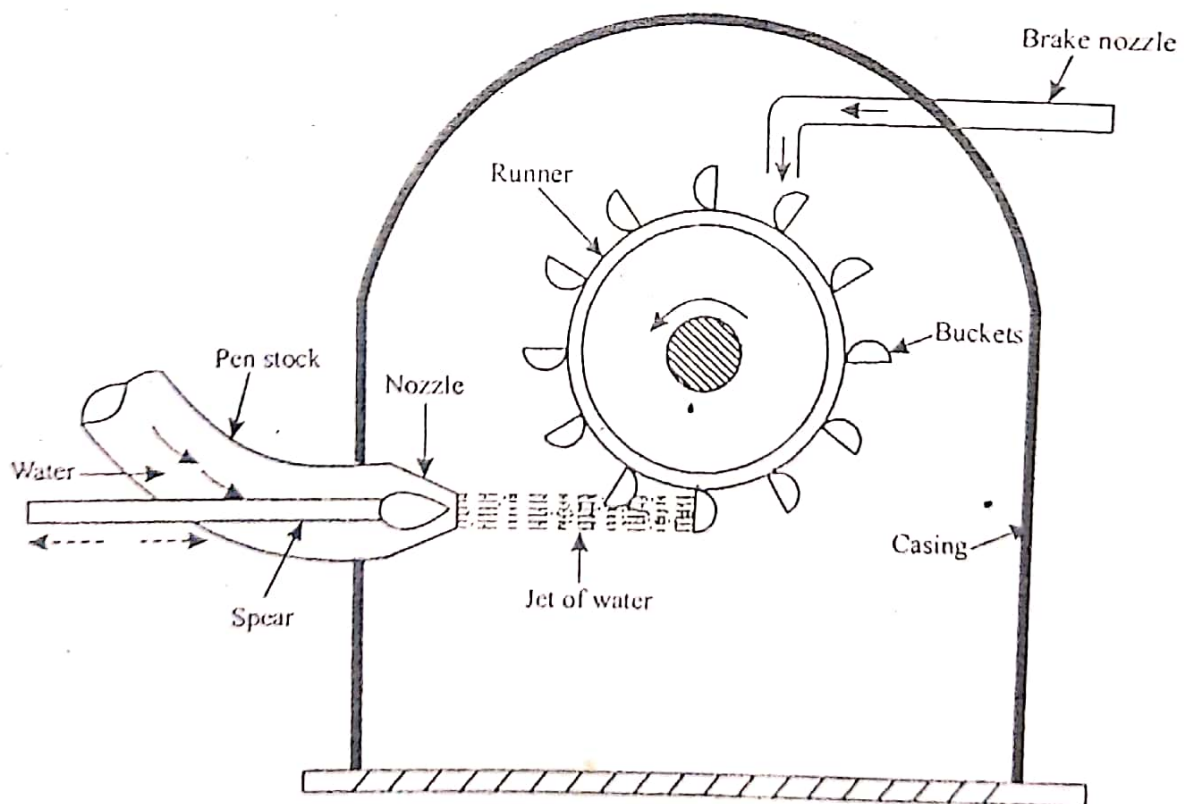
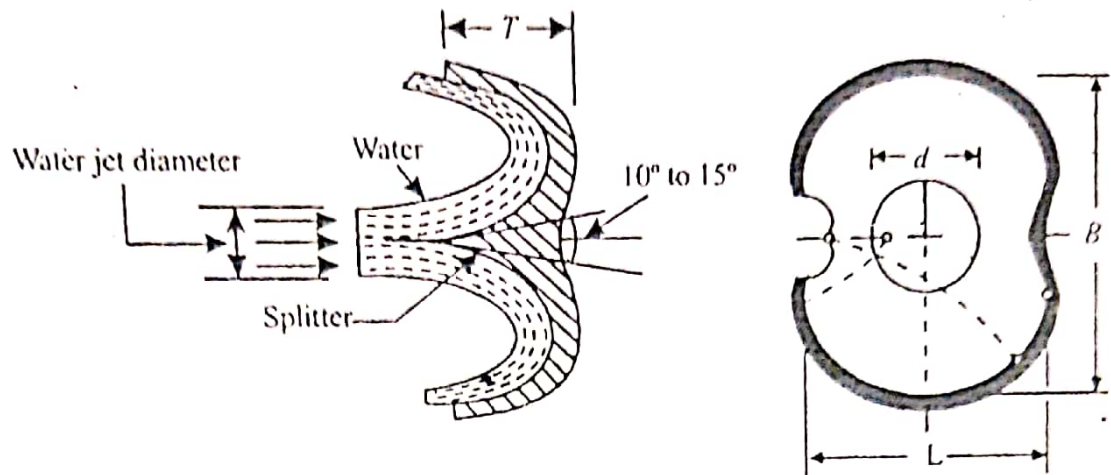


Fig. 18.2. Pelton wheel.

Speed of the turbine runner is kept constant by a governing mechanism that automatically regulates the quantity of water flowing through the runner in accordance with any variation of load.

Fig. 18.2. shows a schematic diagram of a Pelton wheel, while Fig. 18.3. shows two views of its bucket.



$$\frac{L}{d} = 2 \text{ to } 3; \frac{B}{d} = 3 \text{ to } 4; \frac{D}{d} = 11 \text{ to } 16; \frac{T}{d} = 0.8 \text{ to } 1.2; \text{notch (width)} = 1.1d + 5 \text{ mm}$$

Fig. 18.3. The bucket dimensions.

The jet emerging from the nozzle hits the splitter symmetrically and is equally distributed into the two halves of hemispherical bucket as shown. The bucket centre line cannot be made exactly like a mathematical cusp, partly because of manufacturing difficulties and partly because the jet striking the cusp invariably carries particles of sand and other abrasive material which tend to wear it down. The inlet angle of the jet is therefore between 1° and 3° , but it is always assumed to be zero in all calculations. Then the relative velocity of the jet leaving the bucket would be opposite in direction to the relative velocity of the entering jet; this cannot be achieved in practice since the jet leaving the bucket would then strike the back of the succeeding bucket to cause splashing and interference so that overall turbine efficiency would fall to low values. Hence, in practice, the angular deflection of the jet in the bucket is limited to about 165° or 170° , and the bucket is therefore slightly smaller than a hemisphere in size.

Fig. 18.4. shows a section through a horizontal-impulse turbine.

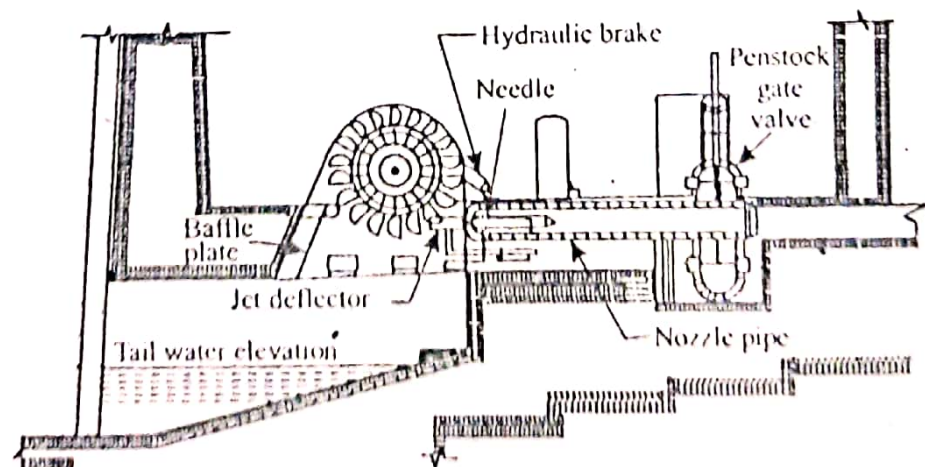


Fig. 18.4. Section through a horizontal-impulse turbine.

18.3.2. Work done and Efficiency of a Pelton Wheel

Fig. 18.5. shows the velocity triangles.

Let, N = Speed of wheel in r.p.m.,
 D = Diameter of the wheel,

d = Diameter of the jet,

u = Peripheral (or circumferential) velocity of runner. It will be same at inlet and outlet of the runners at the mean pitch. (i.e. $u = u_1 = u_2$)

$$= \frac{\pi DN}{60},$$

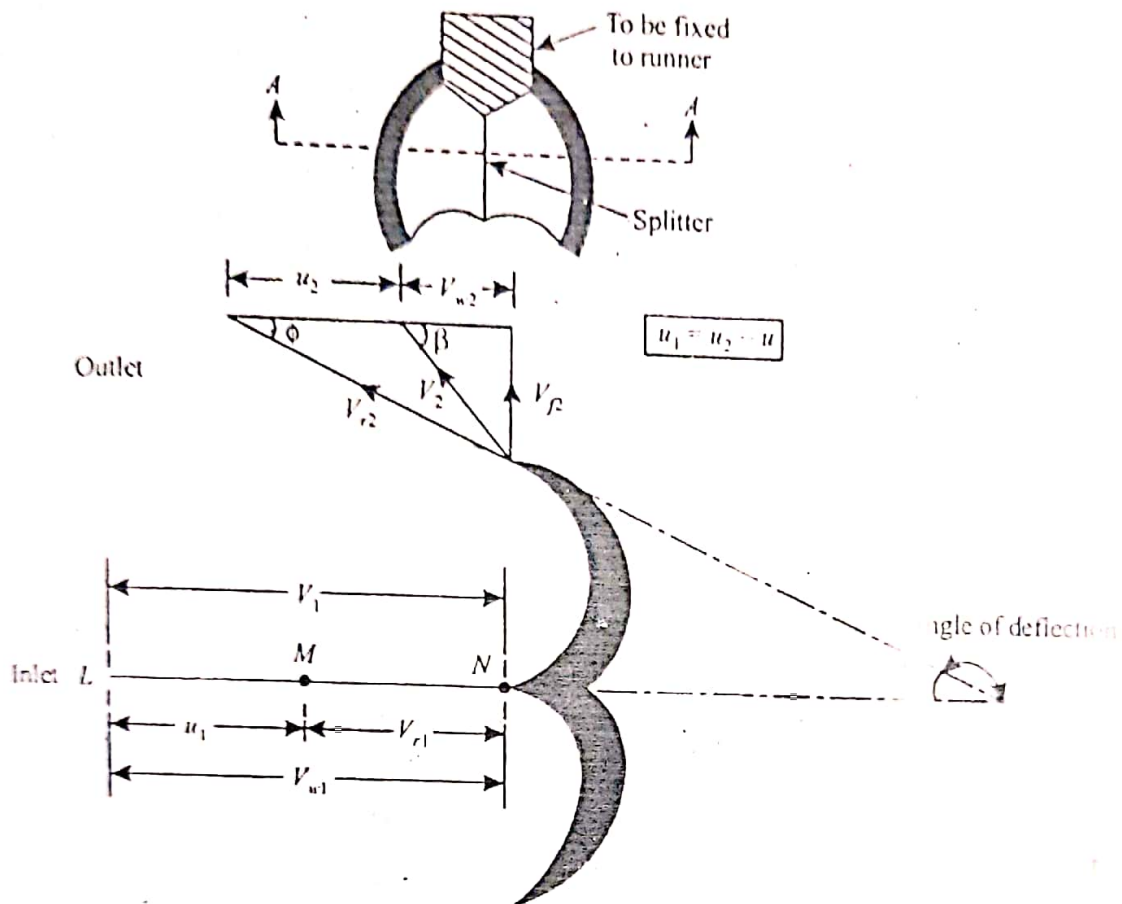


Fig. 18.5. Velocity triangles.

V_1 = Absolute velocity of water at inlet,

V_{r1} = Jet velocity relative to vane/bucket at inlet,

α = Angle between the direction of the jet and direction of motion of the vane/bucket (also called *guide angle*),

θ = Angle made by the relative velocity (V_{r1}) with the direction of motion at inlet (also called *vane angle at inlet*),

V_{w1} and V_f = The components of the velocity of the jet V_1 , in direction of motion and perpendicular to the direction of motion of the vane respectively,

V_{w1} is also known as *velocity of whirl* at inlet,

V_f is also known as *velocity of flow* at inlet,

V_2 = Velocity of jet, leaving the vane or velocity of jet at outlet of the vane,

V_{r2} = Relative velocity of the jet with respect to the vane at outlet,

ϕ = Angle made by the relative velocity V_{r2} with the direction of motion of the vane at outlet and also called *vane angle at outlet*,

β = Angle made by the velocity V_2 with the direction of motion of the vane at outlet, and

V_{w2} and V_f = Components of the velocity V_2 , in the direction of motion of vane and perpendicular to the direction of motion of vane at outlet;

V_{w2} is also called the *velocity of whirl at outlet*, and

V_r is also called the *velocity of flow at outlet*.

Inlet. The velocity triangle at *inlet* will be a *straight line* where

$$V_{r1} = V_1 - u_1 = V_1 - u, \quad V_{w1} = V_1 \quad (\because u_1 = u_2 = u)$$

$$\alpha = 0 \text{ and } \theta = 0$$

Outlet : From velocity triangle at outlet, we have

$$V_{r2} = KV_{r1},$$

where, K = blade friction co-efficient, *slightly less than unity*. Ideally when bucket surfaces are *perfectly smooth* and energy losses due to impact at splitter are *neglected*, $K = 1$

and,
$$V_{w2} = V_{r2} \cos \phi - u_2 = V_{r2} \cos \phi - u \quad (\because u_1 = u_2 = u) \text{ (When } \beta < 90^\circ)$$

Depending upon magnitude of the peripheral speed (u), the unit may have a slow, medium or fast runner and the angle β and V_{w2} will vary as follows :

- | | | |
|-------------------|--------------------|--------------------|
| i) Slow runner | $\beta < 90^\circ$ | (V_{w2} is -ve) |
| ii) Medium runner | $\beta = 90^\circ$ | ($V_{w2} = 0$) |
| iii) Fast runner | $\beta > 90^\circ$ | (V_{w2} is +ve) |

The force exerted by the jet of water in the direction of motion is given as

$$F = \rho a V_1 (V_{w1} + V_{w2}) \quad \dots(18.1)$$

and a are the mass density and area of jet $\left(a = \frac{\pi}{4} d^2\right)$ respectively.

Now work done by the jet on runner per second

$$= F \times u = \rho a V_1 (V_{w1} + V_{w2}) \times u \quad \dots(18.2)$$

Work done per second per unit weight of water striking

$$= \frac{\rho a V_1 (V_{w1} + V_{w2}) \times u}{\text{Weight of water striking}} = \frac{\rho a V_1 (V_{w1} + V_{w2}) \times u}{\rho a V_1 \times g}$$

$$= \frac{1}{g} (V_{w1} + V_{w2}) u \quad \dots[18.2(a)]$$

The energy supplied to the jet at inlet is in the form of K.E. and is equal to $\frac{1}{2} m V_1^2$.

$$\therefore \text{Kinetic energy (K.E.) of jet per second} = \frac{1}{2} (\rho a V_1) \times V_1^2$$

$$\therefore \text{Hydraulic efficiency, } \eta_h = \frac{\text{Work done per second}}{\text{K.E. of jet per second}} = \frac{\rho a V_1 (V_{w1} + V_{w2}) \times u}{\frac{1}{2} (\rho a V_1) \times V_1^2}$$

$$\text{or } \eta_h = \frac{2 (V_{w1} + V_{w2}) \times u}{V_1^2} \quad \dots(18.3)$$

From inlet and outlet velocity triangles, we have

$$V_{w1} = V_1, \quad V_{r1} = V_1 - u_1 = V_1 - u$$

$$V_{w2} = V_{r2} \cos \phi - u_2 = V_{r2} \cos \phi - u = K V_{r1} \cos \phi - u = K (V_1 - u) \cos \phi - u$$

Substituting the values of V_{w1} and V_{w2} in eqn (18-3), we have

$$\eta_h = \frac{2 [V_1 + K (V_1 - u) \cos \phi - u] u}{V_1^2} = \frac{2 [(V_1 - u) (1 + K \cos \phi)] u}{V_1^2} \quad \dots(18.4)$$

The hydraulic efficiency will be *maximum* for given value of V_1 when

$$\frac{d}{du}(\eta_h) = 0$$

$$\text{i.e.,} \quad \frac{d}{du} \left[\frac{2(V_1 - u)(1 + \cos \phi)u}{V_1^2} \right] = 0$$

$$\text{or,} \quad \frac{2(1 + K \cos \phi)}{V_1^2} \times \frac{d}{du}(V_1 u - u^2) = 0$$

$$\text{Since,} \quad \frac{2(1 + K \cos \phi)}{V_1^2} \neq 0, \quad \therefore \quad \frac{d}{du}(V_1 u - u^2) = 0$$

$$\text{or,} \quad V_1 - 2u = 0 \quad \text{or} \quad u = \frac{V_1}{2} \quad \dots(18.5)$$

The above equation states that hydraulic efficiency of a Pelton wheel is maximum when the velocity of the wheel is half the velocity of jet of water at inlet. The maximum efficiency can be obtained by substituting the value of $u = \frac{V_1}{2}$ in eqn. (18.4).

$$(\eta_h)_{\max} = \frac{2 \left(V_1 - \frac{V_1}{2} \right) (1 + K \cos \phi) \frac{V_1}{2}}{V_1^2} = \frac{2 \times \frac{V_1}{2} (1 + K \cos \phi) \times \frac{V_1}{2}}{V_1^2}$$

$$\text{or,} \quad (\eta_h)_{\max} = \frac{(1 + K \cos \phi)}{2} \quad \dots(18.6)$$

If friction factor, $K = 1$ (i.e., assuming no friction), we have

$$(\eta_h)_{\max} = \frac{1 + \cos \phi}{2} \quad \dots[18.6(a)]$$

18-3-3. Definitions of Heads and Efficiencies

Fig. 18-6 shows a general layout of a hydroelectric power plant using an impulse turbine (Pelton wheel).

1. **Gross head.** The gross (total) head is the difference between the water level at the reservoir (also known as the head race) and the water level at the tail race. It is denoted by H_g .

2. **Net or effective head.** The head available at the inlet of the turbine is known as net or effective head. It is denoted by H and is given by

$$H = H_g - h_f - h$$

where, h_f = Total loss of head between the head race and entrance of the turbine

$$= \frac{4fLV^2}{D \times 2g} \quad (L = \text{length of penstock, } D = \text{diameter of penstock, } V = \text{velocity of flow in penstock), and}$$

h = Height of nozzle above the tail race.

3. **Efficiencies.** The following are the important efficiencies of turbine :

(i) **Hydraulic efficiency (η_h).** It is defined as the ratio of power developed by the runner to the power supplied by the jet at entrance to the turbine.

$$\text{Mathematically,} \quad \eta_h = \frac{\text{Power developed by the runner}}{\text{Power supplied at the inlet of turbine}}$$

$$\begin{aligned}
 &= \frac{\rho Q_a (V_{w1} \pm V_{w2}) u}{w Q_a H} = \frac{\left(\frac{w}{g}\right) Q_a (V_{w1} \pm V_{w2}) u}{w Q_a H} \\
 &= \frac{(V_{w1} \pm V_{w2}) u}{g H} = \frac{H_r}{H} \quad \dots(18.7)
 \end{aligned}$$

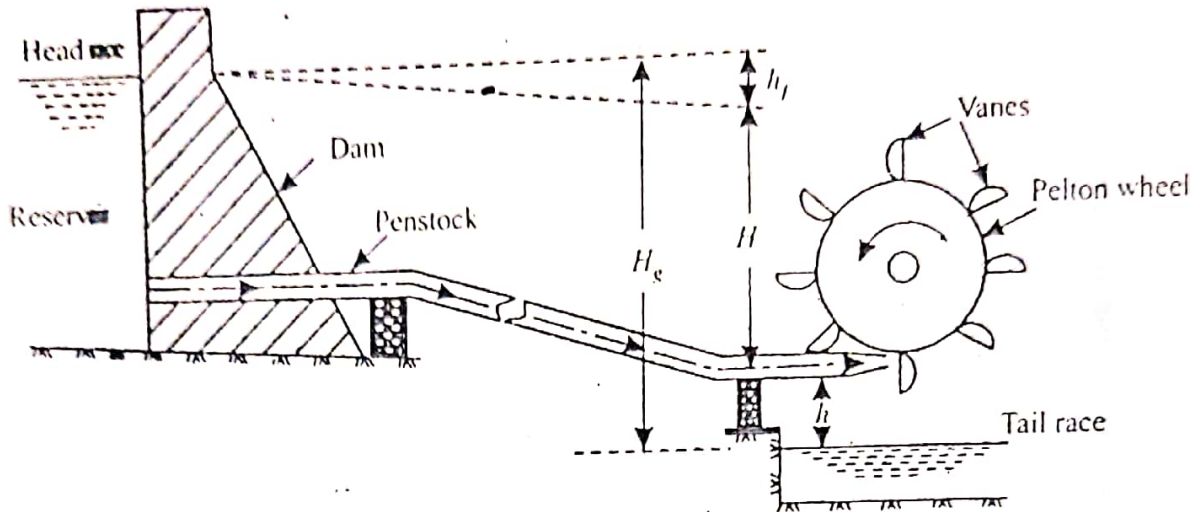


Fig. 18.6. Layout of hydroelectric power plant using an impulse turbine (Pelton wheel).

where, V_{w1}, V_{w2} = Velocities of whirl at inlet and outlet respectively,
 u = Tangential velocity of vane,
 H = Net head on the turbine, and
 Q_a = Actual flow rate to turbine runner (bucket).

The parameter, $H_r = \frac{1}{g} (V_{w1} + V_{w2}) u$ represents the energy transfer per unit weight of water and is referred to as the 'runner head' or 'Euler head'.

$H - H_r = \Delta H$ = Hydraulic losses within the turbine.

(ii) **Mechanical efficiency (η_m)**. It is defined as the ratio of the power obtained from the shaft of the turbine to the power developed by the runner. These two powers differ by the amount of mechanical losses, i.e., bearing friction, etc.

$$\begin{aligned}
 \text{Mathematically, } \eta_m &= \frac{\text{Power available at the turbine shaft}}{\text{Power developed by turbine runner}} = \frac{\text{Shaft power}}{\text{Bucket power}} \\
 &= \frac{P}{w Q_a \left(\frac{V_{w1} + V_{w2}}{g} \right) u} = \frac{P}{w Q_a H_r} \quad \dots(18.8)
 \end{aligned}$$

Values of mechanical efficiency for a Pelton wheel usually lie between 97 to 99 percent depending on sized capacity of the unit.

(iii) **Volumetric efficiency (η_v)**. The volumetric efficiency is the ratio of the volume of water actually striking the runner to the volume of water supplied by the jet to the turbine. That is

$$\eta_v = \frac{\text{Volume of water actually striking the runner } (Q_a)}{\text{Total water supplied by the jet to the turbine } (Q)} \quad \dots(18.9)$$

For Pelton turbines, $\eta_v = 0.97$ to 0.99 .

(iv) **Overall efficiency (η_o).** It is defined as the *ratio of power available at the turbine shaft to the power supplied by the water jet*. That is

$$\eta_o = \frac{\text{Power available at the turbine shaft}}{\text{Power available from the water jet}} = \frac{\text{Shaft power}}{\text{Water power}} = \frac{P}{wQH}$$

(where, Q = the total discharge in m^3/s supplied by the jet.)

The values of overall efficiency for a Pelton wheel lie between 0.85 to 0.90.

The individual efficiencies may be combined to give,

$$\eta_o = \eta_h \times \eta_m \times \eta_v \\ = \frac{H_r}{H} \times \frac{P}{wQ_a H_r} \times \frac{Q_a}{Q} = \frac{P}{wQH}, \text{ which is the same as defined vide eqn (18.1)}$$

If η_g is the efficiency of a generator, then power output of hydroelectric turbine + hydrogenerator =

$$= (wQH) \times \eta_o \times \eta_g$$

The product $\eta_o \times \eta_g$ is known as *hydroelectric plant efficiency*.

18.3.4. Design Aspects of Pelton wheel

The following points should be considered while *designing a Pelton wheel* :

1. **Velocity of jet.** The velocity of jet at inlet is given by,

$$V_1 = C_v \sqrt{2gH} \quad (18.2)$$

where,

C_v = Co-efficient of velocity (= 0.98 or 0.99) and

H = Net head on turbine.

2. **Velocity of wheel.** The velocity of wheel (u) is given by,

$$u = K_u \sqrt{2gH} \quad (18.3)$$

where, K_u = Speed ratio. It varies from 0.43 to 0.48.

3. **Angle of deflection of the jet.** The angle of deflection of the jet through the buckets is taken 165° if no angle of deflection is given.

4. **Mean diameter of the wheel (D).** The mean diameter or pitch diameter D of the Pelton wheel given by,

$$u = \frac{\pi DN}{60} \text{ or } D = \frac{60 u}{\pi N} \quad (18.4)$$

5. **Jet ratio (m).** It is defined as the *ratio of the pitch diameter (D) of the Pelton wheel to the diameter of the jet (d)*. It is denoted by ' m ' and is given as :

$$m = \frac{D}{d} \text{ (lies between 11 and 16 for maximum hydraulic efficiency)} \quad (18.5)$$

Normally, the jet ratio is adopted as 12 in practice.

6. **Bucket dimensions.** Some of the main dimensions of the bucket of a Pelton wheel are follows :

Refer to Fig. 18.3 : $B = 3 \text{ to } 4d$; $L = 2 \text{ to } 3d$; $T = 0.8 \text{ to } 1.2d$.

7. **Number of jets.** Normally a Pelton wheel has one nozzle or one jet. However, a number nozzles may be employed when more power is to be produced with the same wheel. Theoretically, nozzles can be used on Pelton wheel. However, practical considerations limit the use of not more than two jets per runner for a vertical runner and not more than four jets per runner if it is of horizontal configurations.

Number of jets is obtained by *dividing the total rate of flow through the turbine by the rate flow of water through a single jet*.

$$= \frac{0.032 \times 4000 \times (0.0177V_1)^2}{0.6 \times 2 \times 9.81} + \frac{V_1^2}{2 \times 9.81} \times \frac{1}{(0.97)^2} \quad (\because V = 0.0177V_1)$$

$$= 0.0034V_1^2 + 0.054V_1^2 = 0.0574V_1^2$$

$$V_1 = \left(\frac{400}{0.0574} \right)^{1/2} = 83.48 \text{ m/s}$$

$$\therefore \text{Flow rate} = \text{Area of jet} \times \text{velocity of jet} = \frac{\pi}{4} \times (0.08)^2 \times 83.48 = 0.419 \text{ m}^3/\text{s} \text{ (Ans.)}$$

The shaft power :

$$\text{Velocity of bucket, } u_1 = 0.48V_1 = 0.48 \times 83.48 = 40.07 \text{ m/s}$$

$$\text{Refer to Fig. 18.8, } V_{r1} = V_1 - u_1 = 83.48 - 40.07 = 43.4 \text{ m/s}$$

$$V_{w1} = V_1 = 83.48 \text{ m/s}$$

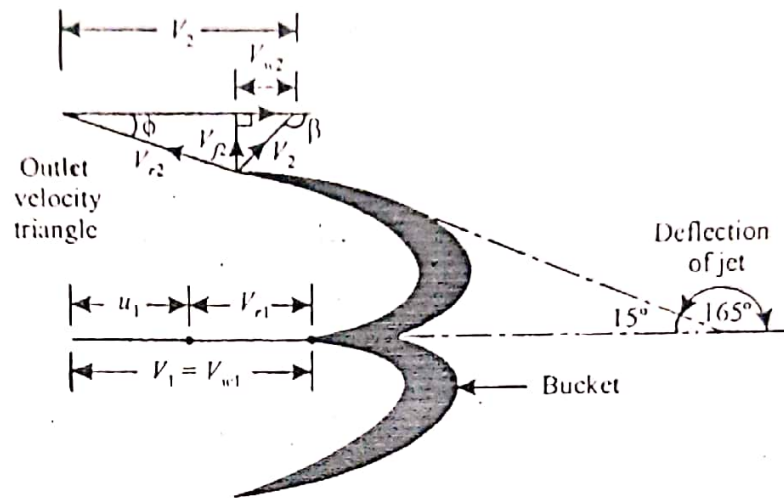


Fig. 18.8

From the outlet velocity triangle, we have

$$V_{r2} = 0.85V_{r1} = 0.85 \times 43.4 = 36.89 \text{ m/s}$$

$$V_{w2} = u_2 - V_{r2} \cos \phi$$

$$= 40.07 - 36.89 \times \cos 15^\circ = 4.44 \text{ m/s}$$

Mechanical efficiency,

$$\eta_m = \frac{\text{Shaft power}}{\text{Power given to runner}}$$

$$\therefore \text{Shaft power, } = \eta_m \times \text{power given to runner}$$

But power given to runner

$$= \frac{w}{g} Q (V_{w1} - V_{w2}) \times u$$

[Here -ve sign is taken since $\beta > 90^\circ$ (i.e., V_{w1} and V_{w2} are in the same direction)]

$$\therefore \text{Shaft power} = \eta_m \times \frac{w}{g} Q (V_{w1} - V_{w2}) \times u_1$$

$$= 0.9 \times \frac{9.81}{9.81} \times 0.419 (83.48 - 4.44) \times 40.07 \text{ kW} \quad (\because w = 9.81 \text{ kN/m}^3)$$

$$= 1194.3 \text{ kW (Ans.)}$$

Example 18.5. The water available for a Pelton wheel is $4 \text{ m}^3/\text{s}$ and the total head from reservoir to the nozzle is 250 m. The turbine has two runners with two jets per runner. All the four jets have the same diameters. The pipe is 3 km long. The efficiency of transmission through the pipeline and the nozzle is 91 % and efficiency of each runner is 90 %. The velocity co-efficient of each nozzle is 0.975 and co-efficient of friction ' $4f$ ' for the pipe is 0.0045. Determine :

[MLU]

- (i) The power developed by the turbine,
- (ii) The diameter of the jet, and
- (iii) The diameter of the pipeline.

Solution. Rate of flow, $Q = 4 \text{ m}^3/\text{s}$
 Total or gross head, $H_g = 250 \text{ m}$
 Total number of jets $= 2 \times 2 = 4$
 Length of pipe, $L = 3 \text{ km} = 3000 \text{ m}$
 Efficiency of transmission, $\eta = 91\%$
 Efficiency of each runner, $\eta_h = 90\%$
 Co-efficient of velocity, $C_v = 0.975$
 Co-efficient of friction, $4f = 0.0045$

- (i) The power developed by the runner :

$$\text{Efficiency of power transmission, } \eta = \frac{H_g - h_f}{H_g}$$

(where, h_f = loss of head due to friction)

$$\text{or, } 0.91 = \frac{250 - h_f}{250}$$

$$\therefore h_f = 250 - 250 \times 0.91 = 22.5 \text{ m}$$

$$\therefore \text{Net head on the turbine, } H = H_g - h_f = 250 - 22.5 = 227.5 \text{ m}$$

$$\text{Velocity of jet, } V_1 = C_v \sqrt{2gH} = 0.975 \sqrt{2 \times 9.81 \times 227.5} = 65.14 \text{ m/s}$$

Now, Water power = Kinetic energy of the jet

$$\begin{aligned} \frac{1}{2} m V_1^2 &= \frac{1}{2} \rho Q V_1^2 = \frac{1}{2} \times 1000 \times 4 \times 65.14^2 = 8486439 \text{ Nm/s} \\ &= 8486439 \text{ J/s or } W = 8486.44 \text{ kW} \end{aligned}$$

$$\text{But, hydraulic efficiency, } \eta_h = \frac{\text{Power developed by the turbine}}{\text{Water power}}$$

$$\text{or, } 0.9 = \frac{\text{Power developed by the turbine}}{8486.44}$$

$$\therefore \text{Power developed by the turbine} = 0.9 \times 8486.44 = 7637.8 \text{ kW (Ans.)}$$

- (ii) The diameter of the jet, d :

$$\text{Discharge per jet, } q = \frac{\text{Total discharge}}{\text{No. of jets}} = \frac{4}{4} = 1.0 \text{ m}^3/\text{s}$$

$$\text{But, } q = \frac{\pi}{4} \times d^2 \times V_1$$

$$\therefore 1.0 = \frac{\pi}{4} \times d^2 \times 65.14 \text{ or } d = \left(\frac{1.0 \times 4}{\pi \times 65.14} \right)^{1/2} = 0.14 \text{ m (Ans.)}$$

energy and when the water flows through the moving parts, there is a *change both in pressure and in the direction and velocity of flow of water*. As the water gives up its energy to the runner, both its pressure and absolute velocity get reduced. The water which acts on the runner blades is under a pressure *above atmospheric* and the runner passages are always completely filled with water.

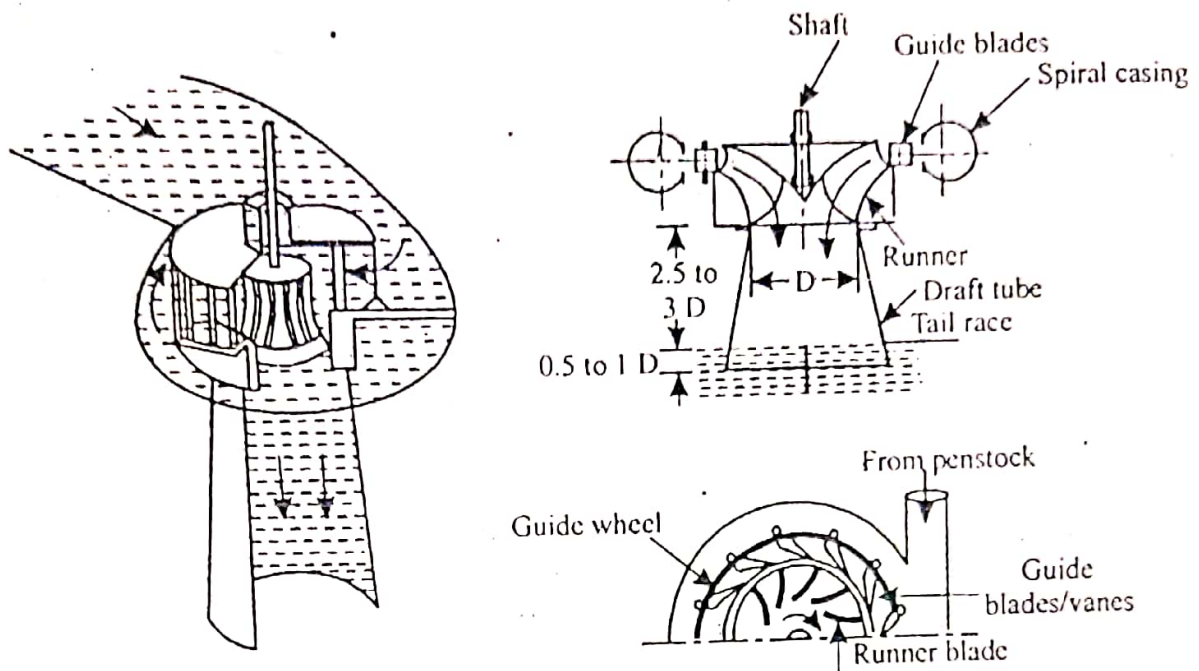


Fig. 18.11. Schematic diagram of a Francis turbine.

Important reaction turbines are *Francis, Kaplan and Propeller*.

18.4.1. Francis Turbine

Fig. 18.11 shows a schematic diagram of a Francis turbine. The *main parts of a Francis turbine* are:

1. *Penstock* ... It is a large size conduit which conveys water from the upstream of the dam/reservoir to the turbine runner.
2. *Spiral/scroll casing* ... It constitutes a closed passage whose cross-sectional area gradually decreases along the flow direction, area is maximum at inlet and nearly zero at exit.
3. *Guide vanes/wicket gates* ... These vanes direct the water onto the runner at an angle appropriate to the design. The motion to them is given by means of a hand wheel or automatically by a governor.
4. *Governing mechanism* ... It changes the position of the guide blades/vanes to affect a variation in water flow rate, when the load conditions on the turbine change.
5. *Runner and runner blades* ... — The driving force on the runner is both due to impulse and reaction effects;
— The number of runner blades usually varies between 16 to 24.
6. *Draft tube* ... It is a gradually expanding tube which discharges water, passing through the runner, to the tail race.

The modern Francis turbine is an *inward mixed flow reaction turbine* (in the earlier stages of development, Francis turbine had a purely radial flow runner), i.e. *water under pressure, enters the*

runner from the guide vanes towards the centre in radial direction and discharges out of the runner axially. The Francis turbine operates under *medium heads* and also requires *medium quantity* of water. It is employed in the medium head power plants. This type of turbine covers a wide range of heads. Water is brought down to the turbine through a *penstock* and directed to a number of stationary orifices fixed all around the circumference of the *runner*. These stationary orifices are commonly called as *guide vanes* or *wicket gates*.

The head acting on the turbine is partly transformed into kinetic energy and the rest remains as pressure head. There is a difference of pressure between the guide vanes and the runner which is called the *reaction pressure* and is responsible for the motion of the runner. That is why a Francis turbine is also known as *reaction turbine*.

In Francis turbine the pressure at inlet is more than that at the outlet. This means that the water in the turbine must flow in a closed conduit. Unlike the Pelton type, where the water strikes only a few of the runner buckets at a time, in the Francis turbine the runner is always full of water. The moment of runner is affected by the change of both the potential and kinetic energies of water. After doing the work the water is discharged to the tail race through a closed tube of gradually enlarging section. This is known as *draft tube*. It does not allow water to fall freely to tail race level as in the Pelton turbine. The free end of the draft tube is submerged deep in tail water making, thus, the entire water passage, right from the head race up to the tail race, *totally enclosed*.

Fig. 18.12 shows general layout of a hydroelectric power plant using a reaction turbine.

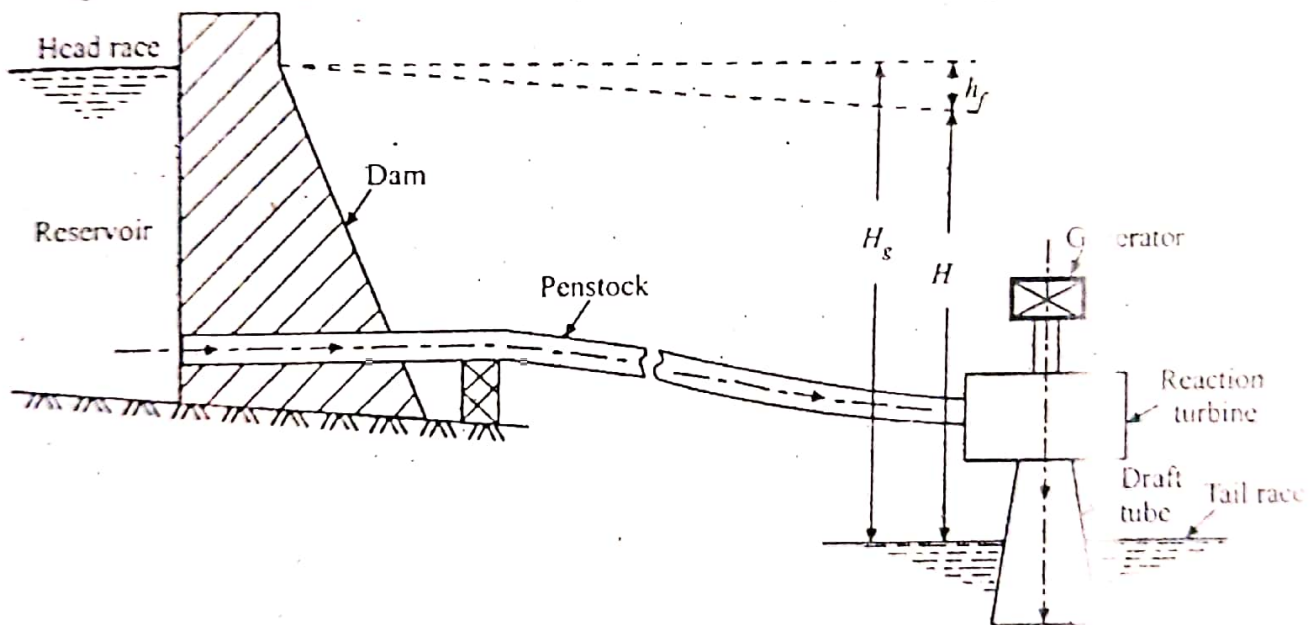


Fig. 18.12. General layout of a hydroelectric power plant using a reaction turbine.

Important Francis Turbine Installations in India :

S.No.	Scheme/Project	Location (State)	Source of water
1.	Bhakra dam project	Bhakra (Punjab)	Sutlej river
2.	Cauvery hydroelectric scheme	Siva Samudram (Karnataka)	Cauvery river
3.	Chambal hydroelectric scheme	Gandhi sagar (Rajasthan)	Chambal river
4.	Hirakud dam project	Hirakud (Orissa)	Hirakud river
5.	Rihand dam project	Rihand (Uttar Pradesh)	Rihand river

Important differences between Inward and Outward Flow Reaction Turbines

The following are the important differences between inward and outward flow reaction turbines :

S. No.	Aspects	Inward flow reaction turbine	Outward flow reaction turbine
	Entry of water	Water enters at the outer periphery, flows inward and towards the centre of the turbine and discharges at the outer periphery.	Water enters at the inner periphery flows outward and discharges at the outer periphery.
	Centrifugal head imparted	Negative (negative centrifugal head reduces the relative velocity of water at the outlet).	Positive (Positive centrifugal head increases the relative velocity of water at the outlet).
	Discharge	Does not increase.	The discharge increases.
	Speed control	Easy and effective.	Very difficult.
	Tendency of the wheel to race	Nil. The turbine adjusts the speed by itself.	If the turbine speed increases the wheel tends to race; the turbine cannot adjust the speed by itself.
	Suitability	Quite suitable for medium high heads; best suitable for large outputs and units.	Quite suitable for low or medium heads.
	Application	For power projects.	Practically obsolete.

18.4.1.1. Work done and efficiency of Francis turbine

Net head at the turbine runner : In the Fig. 18-12,

H_g = Gross head = Difference of water levels between head race and tail race

h_f = Loss of head in the penstock;

H = Net head = $(H_g - h_f)$. The net head is also called *available* or *working* or *operation head*.

Also,

$$H = \left[\begin{array}{c} \text{Total energy available at exit} \\ \text{from the penstock} \end{array} \right] - \left[\begin{array}{c} \text{total energy available at exit} \\ \text{from the draft tube} \end{array} \right]$$

$$= \left(\frac{p}{w} + \frac{V^2}{2g} + z \right)_{\text{penstock}} - \left(\frac{p}{w} + \frac{V^2}{2g} + z \right)_{\text{draft tube}}$$

If the draft tube exit is at tail race level, and the datum is also taken at that level, then,

$$H = \left(\frac{p}{w} + \frac{V^2}{2g} + z \right)_{\text{penstock}} - \frac{V_d^2}{2g}$$

(where, V_d = velocity at the exit of the draft tube)

Neglecting the velocity at the draft tube exit (V_d), we have

$$H = \left(\frac{p}{w} + \frac{V^2}{2g} + z \right) \quad \dots(18-16)$$

Work done by the runner :

Fig. 18-13. shows the runner and the velocity diagrams (inlet and outlet) for an inward flow reaction turbine. The general expression for the work done with usual notations according to the Euler momentum equation, is given by,

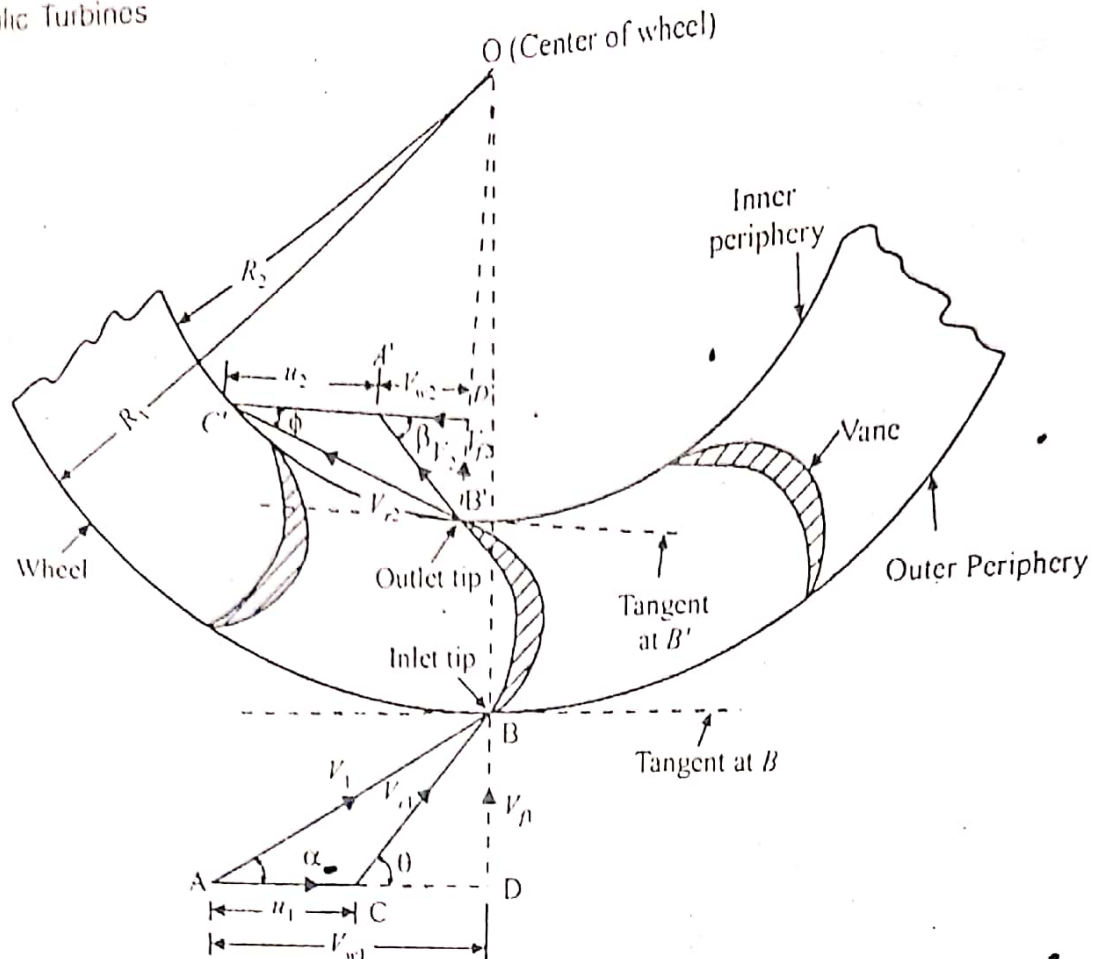


Fig. 18.13. Velocity diagrams for an inward flow reaction turbine.

$$\text{Work done} = \rho Q (V_{w1} u_1 \pm V_{w2} u_2)$$

$$= \frac{wQ}{g} (V_{w1} u_1 \pm V_{w2} u_2)$$

...(18-17)

where,

 Q = Discharge through the runner, m^3/s .The maximum output under given conditions is obtained when $V_{w2} = 0$.

Thus, the maximum work done is given by,

$$\text{Work done} = \frac{wQ}{g} (V_{w1} u_1)$$

...(18-18)

This discharge in this case is radial. For radial discharge, the absolute velocity at exit is radial.

Hydraulic efficiency, η_h :If H is the net head, then input to the turbine = wQH

$$\eta_h = \frac{\text{Power developed by the runner}}{\text{Power supplied to the turbine (water power)}} = \frac{\frac{wQ}{g} (V_{w1} u_1)}{wQH}$$

or,

$$\eta_h = \frac{V_{w1} u_1}{gH}$$

...(18-19)

$$\left[\begin{array}{l} \text{However, if the velocity of whirl at the exit is not zero, then} \\ \eta_h = \frac{V_{w1} u_1 \pm V_{w2} u_2}{gH} \end{array} \right] \quad \dots[18-19(a)]$$

The hydraulic efficiency of the Francis turbine varies from 85 to 90 percent.

Mechanical efficiency, η_m :

The mechanical efficiency is given by.

$$\eta_m = \frac{\text{Shaft power (P)}}{\text{Power developed by the runner}} \quad \dots(18-20)$$

Overall efficiency, η_0 :

The overall efficiency is given as

$$\eta_0 = \frac{\text{Shaft water}}{\text{Water power}} = \frac{P}{wQH} \quad \dots(18-21)$$

$$\text{and,} \quad \eta_0 = \eta_h \times \eta_m \quad [18-21(a)]$$

The overall efficiency varies from 80 to 90 percent.

18.4.1.2. Working proportions of a Francis turbine

The following working proportions pertain to a Francis turbine :

1. Ratio of width to diameter $\left(\frac{B}{D}\right)$:

The ratio of width (B_1) to the diameter of the wheel (D_1) at inlet is represented by n . Thus,

$$n = \frac{B_1}{D_1}$$

The value of n varies from 0.10 to 0.45.

2. Flow ratio (K_f) :

Flow ratio is the *ratio of the velocity of flow at inlet to the theoretical jet velocity*. Thus,

$$\text{Flow ratio,} \quad K_f = \frac{V_{f1}}{\sqrt{2gH}} \quad \dots(18-23)$$

The value of K_f varies from 0.15 to 0.30.

3. Speed ratio (K_u)

Speed ratio is the ratio of the peripheral speed at inlet to the theoretical jet velocity. Thus,

$$\text{Speed ratio,} \quad K_u = \frac{u}{\sqrt{2gH}}$$

The value of K_u ranges from 0.6 to 0.9.

18.4.1.3. Design of Francis turbine runner

The runner of a Francis turbine is required to be designed to develop a known power P , when running at a known speed N r.p.m. under a known head H . The design of the runner involves the determination of its size and the vane angles.

The design of a Francis turbine runner is carried out as follows :

1. Assume suitable values of η_0 , η_h , n , K_f and K_u .
2. Determine the required discharge Q from the relation

$$P = \eta_0 \times wQH \quad \dots(18-24)$$

3. Obtain the velocity of flow from the discharge and flow area.

Let B_1 , D_1 and t_1 respectively be the width, diameter and thickness of runner vane at inlet (Fig. 18.14).

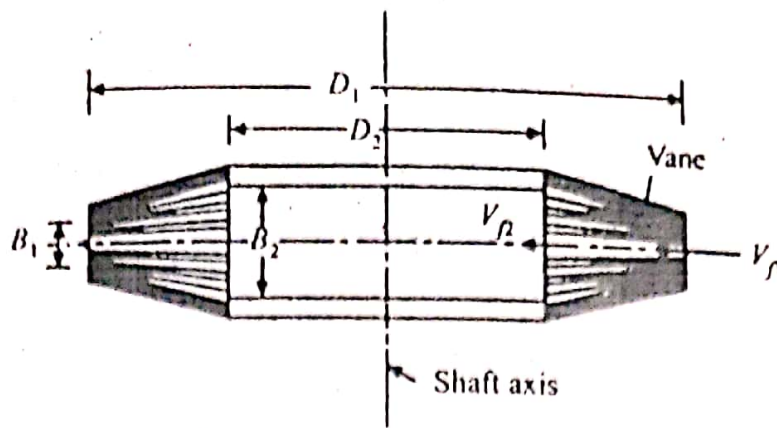


Fig. 18.14. Entry of flow to runner vane.

Then, total area at the outer periphery (i.e., at the runner inlet),

$$A = (\pi D_1 - Z_{r1}) B_1 = K_{r1} \pi D_1 B_1 \quad \dots (18.25)$$

where K_{r1} is known as *vane thickness factor/co-efficient*; its value is always less than unity (usually of the order of 0.95 or so).

Discharge = Area of flow $\times$ velocity of flow

$$\text{i.e.,} \quad Q = K_{r1} \pi D_1 B_1 \times V_{f1} \quad \dots (18.26)$$

$$\therefore \text{Flow velocity, } V_{f1} = \frac{Q}{K_{r1} \pi D_1 B_1} = \frac{Q}{K_r \pi n D_1^2} \quad (\because B_1 = n D_1) \quad \dots [18.26(a)]$$

$$\text{Also,} \quad V_{f1} = K_f \sqrt{2gH}, \quad \therefore K_f \sqrt{2gH} = \frac{Q}{K_r \pi n D_1^2}$$

$$\text{or,} \quad D_1 = \left[\frac{Q}{(K_f \sqrt{2gH}) K_r \pi n} \right]^{1/2} \quad \dots (18.27)$$

$$\text{Then, width } B_1 = n D_1 \quad [\text{Eqn. (18.22)}]$$

4. Find the rim velocity (tangential velocity) u_1 from the relation

$$u_1 = \frac{\pi D_1 N}{60}$$

5. Find the velocity of whirl at inlet (V_{w1}) from Eq [18.19]

$$\eta_h = \frac{V_{w1} u_1}{gH} \quad \text{or} \quad V_{w1} = \frac{\eta_h gH}{u_1} \quad \dots (18.28)$$

6. Obtain the guide vane angle (α) and the runner vane angle (θ) from the following relations obtained from inlet velocity triangle (Fig. 18.13).

$$\tan \alpha = \frac{V_{f1}}{V_{w1}} \quad \dots (18.29)$$

$$\text{and,} \quad \tan \theta = \frac{V_{f1}}{V_{w1} - u_1} \quad \dots (18.30)$$

7. Assume runner diameter D_2 at the outlet to be approximately one-half the diameter at inlet.

$$\text{Thus,} \quad D_2 = \frac{D_1}{2} \quad \text{and} \quad u_2 = \frac{u_1}{2}$$

8. The velocity of flow at the exit (V_{f2}) is obtained as follows :

$$Q = K_{r1} \pi D_1 B_1 V_{f1} = K_{r2} \pi D_2 B_2 V_{f2}$$

...Continuity equation

Thus,
$$\frac{V_{f1}}{V_{f2}} = \frac{K_{f2} \pi D_2 B_2}{K_{f1} \pi D_1 B_1}$$

Usually, it is presumed that $V_{f1} = V_{f2}$ and $K_{f1} = K_{f2}$, that gives $B_2 = 2B_1$

- Find the runner vane angle at exit (ϕ) from the velocity outlet triangle, assuming the discharge at the runner exit to be radial ($\beta = 90^\circ$). Thus,

$$\tan \phi = \frac{V_{f2}}{u_2} \quad \dots(18.31)$$

- The number of vanes varies from 16 to 24. In order to avoid periodic impulse, the number of vanes should be either one more or one less than the number of guide vanes.

18.4.1.4. Advantages and disadvantages of a Francis turbine over a Pelton wheel

Advantages :

The Francis turbine claims the following *advantages* over Pelton wheel :

- In Francis turbine the variation in the operating head can be more easily controlled.
- In Francis turbine the ratio of maximum and minimum operating heads can be even two.
- The operating head can be utilized even when the variation in the tail water level is relatively large when compared to the total head.
- The mechanical efficiency of Pelton wheel decreases faster with wear than Francis turbine.
- The size of the runner, generator and power house required is small and economical if the Francis turbine is used instead of Pelton wheel for same power generation.

Disadvantages/Drawbacks :

As compared with Pelton wheel, the Francis turbine has the following *drawbacks/ shortcomings* :

- Water which is not clean can cause very rapid wear in high head Francis turbine.
- The overhaul and inspection is much more difficult comparatively.
- Cavitation is an ever-present danger.
- The water hammer effect is more troublesome with Francis turbine.
- If Francis turbine is run below 50 percent head for a long period it will not only lose its efficiency but also the cavitation danger will become more serious.

Example 18.16. An inward flow reaction turbine has external and internal diameters as 1.08 m and 0.54 m. The turbine is running at 200 r.p.m. The width of the turbine at inlet is 240 mm and velocity of flow through the runner is constant and is equal to 2.16 m/s. The guide blades make an angle of 10° to the tangent of the wheel and discharge at the outlet of the turbine is radial. Draw the inlet and outlet velocity triangles and determine :

- The absolute velocity of water at inlet of the runner,
- The velocity of whirl at inlet,
- The relative velocity at inlet,
- The runner blade angles,
- Width of runner at outlet,
- Weight of water flowing through the runner per second,
- Head at inlet of the turbine,
- Power developed, and
- Hydraulic efficiency of the turbine.

Hydraulic Turbines

Solution. External diameter, $D_1 = 1.08 \text{ m}$
 Internal diameter, $D_2 = 0.54 \text{ m}$
 Speed, $N = 200 \text{ r.p.m.}$
 Width at inlet, $B_1 = 240 \text{ mm} = 0.24 \text{ m}$
 Velocity of flow, $V_f = V_f = 2.16 \text{ m/s}$
 Guide blade angle, $\alpha = 10^\circ$
 Discharge at outlet radial

$\therefore \beta = 90^\circ$ and $V_{w2} = 0$

Tangential velocity of wheel at inlet, $u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.08 \times 200}{60} = 11.31 \text{ m/s}$

Tangential velocity of wheel at outlet, $u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.54 \times 200}{60} = 5.65 \text{ m/s}$

(i) Absolute velocity of water at inlet of the runner, V_1 :

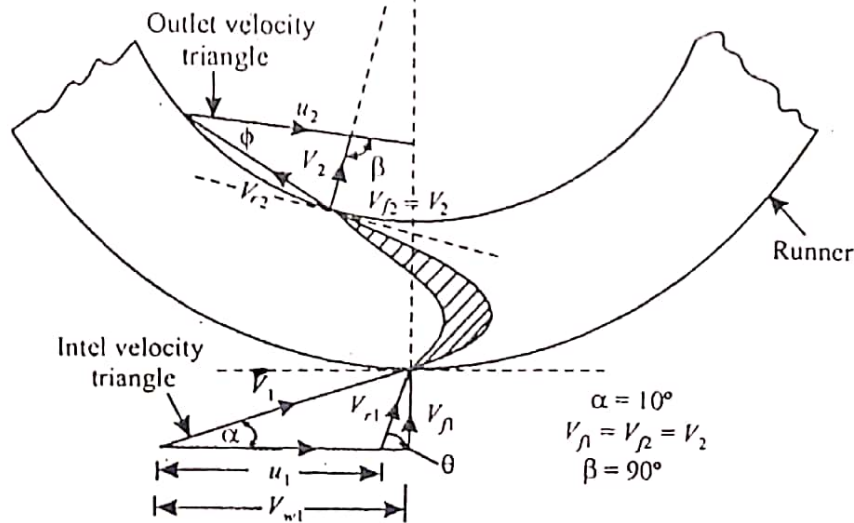
From inlet velocity triangle, we have

$$V_1 \sin \alpha = V_f$$

$$\therefore V_1 = \frac{V_{f1}}{\sin \alpha} = \frac{2.16}{\sin 10^\circ} = 12.44 \text{ m/s (Ans.)}$$

(ii) Velocity of whirl at inlet, V_{w1} :

$$V_{w1} = V_1 \cos \alpha = 12.44 \times \cos 10^\circ = 12.25 \text{ m/s (Ans.)}$$



(iii) Relative velocity at inlet, V_{r1} :

$$\begin{aligned} V_{r1} &= \sqrt{(V_{w1} - u_1)^2 + V_{f1}^2} \\ &= \sqrt{(12.25 - 11.31)^2 + (2.16)^2} \\ &= 2.35 \text{ m/s (Ans.)} \end{aligned}$$

(iv) **Runner blade angles, θ, ϕ :**

Again, from inlet velocity triangle, we have

$$\tan \theta = \frac{V_{f1}}{V_{w1} - u_1} = \frac{2.16}{12.25 - 11.31} = 2.298$$

$$\therefore \theta = \tan^{-1} 2.298 = 66.48^\circ \text{ (Ans.)}$$

From the outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f2}}{u_2} = \frac{2.16}{5.65} = 0.382$$

$$\therefore \phi = \tan^{-1} 0.382 = 20.9^\circ \text{ (Ans.)}$$

(v) **Width of runner at outlet, B_2 :**

$$\pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2} (= Q)$$

...Continuity equation

$$(\because V_{f1} = V_{f2})$$

or,

$$D_1 B_1 = D_2 B_2$$

$$\therefore B_2 = \frac{D_1 B_1}{D_2} = \frac{1.08 \times 0.24}{0.54} = 0.48 \text{ m or } 480 \text{ mm (Ans.)}$$

(vi) **Weight of water flowing through the runner per second:**

Weight of water flowing per second

$$\begin{aligned} &= wQ = w \times \pi D_1 B_1 V_{f1} \\ &= 9.81 \times \pi \times 1.08 \times 0.24 \times 2.16 = 17.25 \text{ kN/s (Ans.)} \end{aligned}$$

(vii) **Head at inlet of turbine, H :**

$$H = \frac{1}{g} (V_{w1} u_1 \pm V_{w2} u_2) + \frac{V_2^2}{2g}$$

$$= \frac{1}{g} (V_{w1} u_1) + \frac{V_2^2}{2g}$$

$$(\because V_{w2} = 0)$$

$$= \frac{1}{9.81} (12.25 \times 11.31) + \frac{2.16^2}{2 \times 9.81}$$

$$(\because V_2 = V_{f2})$$

$$= 14.36 \text{ m (Ans.)}$$

(viii) **Power developed:**

$$\text{Power developed} = \rho Q \times V_{w1} u_1 = \frac{wQ}{g} \times V_{w1} u_1$$

$$= \frac{17.25}{9.81} \times 12.25 \times 11.31 = 243.6 \text{ kW (Ans.)}$$

(ix) **Hydraulic efficiency, η_h :**

$$\eta_h = \frac{V_{w1} u_1}{gH}$$

...[Eqn (18.19)]

$$= \frac{12.25 \times 11.31}{9.81 \times 14.36} = 0.9835 \text{ or } 98.35\% \text{ (Ans.)}$$

Example 18.17. A reaction turbine works at 450 r.p.m. under a head of 120 m. Its diameter at inlet is 1.2 m and the flow area is 0.4 m². The angles made by absolute and relative velocities at inlet are 20° and 60° respectively with the tangential velocity. Determine:

(i) The volume flow rate,

(ii) The power developed, and

(iii) The hydraulic efficiency.

[AMIE, Fluid Power Engg.]

Solution. Speed of turbine, $N = 450$ r.p.m

Head, $H = 120$ m

Diameter at inlet, $D_1 = 1.2$ m

Flow area, $\pi D_1 B_1 = 0.4$ m²

Angle made by absolute velocity, $\alpha = 20^\circ$

Angle made by the relative velocity at inlet, $\theta = 60^\circ$

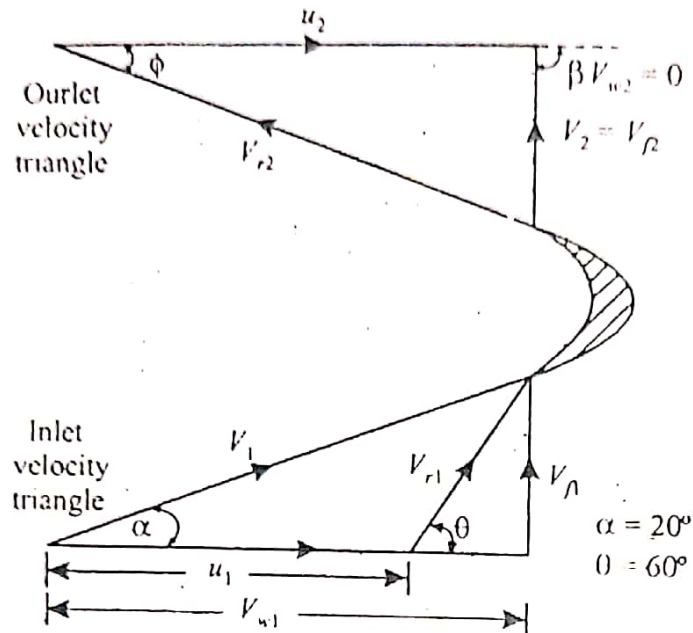


Fig. 18.16

(i) The volume flow rate, Q :

Tangential velocity of the turbine,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 450}{60} = 28.27 \text{ m/s}$$

From inlet velocity triangle, we have:

$$\tan \alpha = \frac{V_{f1}}{V_{w1}} \quad \text{or} \quad \tan 20^\circ = \frac{V_{f1}}{V_{w1}}$$

$$\therefore V_f = V_{w1} \tan 20^\circ = 0.364 V_{w1}$$

$$\text{Also,} \quad \tan \theta = \frac{V_{f1}}{V_{w1} - u_1} = \frac{0.364 V_{w1}}{V_{w1} - 28.27}$$

$$(\because V_f = 0.364 V_{w1})$$

$$\text{or,} \quad \tan 60^\circ = \frac{0.364 V_{w1}}{V_{w1} - 28.27} \quad \text{or} \quad 1.732 = \frac{0.364 V_{w1}}{V_{w1} - 28.27}$$

$$\text{or,} \quad 1.732 (V_{w1} - 28.27) = 0.364 V_{w1} \quad \text{or} \quad 1.732 V_{w1} - 48.96 = 0.364 V_{w1}$$

$$\therefore V_{w1} = \frac{48.96}{(1.732 - 0.364)} = 35.79 \text{ m/s}$$

From eqn. (i), we have

$$V_f = 0.364 \times 35.79 = 13.027 \text{ m/s}$$

$$\therefore \text{Volume flow rate, } Q = \pi D_1 B_1 \times V_f$$

$$\text{But,} \quad \pi D_1 B_1 = 0.4 \text{ m}^2$$

$$\therefore 0.4 \times 13.027 = 5.211 \text{ m}^3/\text{s (Ans.)}$$

...

(i) Power developed :

$$\text{Work done per second} = \rho Q (V_{w1} u_1)$$

[$\because V_{w2} = 0$... Given]

$$= 1000 \times 5.211 \times 35.79 \times 28.27 = 5272402 \text{ Nm/s or J/s}$$

$$\text{Power developed} = 5272402 \text{ J/s or } W = 5272.4 \text{ kW (Ans.)}$$

(ii) The hydraulic efficiency, η_h :

$$\eta_h = \frac{V_{w1} u_1}{gH} \quad \dots [\text{Eqn (18-19)}]$$

$$= \frac{35.79 \times 28.27}{9.81 \times 120} = 0.8595 = 85.95\% \text{ (Ans.)}$$

Example 18-18. An inward flow reaction turbine has an external diameter of 1 m and its breadth at inlet is 250 mm. If the velocity of flow at inlet is 2 m/s, find weight of water passing through the turbine per second. Assume 10 per cent of the area of flow is blocked by blade thickness. If the speed of the runner is 210 r.p.m. and guide blades make an angle of 10° to the wheel tangent, draw the inlet velocity triangle and find :

- The runner vane angle at inlet,
- The velocity of wheel at inlet,
- The absolute velocity of water leaving the guide vanes, and
- The relative velocity of water entering the runner blade.

[AMIE, Fluid Power Engg.]

Solution. External diameter, $D_1 = 1 \text{ m}$

Breadth of inlet, $B_1 = 250 \text{ mm} = 0.25 \text{ m}$

Velocity of flow at inlet, $V_{f1} = 2.0 \text{ m/s}$

Area of flow blocked by blade thickness = 10 %

Speed of the runner, $N = 210 \text{ r.p.m.}$

Guide blade angle, $\alpha = 10^\circ$

Weight of water passing through the turbine :

$$\text{Area blocked by vane thickness} = \frac{10}{100} \times \pi D_1 B_1 = 0.1 \pi D_1 B_1$$

Actual area through which flow takes place,

$$A = \pi D_1 B_1 - 0.1 \pi D_1 B_1 = 0.9 \pi D_1 B_1 = 0.9 \pi \times 1 \times 0.25 = 0.7068 \text{ m}^2$$

Weight of water passing per second through the turbine

$$= w \times A \times V_{f1} = 9.81 \times 0.7068 \times 2 = 13.68 \text{ kN/s (Ans.)}$$

Inlet velocity triangle is shown in Fig 18-17.

(i) The runner vane angle at inlet, θ :

Tangential velocity of wheel at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1 \times 210}{60} = 10.99 \text{ m/s}$$

From inlet velocity triangle, we have

$$\tan \alpha = \frac{V_{f1}}{V_{w1}} = \frac{2.0}{V_{w1}} ; V_{w1} = \frac{2.0}{\tan \alpha} = \frac{2.0}{\tan 10^\circ} = 11.34 \text{ m/s}$$

$$\tan \theta = \frac{V_{f1}}{V_{f1} - u_1} = \frac{2.0}{11.34 - 10.99} = 5.714$$

or,

$$\theta = \tan^{-1} 5.714 = 80.07^\circ \text{ (Ans.)}$$

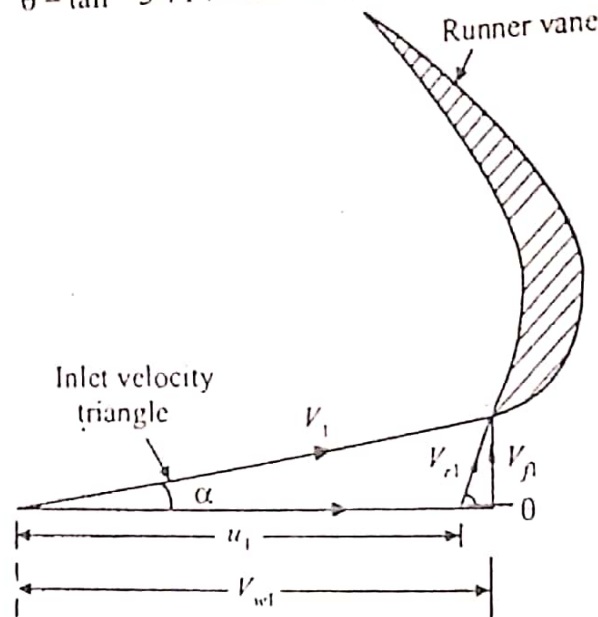


Fig. 18.17

- (ii) The velocity of wheel at inlet u_1 :

The velocity of wheel at inlet $= u_1 = 10.99 \text{ m/s}$ (Ans.)

- (iii) The absolute velocity of water leaving the guide vanes, V_1 :

Again, from inlet velocity triangle, we have

$$\sin \alpha = \frac{V_{f1}}{V_1}, \text{ or, } \sin 10^\circ = \frac{2.0}{V_1}$$

$$\therefore V_1 = \frac{2.0}{\sin 10^\circ} = 11.52 \text{ m/s (Ans.)}$$

- (iv) The relative velocity of water entering the runner blade, V_{r1} :

$$\sin \theta = \frac{V_{f1}}{V_{r1}}, \text{ or, } \sin 80.07^\circ = \frac{2.0}{V_{r1}}$$

$$\therefore V_{r1} = \frac{2.0}{\sin 80.07^\circ} = 2.03 \text{ m/s (Ans.)}$$

Example 18-19. In an inward flow reaction turbine the head on the turbine is 32 m. The external and internal diameters are 1.44 m and 0.72 m respectively. The velocity of flow through the runner is constant and equal to 3 m/s. The guide blade angle is 10° and the runner vanes are rigid at inlet. If the discharge at outlet is radial, determine :

- The speed of the turbine,
- The vane angle at outlet of the runner, and
- Hydraulic efficiency.

Solution. Head on the turbine, $H = 32 \text{ m}$

External diameter, $D_1 = 1.44 \text{ m}$

Internal diameter, $D_2 = 0.72 \text{ m}$

Velocity of flow, $V_f = \text{constant}$; $V_{f1} = V_{f2} = 3 \text{ m/s}$

The guide blade angle, $\alpha = 10^\circ$

Runner vanes are radial at inlet,

$$\therefore V_{f1} = \frac{Q}{\pi D_1 B_1} = \frac{7.2}{\pi \times 2 \times 0.3} = 3.82 \text{ m/s (Ans.)}$$

$$\text{and, } V_{f2} = \frac{Q}{\pi D_2 B_2} = \frac{7.2}{\pi \times 2.75 \times 0.3} = 2.77 \text{ m/s (Ans.)}$$

$$\text{Now, head, } H = \frac{V_{w1} u_1}{g} + \frac{V_2^2}{2g} = \frac{V_{w1} u_1}{g} + \frac{V_{f2}^2}{2g} \quad (\because V_{w2} = 0, V_2 = V_{f2})$$

$$216 = \frac{V_{w1} \times 31.4}{9.81} + \frac{2.77^2}{2 \times 9.81} \quad \text{or } 216 = 3.2 V_{w1} + 0.391$$

$$\therefore V_{w1} = \frac{(216 - 0.391)}{3.2} = 67.38 \text{ m/s}$$

(ii) Vane angles at inlet and outlet, θ, ϕ :

$$\text{From inlet velocity triangle, } \tan \theta = \frac{V_{f1}}{V_{w1} - u_1} = \frac{3.82}{67.38 - 31.4} = 0.1062$$

$$\therefore \theta = \tan^{-1}(0.1062) = 6.06^\circ \text{ (Ans.)}$$

$$\text{From outlet velocity triangle, } \tan \phi = \frac{V_{f2}}{u_2} = \frac{2.77}{43.9} = 0.0641$$

$$\therefore \phi = \tan^{-1}(0.0641) = 3.66^\circ \text{ (Ans.)}$$

18.4.2. Propeller and Kaplan turbines-Axial Flow Reaction Turbines

It has been observed that with increasing specific speed the flow tends to be axial. If water flows parallel to the axis of the rotation of the shaft, the turbine is known as *axial flow turbine*; when the head at inlet of the turbine is the sum of pressure energy and kinetic energy and during the flow of water through runner a part of the pressure energy is converted into kinetic energy, the turbine is known as *reaction turbine*. The shaft of an axial flow reaction turbine is *vertical*. The lower end of the shaft is made larger which is known as '*hub*' or '*boss*'. The vanes are fixed on the hub and it acts as runner for axial flow reaction turbine. Two important axial flow reaction turbines are: (i) *Propeller turbine*, and (ii) *Kaplan turbine*.

In these turbines all parts such as *spiral casing*, *stay vanes*, *guide vanes*, *control vanes*, and *draft tube* are similar to mixed-flow turbines in design. But the water enters the runner in an axial direction and during the process of energy transfer, it travels across the blade passage in axial direction and leaves axially. The pressure at the inlet of the blades is larger than the pressure at the exit of the blades. The energy transfer is due to the reaction effect, i.e. the change in the magnitude of relative velocity across the blades.

In an axial flow turbine the number of blades are fewer and hence the loading on the blade is larger. Smaller contact area causes less frictional loss compared to mixed flow turbines, but the peripheral speed of the turbine is larger. Axial flow rotors do not have a rim at the outer end like the Francis rotors; but the blades are enclosed in a cylindrical casing.

The tip clearance between the blades and the cylindrical casing is small; hence the flow past blades can be considered *two-dimensional*. The water coming out from the guide vanes undergoes a whirl which is assumed to satisfy the law of free vortex ($V_w = Cr$). Accordingly the whirl is largest near the hub and smallest at the outer end of blade. Hence the blade is twisted along its axis.

18.4.2.1. Propeller turbine

The need to utilize low heads where large volume of water is available makes it essential to provide a large flow area and to run the machine at very low speeds. The propeller turbine is a reaction turbine used for heads between 4 m and 80 m. It is purely *axial-flow* device providing the

largest possible flow area that will utilize a large volume of water and still obtain flow velocities which are not too large.

The propeller turbine (Fig. 18.36) consists of an axial-flow runner with four to six or at the most ten blades of air-foil shape. The runner is generally kept horizontal, i.e. the shaft is vertical. The blades resemble the propeller of a ship. In the propeller turbine, as in Francis turbine, the runner blades are fixed and non-adjustable. The spiral casing and guide blades are similar to those in Francis turbine. The guide mechanism is similar to that in a Francis turbine.

18.4.2.2. Kaplan turbine

A propeller turbine is quite suitable when the load on the turbine remains constant. At part load its efficiency is very low; since the blades are fixed, the water enters with shock (at part load) and eddies are formed which reduces the efficiency. This defect of the propeller turbine is removed in Kaplan turbine. In a Kaplan turbine the runner blades are adjustable and can be rotated about pivots fixed to the boss of the runner. The blades are adjusted automatically by servomechanism so that at all loads the flow enters them without shock. Thus, a high efficiency is maintained even at part load. The servomotor cylinder is usually accommodated in the hub. Figs. 18.37 and 18.38 show the Kaplan turbine runner and Kaplan turbine (schematic diagram) respectively.

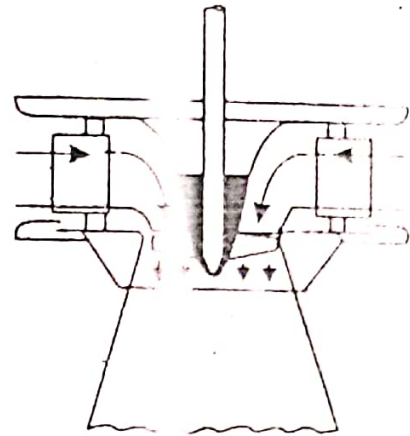


Fig. 18.36 Propeller turbine

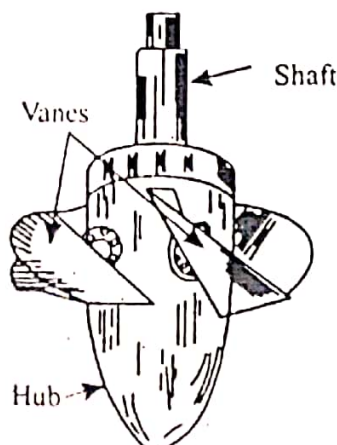


Fig. 18.37. Kaplan turbine runner.

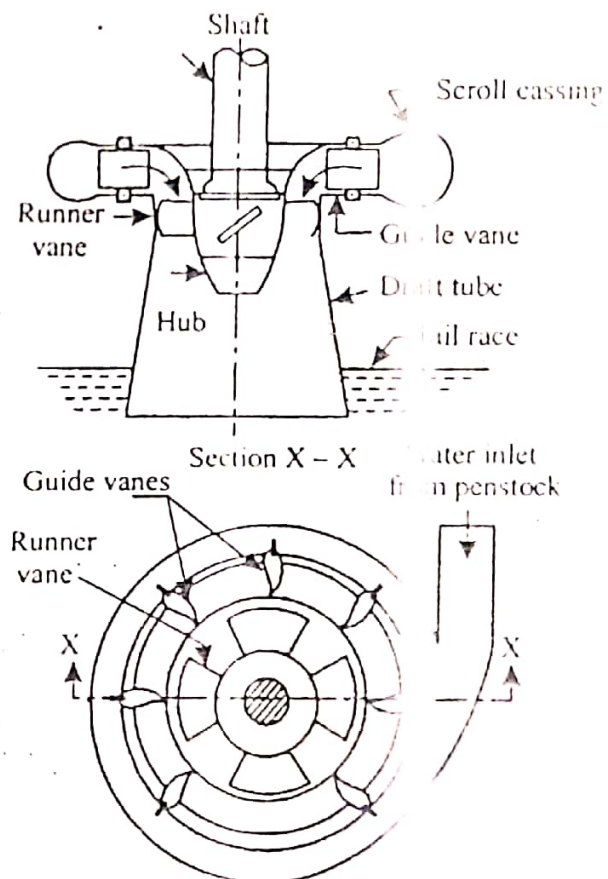


Fig. 18.38

The Kaplan turbine has *purely axial flow*. Usually it has 4 to 6 blades having no outside rim. It is also known as a *variable-pitch propeller turbine* since the pitch of the turbine can be changed because of adjustable vanes. The Kaplan turbine behaves like a propeller turbine at full-load conditions.

The scroll casing, guide mechanism and draft tube are similar to that in the Francis turbine. The shape of runner blades is different from that of Francis turbine. The blades of Kaplan turbine are made of stainless steel.

Kaplan turbine, like every propeller turbine, is a *high speed turbine* and is used for smaller heads; as the speed is high, the number of runner-vanes is small.

Kaplan turbines have taken the place of Francis turbines for certain medium head installations. Kaplan turbines with sloping guide vanes to reduce the overall dimensions are being used.

Important Kaplan Turbine Installations in India :

S.No.	Scheme/Project	Location (State)	Source of water
1.	Bhakra-Nangal Project	Gangwal & Kota (Punjab)	Nangal hydel
2.	Hirakud Dam Project	Hirakud (Orissa)	Mahanadi river
3.	Nizam Sagar Project	Nizam Sagar (Andhra Pradesh)	Nanjira river
4.	Radhanagri Hydroelectric Scheme	Kolhapur (Maharashtra)	Bhagvati river
5.	Tungbhadra Hydroelectric Scheme	Tungbhadra (Karnataka)	Tungbhadra river.

Differences between Francis Turbine and Kaplan Turbine :

No.	Aspects	Francis turbine	Kaplan turbine
1.	Type of turbine	Radially inward or mixed flow.	Partially axial flow.
2.	Disposition of shaft	Horizontal or vertical	Only vertical.
3.	Adjustability of runner vanes	Runner vanes are <i>not</i> adjustable.	Runner vanes are adjustable.
4.	Number of vanes	Large, 16 to 24 blades	Small 3 to 8 blades
5.	Resistance to be overcome	Large, (owing to large number of vanes and greater area of contact with water)	Less (owing to fewer number of vanes and less wetted area)
6.	Head	Medium (60 m to 250 m)	Low (up to 30 m)
7.	Flow rate	Medium.	Large
8.	Specific speed	50-250	250-850
9.	Type of governor	Ordinary	Heavy duty

Working proportions :

The expressions for work done, efficiency and power developed by axial flow propeller and Kaplan turbines are identical to those of a Francis turbine; and the working proportions are obtained in identical fashion. However, the following *deviations need to be noted carefully*:

- In case of a propeller/Kaplan turbine, the ratio n is taken as $\frac{D_b}{D_0}$ (and not $\frac{B}{D}$),

where, D_0 = Outside diameter of the runner, and
 D_b = Diameter of boss (or hub).

Discharge Q = Area of flow $\times$ velocity of flow

$$= \frac{\pi}{4} (D_0^2 - D_b^2) \times V_f$$

$$= \frac{\pi}{4} (D_0^2 - D_b^2) \times K_f \sqrt{2gH}$$

(where, K_f = flow ratio)

or,

$$Q = \frac{\pi}{4} D_0^2 (1 - n^2) \times K_f \sqrt{2gH} \quad \dots (18.31)$$

$$\left(\because n = \frac{D_b}{D_0}, \text{ or, } D_b = nD_0 \right)$$

The value of n ranges from 0.35 to 0.60.

The value of $K_f \approx 0.70$.

- The peripheral velocity u of the runner vanes depends upon the radius of the point under consideration and thus the blade angles vary from the rim to the boss and the vanes are warped; this is necessary to ensure shock free entry and exit.

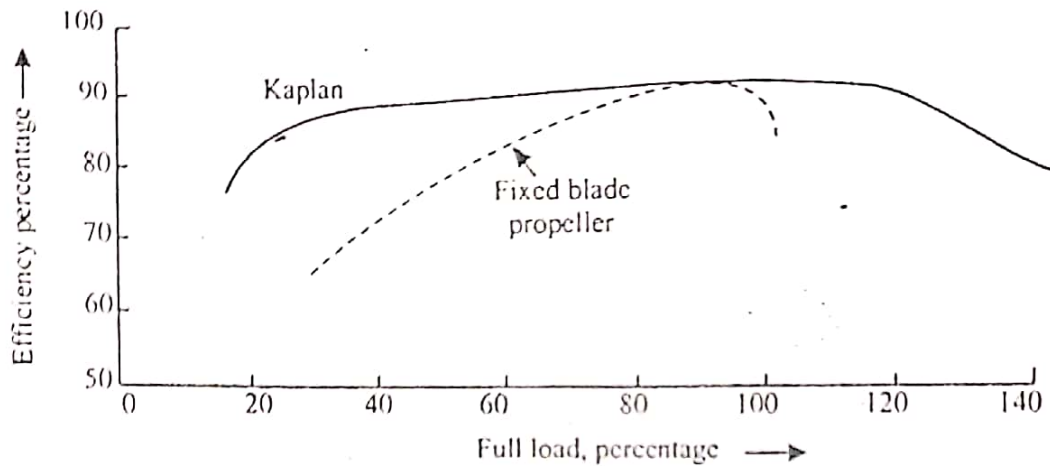


Fig. 18.39. Comparison of efficiencies of propeller (fixed blades) and Kaplan turbines.

- The velocity of flow remains constant throughout.

Fig. 18.39 shows the comparison of efficiencies of propeller (fixed blades) and Kaplan turbines.

18.4.2.3. Kaplan turbine versus Francis turbine :

Kaplan turbine claims the following advantages over Francis turbine :

- For the same power developed Kaplan turbine is more compact in construction and smaller in size.
- Part-load efficiency is considerably high.
- Low frictional losses (because of small number of blades used).

Example 18.37. A Kaplan turbine develops 22000 kW at an average head of 35 m. Assuming a speed ratio of 2, flow ratio of 0.6, diameter of the boss equal to 0.35 times the diameter of the runner and an overall efficiency of 88 per cent, calculate the diameter, speed and specific speed of the turbine.

Solution. Shaft power, $P = 22000$ kW; Head, $H = 35$ m;

Speed ratio, $K_u = 2.0$; Flow ratio, $K_f = 0.6$;

Diameter of boss (D_b) = 0.35 $\times$ diameter of the runner (D_0), i.e. $D_b = 0.35D_0$;

Overall efficiency, $\eta_0 = 88$ per cent.

Diameter of the runner, D_0 :

$$K_u = \frac{u_1}{\sqrt{2gH}} = 2.0; u_1 = 2.0 \times \sqrt{2gH} = 2.0 \times \sqrt{2 \times 9.81 \times 35} = 52.4 \text{ m/s}$$

$$K_f = \frac{V_{f1}}{\sqrt{2gH}} = 0.6, \text{ or, } V_{f1} = 0.6 \times \sqrt{2gH}$$

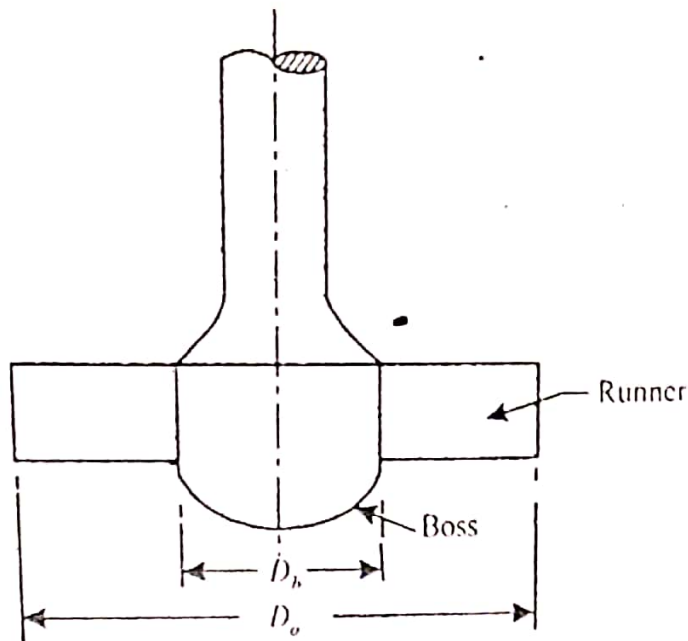


Fig. 18.40

$$= 0.6 \times \sqrt{2 \times 9.81 \times 35} = 15.7 \text{ m/s}$$

Overall efficiency,

$$\eta_0 = \frac{\text{Shaft power (P)}}{\text{Water power}} = \frac{22000}{\rho Q H}$$

or,

$$0.88 = \frac{22000}{9.81 \times Q \times 35}$$

$$Q = \frac{22000}{0.88 \times 9.81 \times 35} = 72.8 \text{ m}^3/\text{s}$$

Also $Q = \text{Area of flow} \times \text{velocity of flow}$

$$= \frac{\pi}{4} \times (D_o^2 - D_b^2) \times V_{f1}$$

or,

$$72.8 = \frac{\pi}{4} [D_o^2 - (0.35 D_o)^2] \times 15.7$$

$$[\because D_b = 0.35 D_o]$$

$$= \frac{\pi}{4} D_o^2 [1 - 0.35^2] \times 15.7 = 10.82 D_o^2$$

$$\therefore D_o = \left(\frac{72.8}{10.82} \right)^{1/2} = 2.6 \text{ m (Ans.)}$$

Speed of the turbine, N :

$$u_1 = \frac{\pi D_o N}{60}, \text{ or, } N = \frac{60 u_1}{\pi D_o} = \frac{60 \times 15.7}{\pi \times 2.6} = 384.9 \text{ r.p.m. (Ans.)}$$

Specific speed of the turbine, N_s :

$$N_s = \frac{N \sqrt{P}}{H^{5/4}} = \frac{384.9 \times \sqrt{22000}}{(35)^{5/4}} = 670.6 \text{ (Ans.)}$$

Example 18.38. The following data pertain to a Kaplan turbine :

Power available at shaft = 22500 kW; Head = 20 m; Speed = 150 r.p.m. Hydraulic efficiency = 95 %; Overall efficiency = 88 %; Outer diameter of runner = 4.5 m; Diameter of the hub = 2 m.

Assuming that the turbine discharges without whirl at exit, determine the runner vane angles at the hub and at the outer periphery.

Solution. Shaft power, $P = 22500 \text{ kW}$
 Head, $H = 20 \text{ m}$
 Speed, $N = 150 \text{ r.p.m.}$
 Hydraulic efficiency, $\eta_h = 95\%$
 Overall efficiency, $\eta_o = 88\%$
 Outer diameter of runner, $D_o = 4.5 \text{ m}$
 Diameter of the hub, $D_b = 2 \text{ m}$
 Overall efficiency,

$$\eta_o = \frac{\text{Shaft power (P)}}{\text{Water power}} = \frac{22500}{wQH}, \text{ or, } Q = \frac{22500}{\eta_o wH}$$

$\therefore$ Discharge,

$$Q = \frac{22500}{0.88 \times 9.81 \times 20} = 130.32 \text{ m}^3/\text{s}$$

Also,
$$Q = \frac{\pi}{4} (D_o^2 - D_b^2) \times V_{f1}$$

or,
$$130.32 = \frac{\pi}{4} (4.5^2 - 2^2) \times V_{f1}$$

$\therefore$
$$V_{f1} = \frac{130.32}{\frac{\pi}{4} \times (4.5^2 - 2^2)} = 10.21 \text{ m/s}$$

Vane angles at hub :

$$u_1 = \frac{\pi D_b N}{60} = \frac{\pi \times 2 \times 150}{60} = 15.7 \text{ m/s}$$

Hydraulic efficiency, $\eta_h = \frac{V_{w1} u_1}{gH}$, or, $V_{w1} = \frac{\eta_h gH}{u_1} = \frac{0.95 \times 9.81 \times 20}{15.7} = 11.87 \text{ m/s}$

From inlet velocity triangle, $\tan (180^\circ - \theta) = \frac{V_{f1}}{u - V_{w1}} = \frac{10.21}{15.7 - 11.87} = 2.66$

$\therefore$ $180^\circ - \theta = \tan^{-1} (2.66) = 69.4^\circ$

$\therefore$ Runner vane angle at inlet, $\theta = 180^\circ - 69.4^\circ = 110.6^\circ$ (Ans.)

From outlet velocity triangle, $\tan \phi = \frac{V_{f2}}{u_2} = \frac{V_{f1}}{u_1} = \frac{10.21}{15.7} = 0.6503$

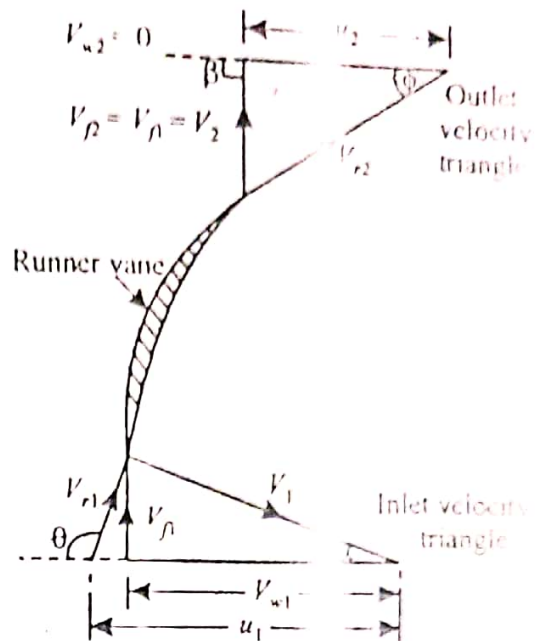
$\therefore$ Runner vane angle at exit, $\phi = \tan^{-1} (0.6503) = 33.03^\circ$ (Ans.)

Vane angles at extreme edge of the runner (outer periphery) :

$$u_1 = \frac{\pi D_o N}{60} = \frac{\pi \times 4.5 \times 150}{60} = 35.34 \text{ m/s}$$

Hydraulic efficiency, $\eta_h = \frac{V_{w1} u_1}{gH}$

$$0.95 = \frac{V_{w1} \times 35.34}{9.81 \times 20}, \text{ or, } V_{w1} = \frac{0.95 \times 9.81 \times 20}{35.34} = 5.27 \text{ m/s}$$



$$\text{From inlet velocity triangle, } \tan (180^\circ - \theta) = \frac{V_{f1}}{u_1 - V_{w1}} = \frac{10.21}{35.34 - 5.27} = 0.339$$

$$(180^\circ - \theta) = \tan^{-1} (0.339) = 18.72^\circ$$

$$\text{Runner vane angle at inlet, } \theta = 180^\circ - 18.72^\circ = 161.28^\circ \text{ (Ans.)}$$

$$\text{From outlet velocity triangle, } \tan \phi = \frac{V_{f2}}{u_2} = \frac{V_{f1}}{u_1} = \frac{10.21}{35.34} = 0.2889$$

$$\text{Runner vane angle at exit, } \phi = \tan^{-1} (0.2889) = 16.11^\circ \text{ (Ans.)}$$

Example 18.39. Calculate the diameter and speed of the runner of a Kaplan turbine developing 6000 kW under an effective head of 5 m. Overall efficiency of the turbine is 90%. The diameter of the boss is 0.4 times the external diameter of the runner. The turbine speed ratio is 2.0 and flow ratio 0.6. What is the specific speed of the turbine? (UPSC)

Solution. Given : Power developed, $P = 6000 \text{ kW}$; $H = 5 \text{ m}$; $\eta_0 = 90\%$;

$D_b = 0.4 D_0$; speed ratio, $K_u = 2.0$; flow ratio, $K_f = 0.6$

Diameter (D_0) and speed (N) of the runner :

$$K_u = 2.0 = \frac{u_1}{\sqrt{2gH}}; \text{ or, } u_1 = 2\sqrt{2 \times 9.81 \times 5} = 19.81 \text{ m/s}$$

$$K_f = 0.6 = \frac{V_{f1}}{\sqrt{2gH}}, \text{ or, } V_{f1} = 0.6\sqrt{2 \times 9.81 \times 5} = 5.94 \text{ m/s}$$

$$\text{Also, } \eta_0 = \frac{P}{\rho Q H}$$

$$Q = \frac{P}{\eta_0 \rho H} = \frac{6000 \times 1000}{0.9 \times 9810 \times 5} = 135.9 \text{ m}^3/\text{s}$$

$$\text{But, } Q = \frac{\pi}{4} (D_0^2 - D_b^2) \times V_{f1}$$

$$\text{or, } 135.9 = \frac{\pi}{4} [D_0^2 - (0.4 D_0)^2] \times 5.94 = \frac{\pi}{4} D_0^2 (1 - 0.16) \times 5.94$$

$$D_0 = \left[\frac{135.9 \times 4}{\pi (1 - 0.16) \times 5.94} \right]^{1/2} = 5.89 \text{ m (Ans.)}$$

$$\text{Now, } u_1 = \frac{\pi D_0 N}{60}$$

$$\text{or, } N = \frac{u_1 \times 60}{\pi D_0} = \frac{19.81 \times 60}{\pi \times 5.89} = 64.23 \text{ r.p.m. (Ans.)}$$

Specific speed of the turbine, N_s :

$$N_s = \frac{N \sqrt{P}}{H^{5/4}} = \frac{64.23 \sqrt{6000}}{(5)^{5/4}} = 665.4 \text{ (Ans.)}$$

Example 18.40. The propeller reaction turbine of runner diameter 4.5 m is running at 48 r.p.m. The guide blade angle at inlet is 145° and the runner blade angle at outlet is 25° to the direction of radius. The axial flow area of water through the runner is 30 m^2 . If the runner blade angle at inlet is 145° , determine :

- (i) Hydraulic efficiency of the turbine,
- (ii) Discharge through the turbine, and
- (iii) Power developed by the runner.

Solution. Diameter of the runner, $D_0 = 4.5$ m

Speed of the runner, $N = 48$ r.p.m.

Guide blade angle, $\alpha = 145^\circ$

Runner blade angle at inlet is radial i.e. $\theta = 90^\circ$, $V_{f1} = V_{r1}$

Runner blade angle at outlet, $\phi = 25^\circ$

Area of flow, $A_f = 30$ m²

- (i) Hydraulic efficiency of the turbine, η_h :

$V_{f1} = V_{f2}$, because area of flow is constant.

The tangential speed of turbine at inlet, $u_1 = \frac{\pi D_0 N}{60} = \frac{\pi \times 4.5 \times 48}{60} = 11.31$ m/s

Also, $u_2 = u_1 = 11.31$ m/s

From inlet velocity triangle, we have

$$\tan (180^\circ - \alpha) = \frac{V_{f1}}{u_1}, \text{ or, } \tan (180^\circ - 145^\circ) = \frac{V_{f1}}{u_1}$$

$$\therefore V_{f1} = u_1 \times \tan 35^\circ = 11.31 \times \tan 35^\circ = 7.92 \text{ m/s}$$

$$\text{Also, } V_{w1} = u_1 = 11.31 \text{ m/s}$$

Outlet velocity triangle

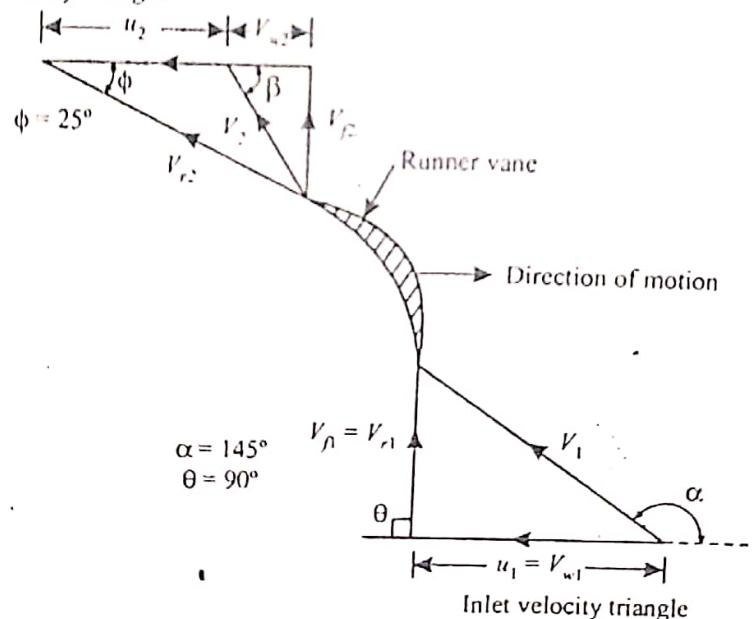


Fig. 18.42

From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f2}}{u_2 + V_{w2}} = \frac{7.92}{11.31 + V_{w2}} \quad (V_{f2} = V_{f1}; u_2 = u_1)$$

$$\therefore \tan 25^\circ = \frac{7.92}{11.31 + V_{w2}}, \text{ or, } 11.31 + V_{w2} = \frac{7.92}{\tan 25^\circ} = 16.98$$

$$\therefore V_{w2} = 16.98 - 11.31 = 5.67 \text{ m/s}$$

$$\text{or, } -3 - \frac{p_{\min}}{w} = 0.1015 \times \frac{(26.18)^2}{2 \times 9.81}, \text{ or, } -3 - \frac{p_{\min}}{w} = 3.545$$

$$\text{or, } \frac{p_{\min}}{w} = -3 - 3.545 = -6.545 \text{ m, or, } 6.545 \text{ m (vacuum)}$$

$$\text{or, } p_{\min} = 9.81 \times 6.545 = 64.2 \text{ kN/m}^2 \text{ (vacuum) (Ans.)}$$

(ii) The value of N_s :

$$N_s = 3.13 N_u \sqrt{Q_u \eta_0}$$

...[Proved at (a)]

$$= 3.13 \times \frac{N}{\sqrt{H}} \sqrt{\frac{Q}{\sqrt{H}}} \times \eta_0$$

$$= 3.13 \times \frac{250}{\sqrt{10}} \sqrt{\frac{24}{\sqrt{10}}} \times 0.9 = 646.7 \text{ (Ans.)}$$

18.12. Scale Effect

However smooth a model is made, the geometric similarity between the prototype and model cannot be extended to surface roughness. This variation of surfaceness with respect to the size of turbine will cause a *small but appreciable variation in the proportion of the effective head lost due to hydraulic friction*. Thus, the *efficiency of prototype will be different from the corresponding model efficiency*. This aspect is referred to as *scale effect*. It has been observed that with increase in size a geometrically similar mixed or axial flow turbine has greater efficiency than that of the model operating under hydraulically similar conditions.

In order to express the difference of efficiencies as found in tests of model and prototypes, various laws have been proposed. One of the earliest and most generally accepted is the semi-empirical formula suggested by Moody;

$$\frac{1 - \eta_p}{1 - \eta_m} = \left(\frac{D_m}{D_p} \right)^{0.2} \quad \dots(18.46)$$

Ackert suggested the following formula, considering the frictional loss as a function of Reynolds number,

$$\frac{1 - \eta_p}{1 - \eta_m} = \frac{1}{2} \left[1 + \left(\frac{D_m}{D_p} \right)^{0.2} \left(\frac{H_m}{H_p} \right)^{0.1} \right] \quad \dots(18.47)$$

where, η_p = Overall efficiency of the prototype,

D_p = Linear dimension of the prototype,

H_p = Head of the prototype,

and, η_m, D_m, H_m = Corresponding values of overall efficiency, linear dimension and head of the model.

These formulae are applicable for the *point of best efficiency*.

Note: (i) Moody's formula is based on the assumptions that all losses are due to fluid friction only and the flow is completely turbulent (and that eliminates any effect of Reynolds number).

(ii) No scale effect has been observed in the case of Pelton wheel; this may be due to the deterioration in the smoothness of the jet with increasing size, which offsets the benefits due to reduced frictional losses.

Example 18.57. A model turbine constructed to a scale of 1:10 when tested under a head of 8 m at 400 r.p.m. gave an efficiency of 77 per cent. Determine the r.p.m. of the prototype and the ratio of powers developed by the model and prototype if the prototype works under a head of 100 m. What will be the efficiency of the prototype if scale effect is considered? {UPSC Exams.}

Solution. Given : Scale ratio = 1 : 10; $\frac{D_p}{D_m} = 10$; $H_m = 8$ m; $N_m = 400$ r.p.m.; $\eta = 77\%$;

$H_p = 100$ m.

Speed of the prototype, N_p :

For similarity between model and prototype :

$$\left(\frac{H}{N^2 D^2} \right)_m = \left(\frac{H}{N^2 D^2} \right)_p, \text{ or, } \frac{H_m}{N_m^2 D_m^2} = \frac{H_p}{N_p^2 D_p^2}, \text{ or, } N_p^2 = N_m^2 \times \frac{D_p^2}{D_m^2} \times \frac{H_m}{H_p}$$

$$\therefore N_p = N_m \times \frac{D_p}{D_m} \times \left(\frac{H_m}{H_p} \right)^{1/2} = 400 \times \frac{1}{10} \times \left(\frac{100}{8} \right)^{1/2} = 141.4 \text{ r.p.m. (Ans.)}$$

Ratio of powers developed, $\frac{P_p}{P_m}$:

$$\left(\frac{P}{N^3 D^5} \right)_m = \left(\frac{P}{N^3 D^5} \right)_p, \text{ or, } \frac{P_m}{N_m^3 D_m^5} = \frac{P_p}{N_p^3 D_p^5}$$

$$\therefore \frac{P_p}{P_m} = \left(\frac{N_p}{N_m} \right)^3 \times \left(\frac{D_p}{D_m} \right)^5 = \left(\frac{141.42}{400} \right)^3 \times (10)^5 = 4419.3 \text{ (Ans.)}$$

Efficiency of the prototype when scale effect is considered :

The efficiencies are related by,

$$\frac{N_p}{N_m} = \frac{D_m}{D_p} \sqrt{\frac{H_p}{H_m}} \sqrt{\frac{\eta_p}{\eta_m}}$$

$$\frac{\eta_p}{\eta_m} = \left(\frac{N_p}{N_m} \right)^2 \left(\frac{D_p}{D_m} \right)^2 \frac{H_m}{H_p} = \left(\frac{141.42}{400} \right)^2 (10)^2 \times \frac{8}{100} = 0.9999$$

$$\therefore \eta_p = \eta_m \times 0.9999 = 0.77 \times 0.9999 = 0.7699 \text{ or } 76.99\% \text{ (Ans.)}$$

18-13. Performance Characteristics of Hydraulic Turbines

The turbines are normally designed for specific values of head, speed, discharge, power and efficiency (known as the *designed conditions*). But oftenly turbines may be required to operate under conditions different from those for which these have been designed. Thus, to know about their exact behaviour under varying conditions it becomes necessary to conduct tests either on the actual turbines at the site or on their small scale models in a research laboratory. The results so obtained are usually represented graphically and the curves obtained are known as "Characteristic curves". These curves are usually plotted in terms of unit quantities (for sake of convenience). The characteristic curves are of the following types :

1. Main or constant head characteristic curves.
2. Operating or constant speed characteristic curves.
3. Constant efficiency or iso-efficiency or Muschel curves.

18-13-1. Main or Constant Head Characteristic Curves

- Head and gate opening are maintained constant.
- Speed is varied by allowing a variable quantity of water to flow through the inlet opening.
- The brake power (P) is then measured mechanically by means of a dynamometer.
- The overall efficiency and unit quantities are then calculated by using the basic data; these are then plotted against unit speed as abscissa.

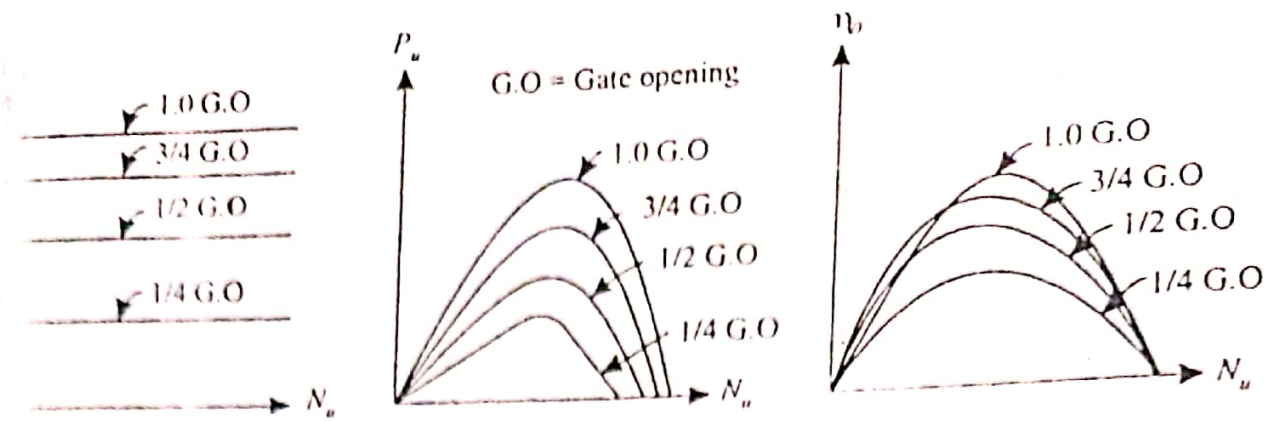


Fig. 18.53. Main characteristic curves of Pelton wheel

Figs. 18.53, 18.54 and 18.55 show the main characteristic curves of Pelton wheel, Francis turbine and Kaplan turbine respectively.

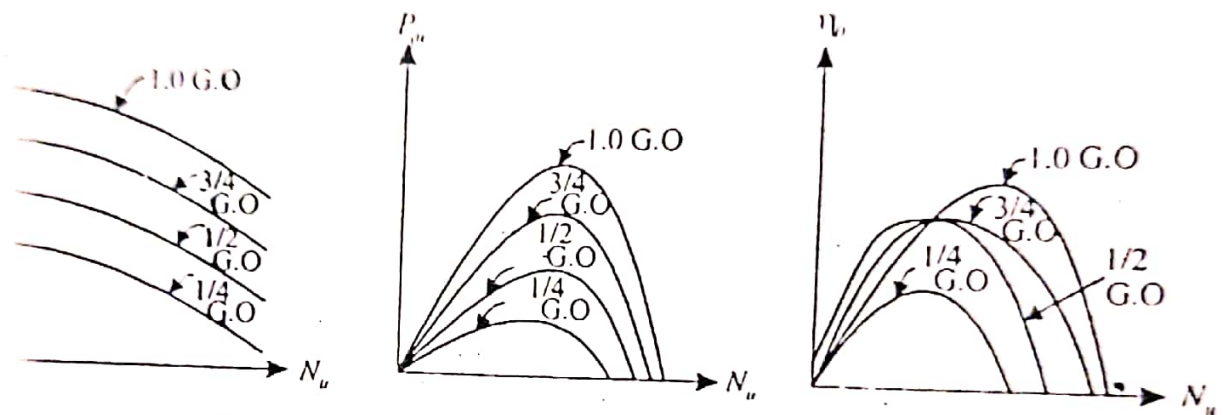


Fig. 18.54. Main characteristic curves of Francis turbine.

The main characteristic curves yield the following information :

- The discharge Q_u for a *Pelton wheel* depends only upon the gate opening and is independent of N_u ; the curves for Q_u are horizontal.
- The curves between Q_u and N_u for a *Francis turbine* are *falling curves*. This is due to the fact that a *centrifugal head* develops which acts outwards and *opposes* the external head causing flow, eventually decreasing the discharge as the speed increases.
- The curves between Q_u and N_u for a *Kaplan turbine* are *rising curves*; the discharge increases with the increase in speed.

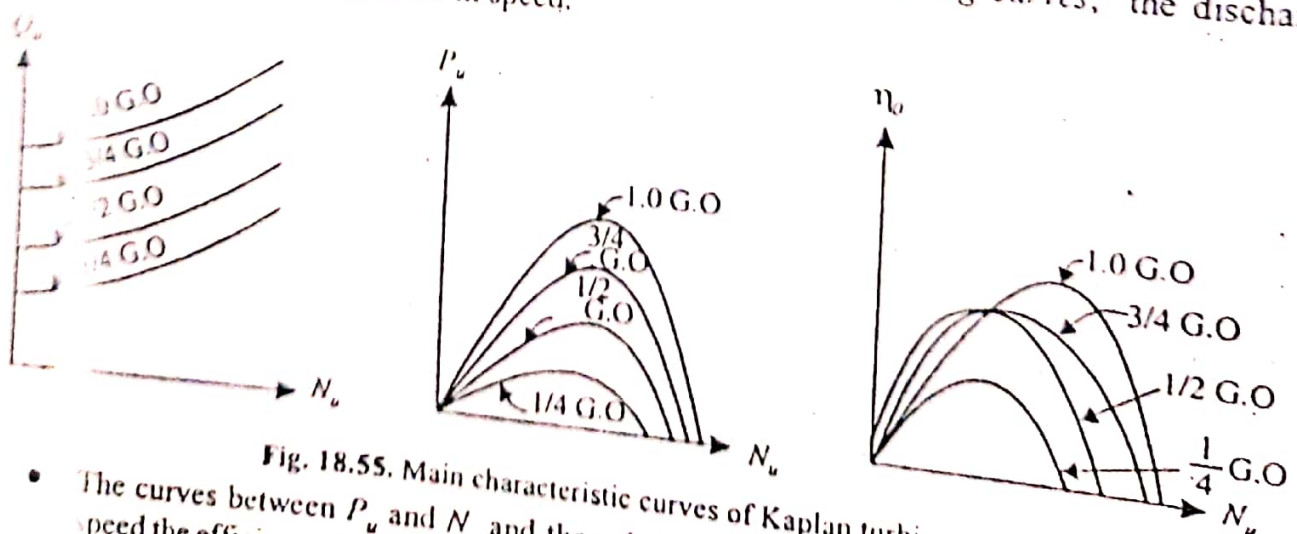


Fig. 18.55. Main characteristic curves of Kaplan turbine.

- The curves between P_u and N_u and those between η_0 and N_u indicate that at a particular speed the efficiency is maximum.

The maximum efficiency for a Pelton wheel usually occurs at the same speed for all gate openings; this speed usually corresponds to a speed ratio of 0.45. However, the maximum efficiency for a reaction turbine usually occurs at different speeds for different gate openings.

18.3.2. Operating or Constant Speed Characteristic Curves

These curves are obtained as follows :

(a) Percentage of full load v/s overall efficiency (η_o) curves :

- For each gate opening speed is kept constant. The constant speed is attained by regulating the gate opening thereby varying the discharge flowing through the turbine as the load varies; the head may or may not remain constant.
- The brake power (P) is measured mechanically by means of a dynamometer.
- The overall efficiency (η_o) is then calculated from the measured values of discharge, head and power.
- Further knowing the total load capacity of the turbine the percentage of full load is computed from the measured power and a plot of η_o v/s percentage of full load is prepared.

Fig. 18.56 shows the graphs plotted between percentage of full load v/s η_o for different types of turbines. The following points are worth noting :

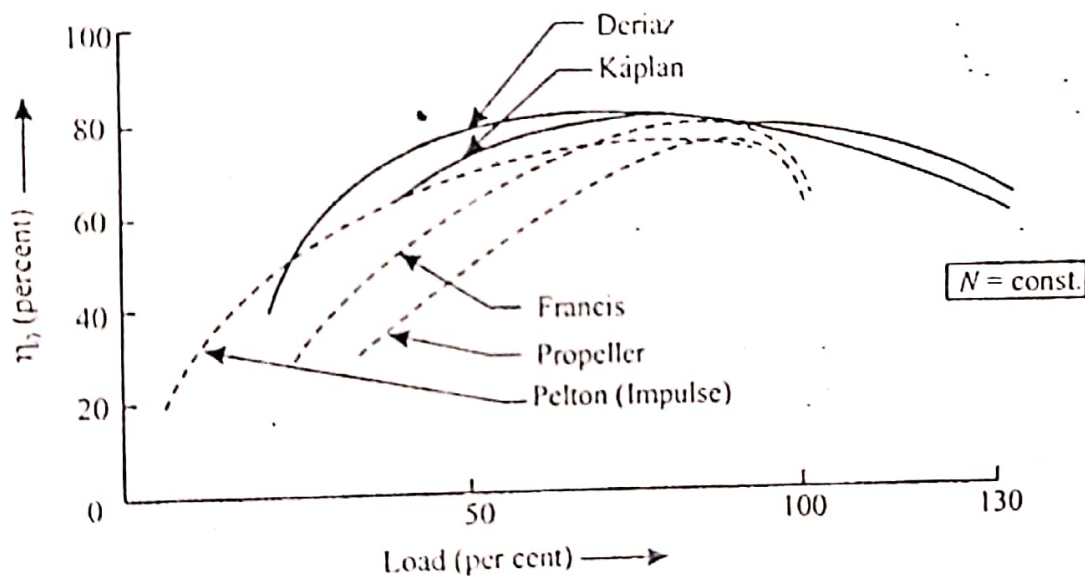


Fig. 18.56. Percentage of full load v/s η_o curves for hydraulic turbines.

- As the percentage full load increases η_o also increases (In other words, at reduced loads η_o is also less).
- At 100 per cent full load η_o is near about the maximum efficiency in all cases.
- The Kaplan, the Deriaz and the Pelton wheel maintain a high efficiency over a longer range of *part load* as compared with either the Francis or the fixed blade propeller turbine.
- The maximum overall efficiency of all the turbines is almost the same (about 85%).

(b) Overall efficiency (η_o) and output (shaft) power (P) v/s discharge (Q) curves:

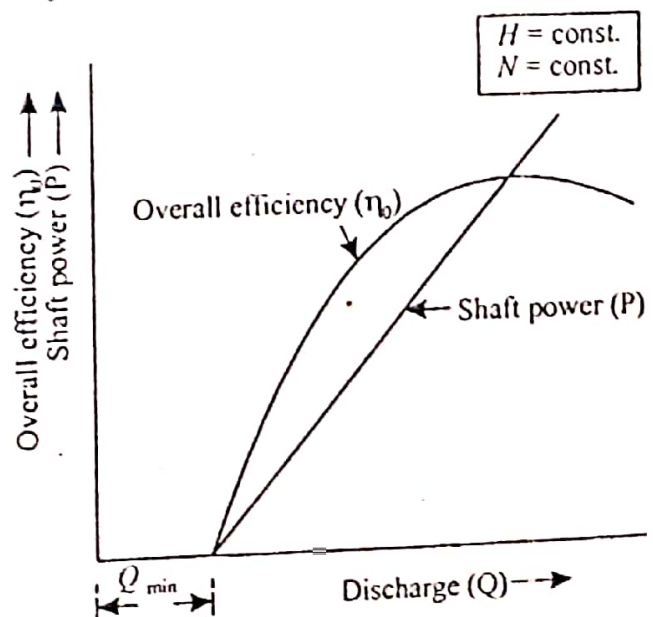


Fig. 18.57. η_o and P v/s discharge curves.

Fig. 18-57 shows overall efficiency (η_0) and shaft power (P) v/s discharge curves. Q_{min} is the minimum discharge required to set the turbine runner into motion from its state of rest. These curves yield the following information :

- Shaft power or output power (P) is a straight line, since $P \propto Q$ if H (head) is constant.
- η_0 v/s discharge (Q) graph is curvilinear and η_0 increases with Q and remains *nearly* constant beyond a particular value of discharge.

18-13.3. Constant efficiency or iso-efficiency or Muschel curves

Refer to Fig. 18-58. As η - N curve is of parabolic nature, there exists two speeds for one value of efficiency except for maximum efficiency which occurs at one speed only. Corresponding to these values of speeds there are also two values of discharge for each value of efficiency (Q - N curve). Hence on Q - N curve we can plot two points for each value of efficiency and one point for maximum efficiency. By adopting this procedure for different gate openings or heads we can get number of Q - N curves and we can plot on them efficiency points (as described above). The points denoting the same efficiency can now be joined to get constant iso-efficiency curves or Muschel curves (The German word 'Muschel' means shell, indicating shape of curve). The diagram showing these curves is also called Hill diagram (since it looks like top view of a hill). In actual practice unit speed and unit discharge are taken along the co-ordinate axes.

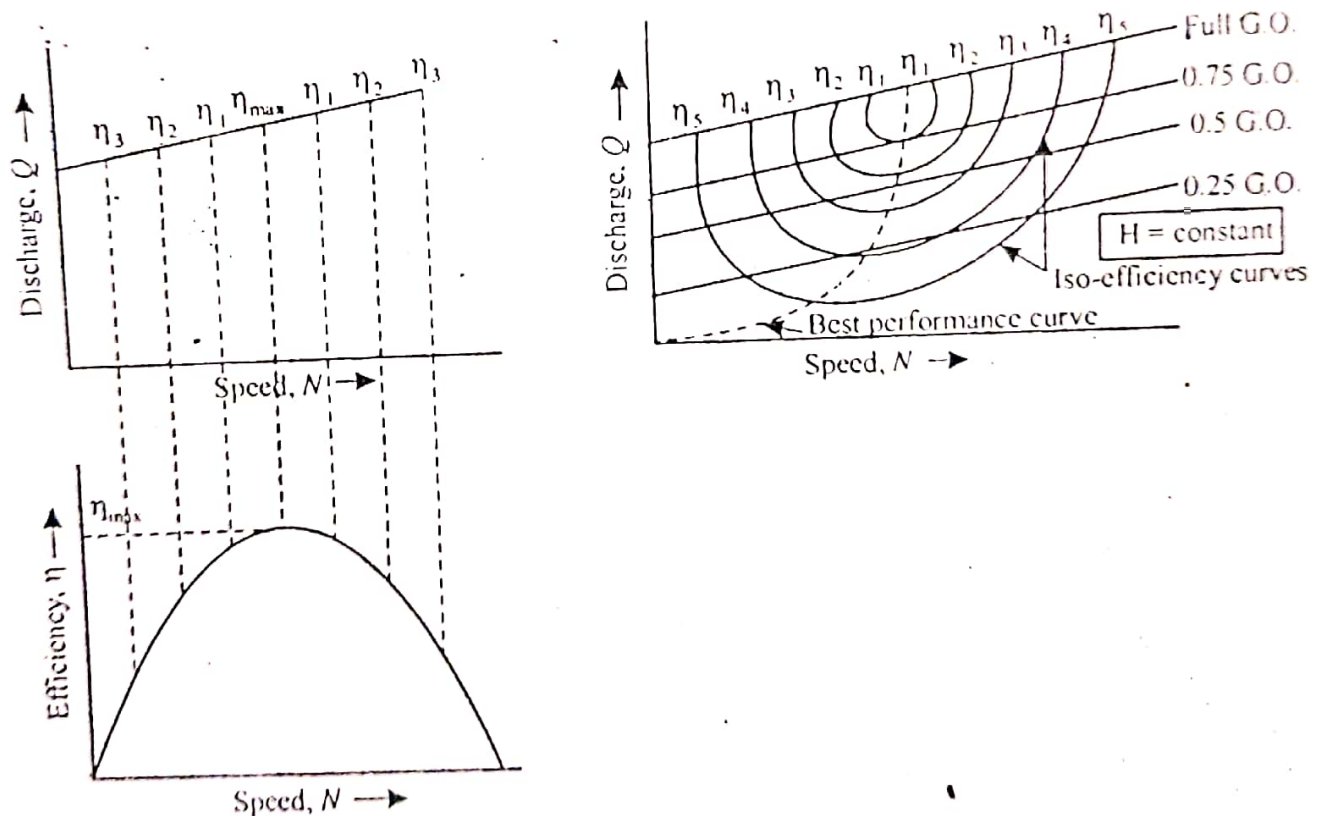


Fig. 18.58. Constant efficiency curves for turbines.

The curve for the best performance is obtained by joining the peak points of the various efficiency curves.

The constant efficiency curves are helpful for determining the zone of constant efficiency and for predicting the performance of the turbine at various efficiencies.

18-14. Governing of Hydraulic Turbines

Governing of hydraulic turbine means *speed regulation*. Governing of a turbine is necessary as a turbine is directly coupled to an electric generator, which is required to run at constant speed under all fluctuating load conditions. This is achieved by means of a *governor* called oil pressure governor.

18.14.1. Governing of Impulse Turbines

In order to regulate the quantity of water rejected from the turbine nozzle and from striking the buckets one of the following methods of regulation may be adapted :

1. Spear regulation. 2. Deflector regulation. 3. Combined spear and deflector regulation.

1. **Spear regulation.** Refer to Fig. 18.59. In this method the rate of flow is regulated by altering the cross-sectional area of stream by moving the spear to and from inside nozzle. (This method of speed regulation is suitable when the fluctuation of load is small and a relatively large penstock feeds a small turbine.) The disadvantages of this method is that when the load falls all of sudden, the turbine nozzle has to close suddenly which may cause water hammer in the penstock.

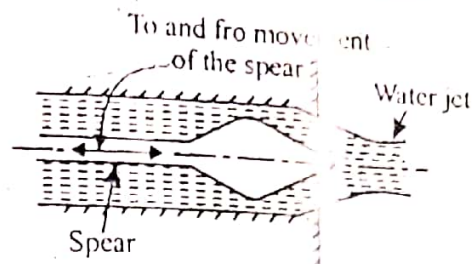


Fig 18.59. Spear regulation in Pelton wheel.

2. **Deflector regulation.** Refer to Fig. 18.60. (The deflector is generally a plate connected to the oil pressure governor by means of levers.) When necessity arises to deflect the jet, the plate can be brought in between the nozzle and buckets, thereby diverting the water away from the runner and directing into the tail race. The use of deflector regulation is restricted to when the supply of water is constant but the load fluctuates. The position of spear can be adjusted by hand. As the nozzle has always a constant opening, it results in wastage of water and can be employed only when there is an abundant water supply.

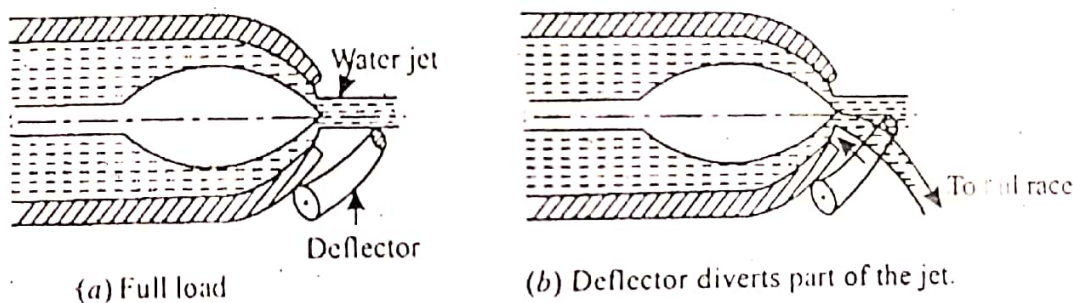


Fig. 18.60. Deflector regulation.

3. **Combined spear and deflector regulation.** As the above mentioned methods have some disadvantages, the modern turbines make use of combined spear and deflector regulation. (the spear regulates the speed and the deflector arrangement regulates the pressure.) Fig. 18.61 shows such an arrangement for governing of Pelton turbine when the turbine is running at normal speed. The working of the system is as follows :

- When the load on the turbine increases the speed of the runner falls and consequently balls of the centrifugal governor move inwards; the governor sleeve moves downwards.
- The downward movement of the sleeve is transmitted to a relay or control valve (through suitable linkages) which admits oil under pressure to a servomotor. The oil exerts a force on the piston of the servomotor, and that pushes the spear to a position which increases the annular area of the nozzle flow passage; the quantity of water striking the buckets is then increased and the turbine regains its normal speed.
- When the load on the turbine decreases the direction of movement of the servomotor is such the nozzle area decreases and that allows a smaller quantity of water to strike the runner of the turbine.
- A deflector arrangement safeguards against excessive water hammer pressure.

18.14.2. Governing of Reaction Turbines

In a reaction turbine the discharge is controlled by varying the area of flow between adjacent guide vanes. The guide vanes are connected to the regulating ring through links. The regulating ring

7. The applied torque is more uniform to the driven shaft.
8. A steam turbine requires less attention during running. Moreover, the repair costs are generally less.

22.3. Classification of Steam Turbines

The steam turbines may be classified into the following types :

1. According to the mode of steam action
 - (i) Impulse turbine, and (ii) Reaction turbine.
2. According to the direction of steam flow.
 - (i) Axial flow turbine, and (ii) Radial flow turbine.
3. According to the exhaust condition of steam
 - (i) Condensing turbine, and (ii) Non-condensing turbine.
4. According to the pressure of steam
 - (i) High pressure turbine, (ii) Medium pressure turbine, and (iii) Low pressure turbine.
5. According to the number of stages
 - (i) Single stage turbine, and (ii) Multi-stage turbine.

In this chapter, we shall discuss impulse turbines only. All the above mentioned other steam turbines will be discussed at the appropriate places in the book.

22.4. Impulse Turbine

An *impulse turbine, as the name indicates, is a turbine which runs by the impulse of steam jet. In this turbine, the steam is first made to flow through a nozzle. Then the steam jet impinges on the turbine blades (which are curved like buckets) and are mounted on the circumference of the wheel. The steam jet after impinging glides over the concave surface of the blades and finally leave the turbine.

Note : The action of the jet of steam, impinging on the blades, is said to be an *impulse* and the rotation of the rotor is due to the impulsive forces of the steam jets.

22.5. De-Laval Impulse Turbine

A **De-Laval turbine is the simplest type of impulse steam turbine, and is commonly used. It has the following main components :

1. **Nozzle.** It is a circular guide mechanism, which guides the steam to flow at the designed direction and velocity. It also regulates the flow of steam. The nozzle is kept very close to the blades, in order to minimise the losses due to windage.

2. **Runner and blades.** The runner of a De-Laval impulse turbine essentially consists of a circular disc fixed to a horizontal shaft. On the periphery of the runner, a number of blades are fixed uniformly. The steam jet impinges on the buckets, which move in the direction of the jet. This movement of the blades makes the runner to rotate.

The surface of the blades is made very smooth to minimise the frictional losses. The blades are generally

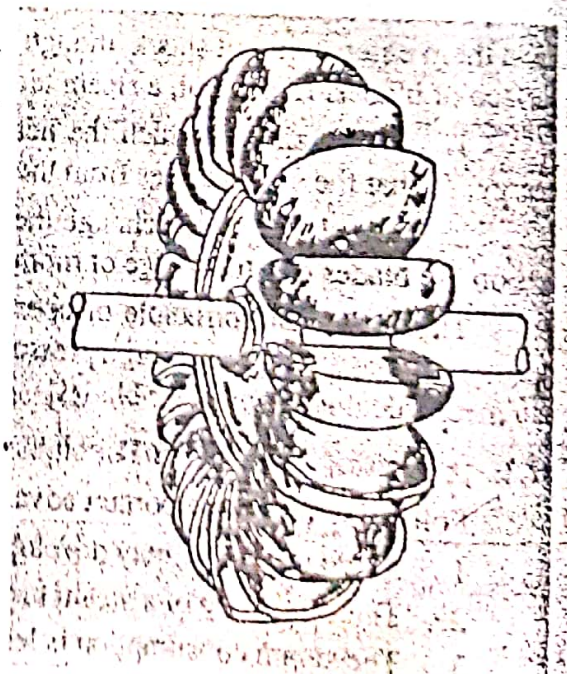


Fig. 22.1 Runner and bucket of impulse turbine.

The first impulse turbine was devised by Giovanni Branca in 1629.

* Named after the Swedish engineer De-Laval, who devised this turbine in 1881.

Impulse Turbines

made of special steel alloys. In most of the cases, the blades are bolted to the runner disc. But sometimes the blades and disc are cast as a single unit.

It has been experienced that all the blades do not wear out equally with the time. A few of them get worn out and damaged early and need replacement. This can be done only if the blades are bolted to the disc.

3. **Casing.** It is an air-tight metallic case, which contains the turbine runner and blades. It controls the movement of steam from the blades to the condenser, and does not permit it to move into the space. Moreover, it is essential to safeguard the runner against any accident.

22.6. Pressure and Velocity of Steam in an Impulse Turbine

The pressure of steam jet is reduced in the nozzle and remains constant while passing through the moving blade. The velocity of steam is increased in the nozzle, and is reduced while passing through the moving blades.

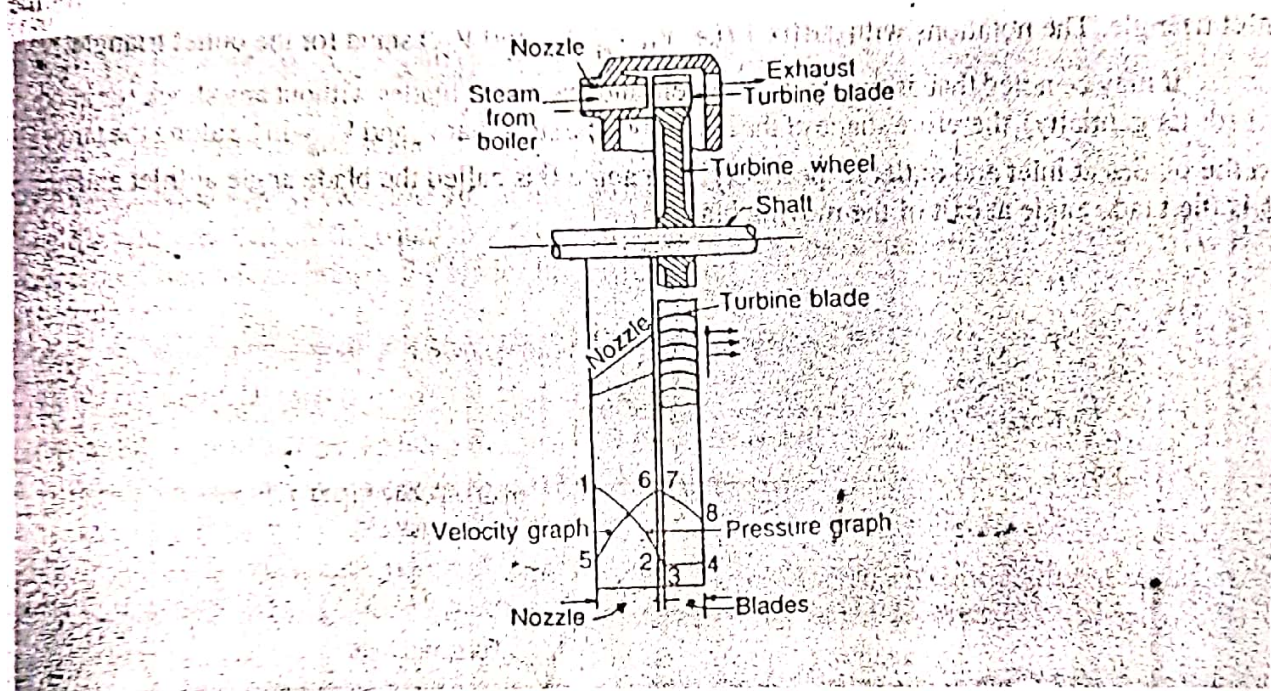


Fig. 22.2 Pressure and velocity graph of steam in a simple impulse turbine.

Fig. 22.2 shows the pressure and velocity graphs of the steam in a simple impulse turbine while it flows in the nozzle and blades. The pressure graph 1-2-3-4 represents steam pressure at entrance of the nozzle, exit of the nozzle, entrance of the blades and exit of the blades respectively. Similarly, velocity graph 5-6-7-8 represents the velocity of steam at entrance of the nozzle, exit of the nozzle, entrance of the blades and exit of the blades respectively.

22.7. Velocity Triangles for Moving Blade of an Impulse Turbine

We have already discussed that in an impulse turbine, the steam jet after leaving the nozzle impinges on one end of the blade. The jet then glides over the inside surface of the blade and finally leaves from the other edge, as shown in Fig. 22.3. It may be noted that the jet enters and leaves the blades tangentially for shockless entry and exit.

Consider a steam jet entering a curved blade after leaving the nozzle at C. Now let the jet glides over the inside surface and leaves the blade at D, as shown in Fig. 22.3. Now let us draw the velocity triangles at inlet and outlet tips of the moving blade, as shown in Fig. 22.3.

Let V_b = Linear velocity of the moving blade (AB),
 V = Absolute velocity of steam entering the moving blade (AC).

V_r = Relative velocity of jet to the moving blade (BC). It is the vectorial difference between V_b and V .

V_f = Velocity of flow at entrance of the moving blade. It is the vertical component of V .

V_w = Velocity of whirl at entrance of the moving blade. It is the horizontal component of V .

θ = Angle which the relative velocity of jet to the moving blade (V_r) makes with the direction of motion of the blade.

α = Angle with the direction of motion of the blade at which the jet enters the blade.

$V_1, V_{r1}, V_{f1}, V_{w1}, \beta, \phi$ = Corresponding values at exit of the moving blade.

It may be seen from the above, that the original notations (i.e. V, V_r, V_f and V_w) stand for the inlet triangle. The notations with suffix 1 (i.e. V_1, V_{r1}, V_{f1} and V_{w1}) stand for the outlet triangle.

It may be noted that as the steam jet enters and leaves the blades without any shock (or in other words tangentially), therefore shape of the blades will be such that V_r and V_{r1} will be along the tangents to the blades at inlet and outlet respectively. The angle θ is called the blade angle at inlet and angle ϕ is the blade angle at exit of the moving blade.

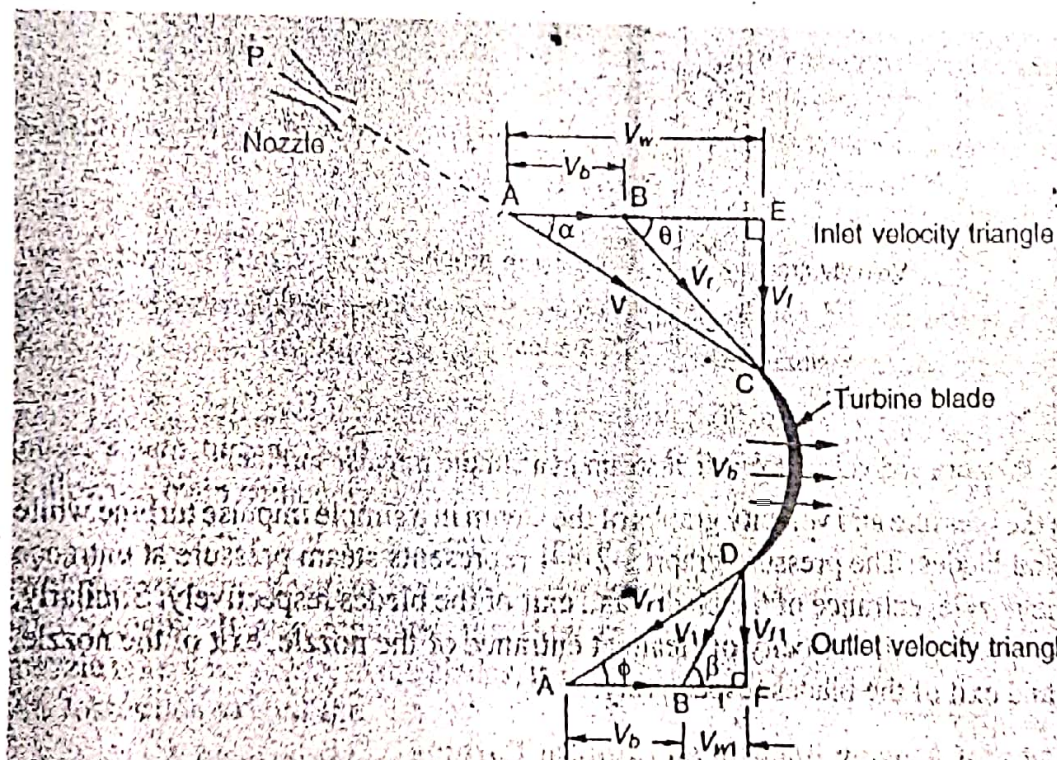


Fig. 22.3 Velocity triangles of an impulse turbine.

In Fig. 22.3, PC is the axis of the nozzle, which delivers the steam jet with a high velocity (V) at an angle α with the direction of motion of the blade. The jet impinges on a series of turbine blades mounted on the runner disc.

The axial component of V (i.e. EC) which does no work on the blade, is known as velocity of flow (V_f). It causes the steam to flow through the turbine and also an axial thrust on the rotor. The tangential component of V (represented by BE) is known as velocity of whirl at inlet (V_w). The linear velocity or mean velocity of the blade (i.e. V_b) is represented by AB in magnitude and direction. The

The jet now glides over and leaves the blade with a relative velocity V_{r1} which is represented by DA . The absolute velocity of jet (V_1) as it leaves the blade, is represented by DB inclined at an angle β with the direction of the blade motion. The tangential component of V_1 (represented by BF) is known as velocity of whirl at exit (V_{w1}). The axial component of V_1 (represented by DF) is known as velocity of flow at exit (V_{f1}).

- Notes : 1. The inlet triangle of velocities is represented by BEC , whereas the outlet triangle by AFD .
2. The relations between inlet and outlet velocity triangle (until and unless given) is :

$$V_r = V_{r1}$$

22.8. Combined Velocity Triangle for Moving Blades

In the last article, we have discussed the inlet and outlet velocity triangle separately. For the sake of simplification, a combined velocity triangle for the moving blade is drawn, for solving problems on steam turbines, as shown in Fig. 22.4, and as discussed below :

1. First of all, draw a horizontal line, and cut off AB equal to velocity of blade (V_b) to some suitable scale.

2. Now at B , draw a line BC at an angle α with AB . Cut off BC equal to V (i.e. velocity of steam jet at inlet of the blade) to the scale.

3. Join AC , which represents the relative velocity at inlet (V_r). Now at A , draw a line AD at an angle ϕ with AB .

4. Now with A as centre and radius equal to AC , draw an arc meeting the line through A at D , such that $AC = AD$ or $V_r = V_{r1}$.

5. Join BD , which represents velocity of jet at exit (V_1) to the scale.

6. From C and D , draw perpendiculars meeting the line AB produced at E and F respectively.

7. Now EB and CE represents the velocity of whirl and velocity of flow at inlet (V_w and V_f) to the scale. Similarly, BF and DF represents the velocity of whirl and velocity of flow at outlet (V_{w1} and V_{f1}) to the scale.

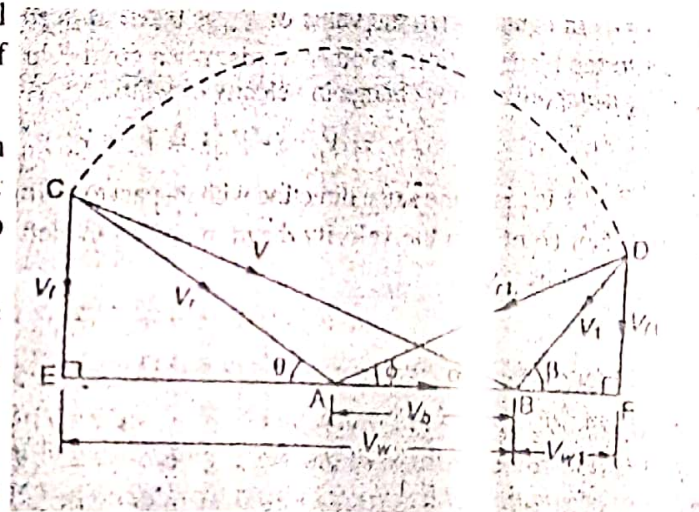


Fig. 22.4 Combined velocity triangle for an impulse turbine.

22.9. Power Produced by an Impulse Turbine

Consider an impulse turbine working under the action of a steam jet. Let us draw a combined velocity triangle, for the impulse turbine as shown in Fig. 22.4.

Let m = Mass of the steam flowing through the turbine in kg/s ,

$(V_w + V_{w1})$ = Change in the velocity of whirl in m/s .

We know that according to the Newton's second law of motion, force in the direction of motion of the blades,

$$\begin{aligned} F_x &= \text{Mass of steam flowing per second} \times \text{Change in the velocity of whirl} \\ &= m [V_w - (-V_{w1})] \\ &= m [V_w + V_{w1}] = m \times EFN \end{aligned} \quad \dots (i)$$

and work done in the direction of motion of the blades

$$\begin{aligned}
 &= \text{Force} \times \text{Distance} \\
 &= m [V_w + V_{w1}] V_b \text{ N-m/s} \quad \dots (ii) \\
 &= m \times EF \times AB \text{ N-m/s}
 \end{aligned}$$

∴ Power produced by the turbine,

$$\begin{aligned}
 P &= m \times EF \times AB \text{ watts} \quad \dots (\because 1 \text{ N-m/s} = 1 \text{ watt}) \\
 &= m (V_w + V_{w1}) V_b \text{ watts}
 \end{aligned}$$

Similarly, we can find out the axial thrust on the wheel which is due to the difference of velocities of flow at inlet and outlet. Mathematically, axial thrust on the wheel,

$$\begin{aligned}
 F_Y &= \text{Mass of steam flowing per second} \times \text{Change in the velocity of flow} \\
 &= m (V_f - V_{f1}) = m (CE - DF) \text{ N} \quad \dots (iii)
 \end{aligned}$$

Note : 1. In equation (i), the value of V_{w1} is taken as *negative* because of the opposite direction of V_w with respect to the blade motion. In other words, when point F in the velocity diagram lies on the right of point B , the V_{w1} is negative. Thus change in velocity of whirl,

$$= V_w - (-V_{w1}) = V_w + V_{w1}$$

2. If V_{w1} is in the same direction with respect to the blade motion, then V_{w1} is taken as positive. In other words, when point F in the velocity diagram lies on the left of point B , then V_{w1} is positive. Thus change in velocity of whirl

$$= V_w - (+V_{w1}) = V_w - V_{w1} \quad (\text{See Example 22.3})$$

Example 22.1. In a De-laval turbine, the steam enters the wheel through a nozzle with a velocity of 500 m/s and at an angle of 20° to the direction of motion of the blade. The blade speed is 200 m/s and the exit angle of the moving blade is 25° . Find the inlet angle of the moving blade, exit velocity of steam and its direction and work done per kg of steam.

Solution. Given : $V = 500 \text{ m/s}$; $\alpha = 20^\circ$; $V_b = 200 \text{ m/s}$; $\phi = 25^\circ$

Now let us draw the combined velocity triangle, as shown in Fig. 22.5, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to 200 m/s, to some suitable scale, representing the blade speed (V_b).

2. Now at B , draw a line BC at an angle of 20° (nozzle angle, α) and cut off BC equal to 500 m/s to the scale to represent the velocity of steam jet entering the blade (V).

3. Join AC , which represents the relative velocity at inlet (V_r).

4. At A , draw a line AD at an angle of 25° (exit angle of the moving blade, ϕ). Now with A as centre, and radius equal to AC , draw an arc meeting the line through A at D .

5. Join BD , which represents the velocity of steam jet at outlet (V_1).

6. From C and D , draw perpendiculars meeting the line AB produced at E and F respectively. CE and DF represents the velocity of flow at inlet (V_f) and outlet (V_{f1}) respectively.

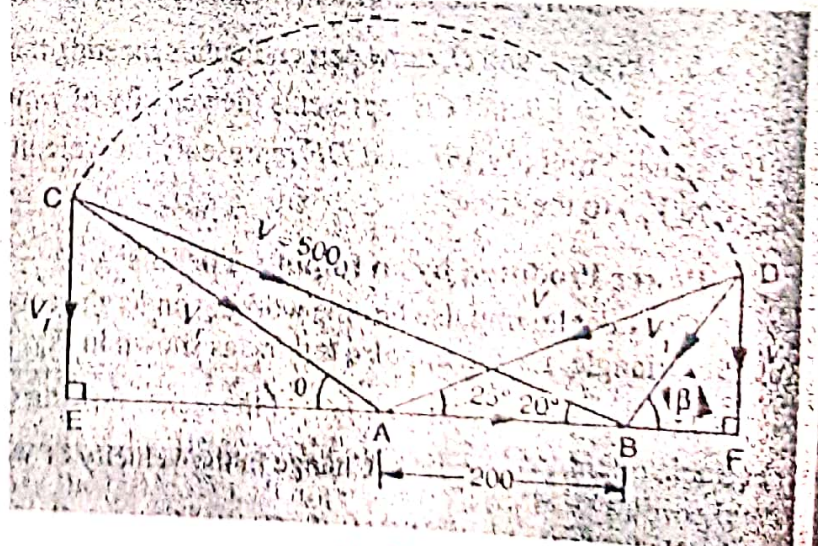


Fig. 22.5

The following values are *measured from the velocity diagram :

$$\theta = 32^\circ ; \beta = 59^\circ ; V_1 = BD = 165 \text{ m/s}$$

$$V_w = BE = 470 \text{ m/s and } V_{w1} = BF = 90 \text{ m/s}$$

Inlet angle of moving blade

By measurement from the velocity diagram, we find that the inlet angle of the moving blade,

$$\theta = 32^\circ \text{ Ans.}$$

Exit velocity of steam

By measurement from the velocity diagram, we also find that the exit velocity of steam,

$$V_1 = 165 \text{ m/s}$$

Direction of the exit steam

By measurement from the velocity diagram, we also find that the direction of the exit steam,

$$\beta = 59^\circ \text{ Ans.}$$

Workdone per kg of steam

We know that workdone per kg of steam

$$= m (V_w + V_{w1})$$

$$= 1 (470 + 90) = 560 \text{ N-m Ans.}$$

... ($\because m = 1 \text{ kg}$)

22.10. Effect of Friction on the Combined Velocity Triangle

In the last article, we have discussed that the relative velocity of steam jet is the same at the inlet and outlet tips of the blade. In other words, we have assumed that the inner side of the curved blade offers no resistance to the steam jet. But in actual practice, some resistance is always offered by the blade surface to the gliding steam jet, whose effect is to reduce the relative velocity of the jet. i.e. to make V_{r1} less than V_r . The ratio of V_{r1} to V_r is known as *blade velocity coefficient* or coefficient of velocity or friction factor, (usually denoted by K). Mathematically, blade velocity coefficient,

$$K = \frac{V_{r1}}{V_r}$$

These values may also be found out from the geometry of the velocity diagram as discussed below :

$$V_w = V \cos 20^\circ = 500 \times 0.9397 = 469.9 \text{ m/s}$$

$$V_f = V \sin 20^\circ = 500 \times 0.342 = 171 \text{ m/s}$$

$$\tan \theta = \frac{V_f}{V_w - V_b} = \frac{171}{469.9 - 200} = 0.6335 \text{ or } \theta = 32.35^\circ$$

$$V_r = V_{r1} = \frac{V_f}{\sin \theta} = \frac{171}{\sin 32.35^\circ} = \frac{171}{0.5351} = 319.4 \text{ m/s}$$

$$V_{r1} = V_{r1} \sin 25^\circ = 319.4 \times 0.4226 = 135 \text{ m/s}$$

$$V_{w1} = (V_{r1} \cos 25^\circ) - 200 = (319.4 \times 0.9063) - 200 = 89.5 \text{ m/s}$$

$$\tan \beta = \frac{V_{r1}}{V_{w1}} = \frac{135}{89.5} = 1.51 \text{ or } \beta = 56.47^\circ$$

$$V_1 = \frac{V_{r1}}{\sin \beta} = \frac{135}{\sin 56.47^\circ} = \frac{135}{0.8336} = 162 \text{ m/s}$$

It may be noted that the effect of friction on the combined velocity triangle will be to reduce the relative velocity at outlet (V_{r1}) as shown in Fig. 22.6.

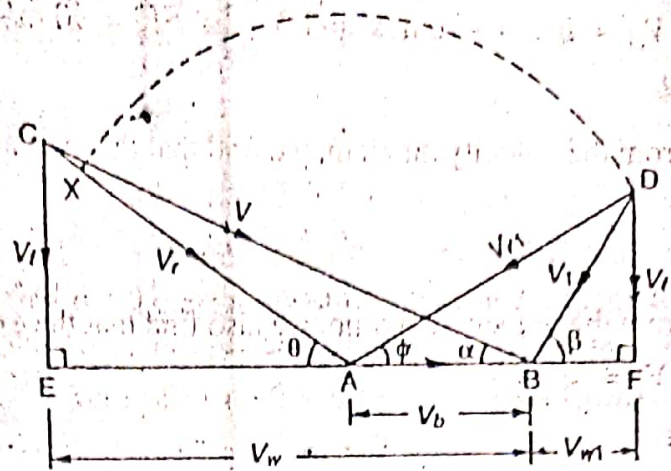


Fig. 22.6. Effect of friction on the combined velocity triangle.

Notes : 1. Since V_{r1} is decreased due to friction, therefore work done per kg of steam is also reduced.

2. The value of K varies from 0.75 to 0.85 depending upon the shape of the blades.

Example 22.2. The velocity of steam, leaving the nozzles of an impulse turbine, is 1200 m/s and the nozzle angle is 20° . The blade velocity is 375 m/s and the blade velocity coefficient is 0.75. Assuming no loss due to shock at inlet, calculate for a mass flow of 0.5 kg/s and symmetrical blading: (a) blade inlet angle; (b) driving force on the wheel; (c) axial thrust on the wheel; and (d) power developed by the turbine.

Solution. Given : $V = 1200$ m/s ; $\alpha = 20^\circ$; $V_b = 375$ m/s ; $K = V_{r1}/V_r = 0.75$; $m = 0.5$ kg/s ; $\theta = \phi$, for symmetrical blading.

Now draw the combined velocity triangle, as shown in Fig. 22.7, as discussed below :

1. First of all, draw a horizontal line, and cut off AB equal to 375 m/s to some suitable scale representing the velocity of blade (V_b).

2. Now at B , draw a line BC at an angle of 20° (Nozzle angle, α) and cut off BC equal to 1200 m/s to the scale to represent the velocity of steam jet entering the blade (V).

3. Join CA , which represents the relative velocity at inlet (V_r). By measurement, we find that $CA = V_r = 860$ m/s. Now cut off AX equal to $860 \times 0.75 = 645$ m/s to the scale to represent the relative velocity at exit (V_{r1}).

4. At A , draw a line AD at an angle ϕ equal to the angle θ , for symmetrical blading. Now with A as centre, and radius equal to AX , draw an arc meeting the line through A at D , such that $AD = V_{r1}$.

5. Join BD , which represents the velocity of steam jet at outlet (V_o).

6. From C and D , draw perpendiculars meeting the line AB produced at E and F respectively. CE and DF represents the velocity of flow at inlet (V_i) and outlet (V_o1) respectively.

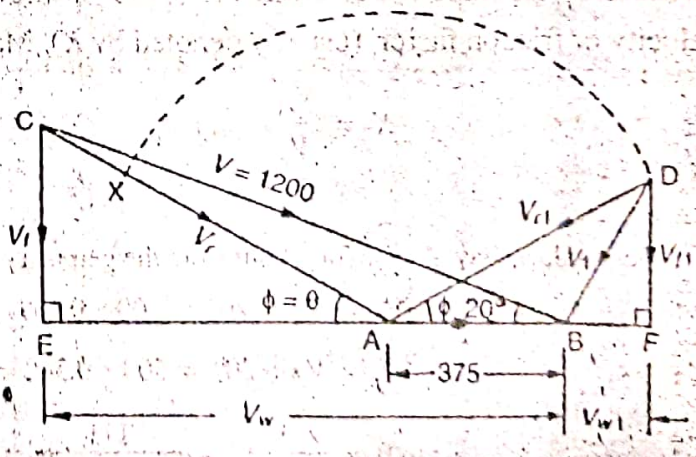


Fig. 22.7

Impulse Turbines

The following values are *measured from the velocity diagram :

$$\theta = 29^\circ ; V_w = BE = 1130 \text{ m/s} ; V_{w1} = BF = 190 \text{ m/s}$$

$$V_f = CE = 410 \text{ m/s and } V_{f1} = DF = 310 \text{ m/s}$$

(a) Blade inlet angle

By measurement from the velocity diagram, we find that the blade angle at inlet,

$$\theta = 29^\circ \text{ Ans.}$$

(b) Driving force on the wheel

We know that driving force on the wheel,

$$F_x = m (V_w + V_{w1}) = 0.5 (1130 + 190) = 660 \text{ N Ans.}$$

(c) Axial thrust on the wheel

We know that axial thrust on the wheel,

$$F_y = m (V_f - V_{f1}) = 0.5 (410 - 310) = 50 \text{ N Ans.}$$

(d) Power developed by the turbine

We know that power developed by the turbine,

$$P = m (V_w + V_{w1}) V_b = 0.5 (1130 + 190) 375 = 17500 \text{ W} \\ = 17.5 \text{ kW Ans.}$$

Example 22.3. A steam jet enters the row of blades with a velocity of 375 m/s at an angle of 20° with the direction of motion of the moving blades. If the blade speed is 165 m/s, find the suitable inlet and outlet blade angles assuming that there is no thrust on the blades. The velocity of steam passing over the blades is reduced by 15%. Also determine power developed by the turbine per kg of steam flowing over the blades per second.

Solution. Given : $V = 375 \text{ m/s} ; \alpha = 20^\circ ; V_b = 165 \text{ m/s} ; m = 1 \text{ kg/s}$

Since there is no thrust on the blades, therefore

$$V_f = V_{f1}$$

Also, the velocity of steam passing over the blades is reduced by 15% therefore

$$V_{f1} = 0.85 V_f$$

These values may also be found out from the geometry of the velocity diagram discussed below

$$V_w = V \cos 20^\circ = 375 \times 0.9397 = 352.5 \text{ m/s}$$

$$V_f = V \sin 20^\circ = 375 \times 0.3420 = 128.25 \text{ m/s}$$

$$\tan \theta = \frac{V_f}{V_w - V_b} = \frac{128.25}{352.5 - 165} = 0.5450 \text{ or } \theta = 28.6^\circ$$

$$V_r = \frac{V_f}{\sin \theta} = \frac{128.25}{\sin 28.6^\circ} = \frac{128.25}{0.4787} = 268.1 \text{ m/s}$$

$$V_{f1} = 0.85 \times 268.1 = 227.9 \text{ m/s}$$

$$V_{f1} = V_{f1} \sin 28.6^\circ = 227.9 \times 0.4787 = 109.1 \text{ m/s}$$

$$V_{w1} = V_{f1} \cos 28.6^\circ = 109.1 \times 0.8780 = 95.8 \text{ m/s}$$

Now draw the combined velocity triangle, as shown in Fig. 22.8, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to 165 m/s to some suitable scale representing the blade speed (V_b).

2. Now draw inlet velocity triangle ABC on the base AB with $\alpha = 20^\circ$ and $V = 375$ m/s to the scale. From the velocity triangle, we find that $V_f = 30$ m/s and $V_r = 230$ m/s.

3. Similarly, draw outlet velocity triangle ABD on the same base AB with $V_{r1} = 0.85 V_r = 0.85 \times 230 = 195.5$ m/s to the scale and $V_{f1} = V_f = 130$ m/s.

4. From C and D draw perpendiculars to meet the line AB produced at E and F . From the geometry of the figure, we find that V_{w1} is in the opposite direction of V_w .

The following values are measured from the combined velocity triangle :

$$\theta = 34^\circ; \phi = 41^\circ \text{ and } (V_w - V_{w1}) = 320 \text{ m/s}$$

Inlet and outlet blade angle

By measurement from the velocity diagram, we find that inlet blade angle,

$$\theta = 34^\circ \text{ Ans.}$$

and outlet blade angle,

$$\phi = 41^\circ \text{ Ans.}$$

Power developed by the turbine

We know that power developed by the turbine,

$$\begin{aligned} P &= m (V_w - V_{w1}) V_b \\ &= 1 \times 320 \times 165 = 52800 \text{ W} = 52.8 \text{ kW Ans.} \end{aligned}$$

Example 22.4. The blade speed of a single ring impulse blading is 250 m/s and nozzle angle is 20° . The heat drop is 550 kJ/kg and nozzle efficiency is 0.85. The blade discharge angle is 30° and the machine develops 30 kW, when consuming 360 kg of steam per hour. Draw the velocity diagram and calculate : 1. axial thrust on the blading, and 2. the heat equivalent per kg of steam friction of the blading.

Solution. Given : $V_b = 250$ m/s ;
 $\alpha = 20^\circ$; $h_d = 550$ kJ/kg ; $K = 0.85$; $\phi = 30^\circ$;
 $P = 30$ kW = 30×10^3 W ; $m = 360$ kg/h =
 0.1 kg/s

We know that the velocity of steam entering the blades,

$$\begin{aligned} V &= 44.72 \sqrt{K h_d} \\ &= 44.72 \sqrt{0.85 \times 550} = 967 \text{ m/s} \end{aligned}$$

and power developed (P),

$$30 \times 10^3 = m (V_w + V_{w1}) V_b = 0.1 (V_w + V_{w1}) 250$$

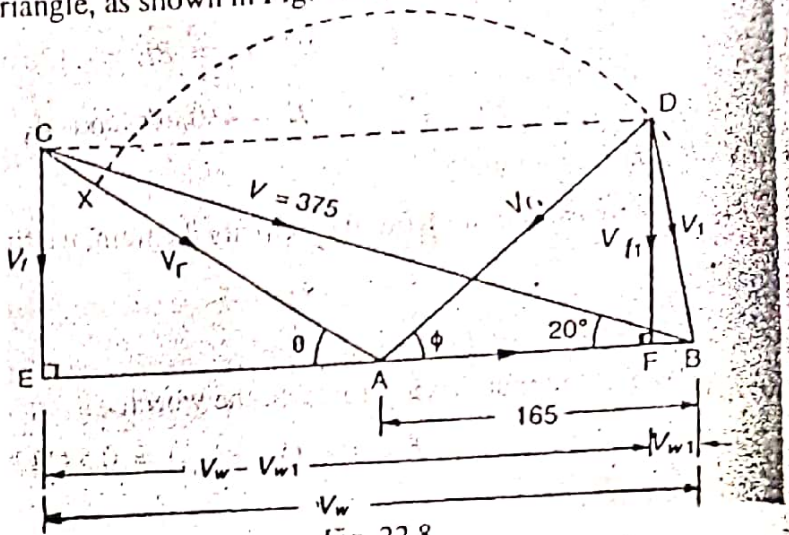


Fig. 22.8

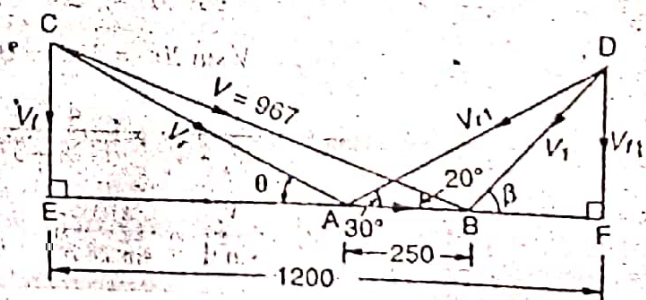


Fig. 22.9

$$V_w + V_{w1} = 1200 \text{ m/s}$$

Now draw the combined velocity triangle, as shown in Fig. 22.9, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to 250 m/s to some suitable scale representing the blade speed (V_b).

2. Now draw inlet velocity triangle ABC on the base AB with $\alpha = 20^\circ$ and $V = 967 \text{ m/s}$ to the scale.

3. From point C , draw CE perpendicular to AB produced and take

$$EF = V_w + V_{w1} = 1200 \text{ m/s}$$

4. Draw a line at point A at an angle 30° intersecting the perpendicular drawn through F , at D .

Now complete the outlet velocity triangle ABD as shown in the figure.

The following values are measured from the combined velocity triangle :

$$V_f = 400 \text{ m/s} ; V_{f1} = 325 \text{ m/s} ; V_r = 895 \text{ m/s} \text{ and } V_{r1} = 650 \text{ m/s}$$

1. Axial thrust on the blading

We know that axial thrust on the blading,

$$F_Y = m (V_f - V_{f1}) = 0.1 (400 - 325) = 7.5 \text{ N Ans.}$$

2. Heat equivalent per kg of steam of friction of the blading

We know that work lost in friction of the blading

$$= \frac{(V_r)^2 - (V_{r1})^2}{2000} = \frac{(895)^2 - (650)^2}{2000} = 189.26 \text{ kJ}$$

$\therefore$ Heat equivalent per kg of steam friction of the blading

$$= 189.26 \text{ kJ Ans.}$$

Example 22.5. Steam at 5 bar and 200°C is first made to pass through nozzles. It is then supplied to an impulse turbine at the rate of 30 kg/minute. The steam is finally exhausted to a condenser at 0.2 bar. The blade speed is 300 m/s. The nozzles are inclined at 25° with the direction of motion of the blades and the outlet blade angle is 35° . Neglecting friction, find the theoretical power developed by the turbine.

Solution. Given : $p_1 = 5 \text{ bar}$; $T_1 = 200^\circ \text{C}$;
 $m = 30 \text{ kg/min} = 0.5 \text{ kg/s}$; $p_2 = 0.2 \text{ bar}$; $V_b = 300 \text{ m/s}$;
 $\alpha = 25^\circ$; $\phi = 35^\circ$

First of all, let us draw Mollier diagram for the flow of steam through the nozzle, as shown in Fig. 22.10. From this diagram, we find that heat drop during the flow,

$$\begin{aligned} h_d &= h_1 - h_2 \\ &= 2850 - 2340 = 510 \text{ kJ/kg} \end{aligned}$$

$\therefore$ Velocity of steam at inlet of the blade,

$$V = 44.72 \sqrt{510} = 1010 \text{ m/s}$$

Now draw the combined velocity triangle, as shown in Fig. 22.11, as discussed below :

1. First of all, draw a horizontal line and cut off AB equal to 300 m/s, to some suitable scale, to represent the blade speed (V_b).

2. Now draw inlet velocity triangle ABC on the base AB with $\alpha = 25^\circ$ and $V = 1010 \text{ m/s}$. From the velocity triangle, we find that $V_r = 850 \text{ m/s}$ to the scale.

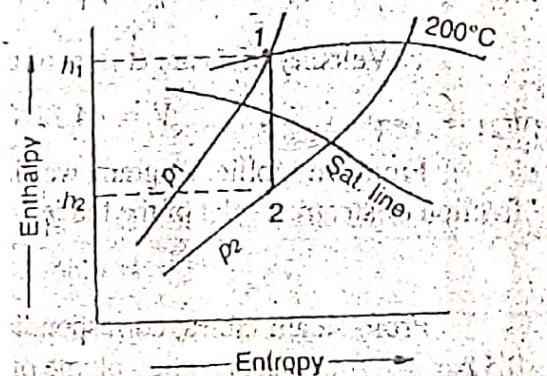


Fig. 22.10

3. Similarly, draw the outlet velocity triangle ABD on the same base AB with $\phi = 35^\circ$ and $V_{r1} = V_r = 850 \text{ m/s}$ to the scale.

4. From C and D draw perpendiculars to meet the line AB produced at E and F .

By measurement from the velocity diagram, we find that

$$EF = V_w + V_{w1} = 1400 \text{ m/s}$$

We know theoretical power developed by the turbine,

$$\begin{aligned} P &= m (V_w + V_{w1}) V_b \\ &= 0.5 \times 1400 \times 300 \\ &= 210\,000 \text{ W} \\ &= 210 \text{ kW Ans.} \end{aligned}$$

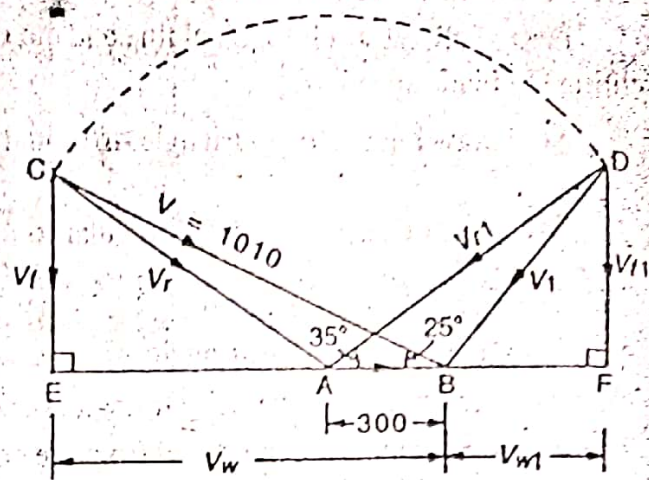


Fig. 22.11

Example 22.6. In a certain stage of an impulse turbine, the nozzle angle is 20° with the plane of the wheel. The mean diameter of the blade ring is 2.8 metres. It develops 55 kW at 2400 r.p.m. Four nozzles, each of 10 mm diameter expand steam isentropically from 15 bar and 250°C to 0.5 bar. The axial thrust is 3.5 N. Calculate :

1. blade angles at entrance and exit, and 2. power lost in blade friction.

Solution. Given : $\alpha = 20^\circ$; $D = 2.8 \text{ m}$; $P = 55 \text{ kW} = 55 \times 10^3 \text{ W}$; $N = 2400 \text{ r.p.m.}$; $n = 4$; $d = 10 \text{ mm}$; $p_1 = 15 \text{ bar}$; $T_1 = 250^\circ \text{C}$; $p_2 = 0.5 \text{ bar}$; $F_y = 3.5 \text{ N}$

We know that blade speed,

$$V_b = \pi DN/60 = \pi \times 2.8 \times 2400/60 = 352 \text{ m/s}$$

1. Blade angles at entrance and exit

First of all, let us draw Mollier diagram for the flow of steam through the nozzle, as shown in Fig. 22.12. From this diagram, we find that heat drop during the flow,

$$h_d = h_1 - h_2 = 2920 - 2330 = 590 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at inlet of the blade,

$$V = 44.72 \sqrt{590} = 1086 \text{ m/s}$$

From the Mollier diagram, we also find that dryness fraction of steam at inlet of the blade,

$$x = 0.86$$

From steam tables, corresponding to a pressure of 0.5 bar, we find that specific volume of steam,

$$v_g = 3.24 \text{ m}^3/\text{kg}$$

We know that area of each nozzle

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 10^2 = 78.55 \text{ mm}^2 = 78.55 \times 10^{-6} \text{ m}^2$$

$\therefore$ Mass of steam discharged through the nozzle,

$$m = \frac{n A V}{x v_g} = \frac{4 \times 78.55 \times 10^{-6} \times 1086}{0.86 \times 3.24} = 0.12 \text{ kg/s}$$

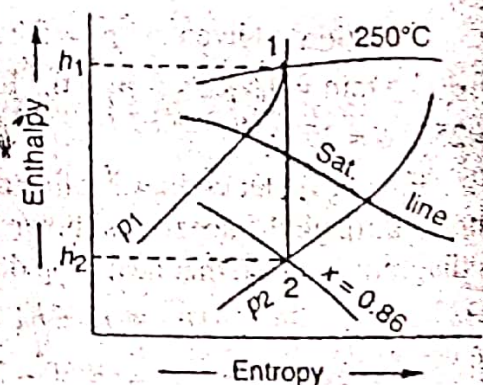


Fig. 22.12

We know that power developed (P),

$$55 \times 10^3 = m (V_w + V_{w1}) V_b = 0.12 (V_w + V_{w1}) 352 = 42.24 (V_w + V_{w1})$$

$$\therefore V_w + V_{w1} = 1302 \text{ m/s}$$

We also know that axial thrust (F_A),

$$3.5 = m (V_f - V_{f1}) = 0.12 (V_f - V_{f1})$$

$$\therefore V_f - V_{f1} = 29.2 \text{ m/s or } V_{f1} = (V_f - 29.2) \text{ m/s}$$

Now draw the combined velocity triangle, as shown in Fig. 22.13, as discussed below:

1. First of all, draw a horizontal line and cut off AB equal to 352 m/s, to some suitable scale, representing the blade speed (V_b).

2. Now draw the inlet velocity triangle ABC on the base AB with $\alpha = 20^\circ$ and $V = 1086 \text{ m/s}$ to the scale. By measurement, we find that $V_f = 370 \text{ m/s}$.

3. Similarly, draw outlet velocity triangle ABD on the same base AB with EF (i.e. $V_w + V_{w1}$) = 1302 m/s to the scale and $V_{f1} = V_f - 29.2 = 370 - 29.2 = 340.8 \text{ m/s}$.

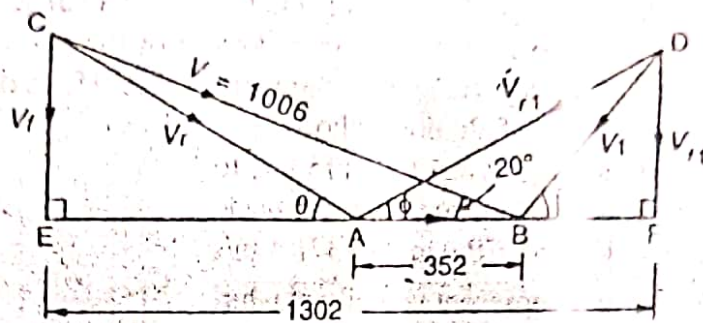


Fig. 22.13

4. From C and D , draw perpendiculars to meet the line AB produced at E and F .

By measurement from the velocity diagram, we find that

$$\theta = 29^\circ; \text{ and } \phi = 28^\circ \text{ Ans.}$$

2. Power lost in friction

By measurement from the velocity diagram, we also find that

$$V_r = 765 \text{ m/s}; \text{ and } V_{r1} = 730 \text{ m/s}$$

We know that power lost in blade friction

$$= \frac{(V_r)^2 - (V_{r1})^2}{2000} = \frac{(765)^2 - (730)^2}{2000} = 26.16 \text{ kJ/s}$$

$$= 26.16 \text{ kW Ans.}$$

$$\dots (1 \text{ kJ/s} = 1 \text{ kW})$$

22.11. Combined Velocity Diagram for Axial Discharge

Sometimes, the steam leaves the blade at its exit tip at 90° to the direction of the blade motion. In such a case, the turbine is said to have an axial discharge. The combined velocity diagram for axial discharge is drawn as shown in Fig. 22.14. It may be noted that in such a turbine, velocity of whirl at outlet (V_{w1}) is equal to zero. Therefore power developed by the turbine,

$$P = m \times V_w \times V_b \text{ watts}$$

$$\dots (\because V_{w1} = 0)$$

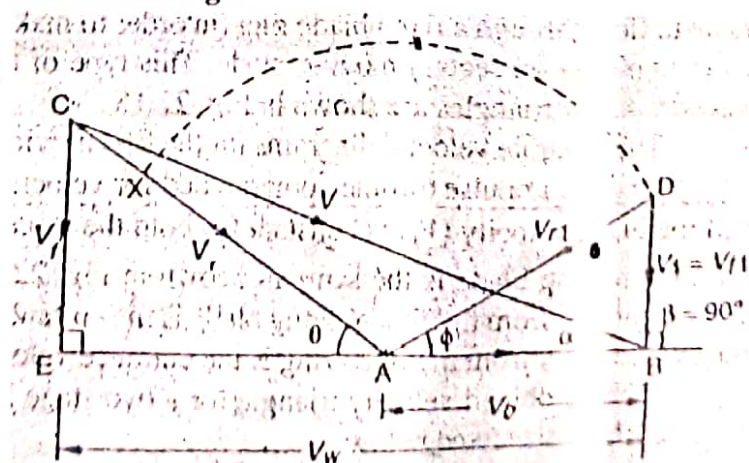


Fig. 22.14. Combined velocity diagram for axial discharge