CHAPTER 17

IMPACT OF FREE JETS

17·1. Introduction. 17·2. Forces exerted on a stationary flat plate he... normal to the jet. 17·3 Force exerted on a stationary flat plate held inclined to th... et 17·4 Force... exerted on a stationary curved plate. 17.5. Force exerted on a movi... flat plate held normal to jet. 17·6. Force exerted on a moving plate inclined to the... rection of j... 17·7. Force exerted on a moving plate inclined to the direction of jet... 8. Jet strikin... a moving curved vane tangentially at one tip and leaving at the... their. 17·9. Jet... propulsion of ships. Highlights— Objective Type Questions—Theoret... l Questions... Unsolved Examples.

17·1. Introduction

A fluid **jet** *is a stream of fluid issuing from a nozzle with a high velocity an... hence a high... energy.* When a jet impinges on a plate or vane, it exerts a force on it (due to c... nge in momentum... This force (hydrodynamic) can be evaluted by using '*Impulse- momentum p... ciple*'. This chapter deals with the application of the impulse-momentum equation for evaluating th... ydrodynamic for... on the stationary and moving vanes. The following cases of impact of jet will... considered

A. Force exerted by the jet on the stationary plate:

1. When flat plate is held normal to the jet;
2. When flat plate is held inclined to the jet;
3. When plate is curved.

B. Force exerted by the jet on the moving plate:

1. When plate is held normal to the jet;
2. When plate is held inclined to the jet;
3. When plate is curved.

A. FORCE EXERTED BY THE JET ON THE STATIONARY PLATE

17·2. Force Exerted on a Stationary Flat Plate held Normal to the ...t

Fig. 17·1 shows a fluid jet striking a stationary flat plate held perpendicula... o the flow direction... Let a and V be cross-sectional area and velocity of the jet respectively. The jet... er striking the plate... (vertical), will get its direction changed through 90°; but, it will move on... d off the plate with... velocity V, if we *neglect* the friction between the jet and the plate as is possible w... n the plate is smooth... If the friction is considered, the velocity of liquid coming off the plate will be s... htly *less* than V...

The force exerted by the jet on the plate (assuming it *smooth*) in the direct... of jet (X-direction)...

F_x = Rate of change of momentum (in the direction ... force)

= (Initial momentum—final momentum) —*Impuls... momentum principle*

= (Mass/sec) × [velocity of jet before striking the ...late – velocity of jet after striking the plate]

= $\rho aV(V-0)$ [... ass/second ... part ...

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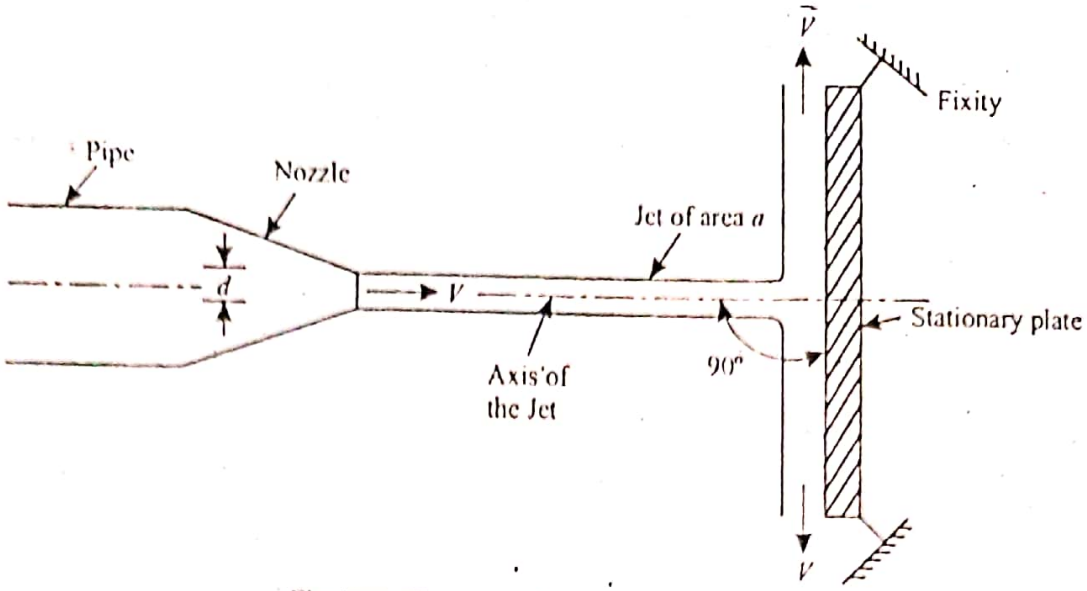


Fig. 17.1. Fluid jet striking a stationary plate.

or, $$F_x = \rho a V^2 \qquad ...(17\cdot1)$$

(where, ρ = mass density of liquid; a = area of jet $= \frac{\pi}{4} d^2$, d being diameter of the jet).

It may be noted that a jet leaves in the direction normal to X-axis, the final velocity in the direction is *zero*.

Since the plate is *stationary*, therefore, the *work done on the plate is zero.*

3. Force exerted on a Stationary Flat Plate held inclined to the Jet

Fig. 17·2 shows the stationary flat plate inclined at θ° to the direction of the horizontal jet. If a and V are the cross-sectional area and velocity of the jet respectively, then the mass of liquid per second striking the plate

$$= \rho \times aV \qquad \text{(where } a = \frac{\pi}{4} d^2, d \text{ being diameter of the jet)}$$

After striking the plate (assuming it *smooth*),the jet leaves the plate with a velocity equal to initial velocity (V).

Let us apply the *impulse- momentum equation* in the direction normal to the plate.

Force in normal direction, $F_n = \rho a V (V \sin\theta - 0)$

$$= \rho a V^2 \sin\theta \qquad ...(17\cdot2)$$

This normal force can be resolved into two components; component F_x, *parallel* to the direction of jet and component F_y, *normal* to the direction of jet.

$$F_x = F_n \sin\theta = \rho a V^2 \sin\theta \times \sin\theta = \rho a V^2 \sin^2\theta \qquad ...(17\cdot3)$$

$$F_y = F_n \cos\theta = \rho a V^2 \sin\theta \times \cos\theta = \rho a V^2 \sin\theta \cos\theta \qquad ...(17\cdot4)$$

In this case also, the work done is *zero* as the plate is stationary. When the fluid strikes the plate, it is divided into two streams Q_1 and Q_2. Since the frictional resistance has been assumed to be negligible, the resultant force in the direction tangential to the plate is zero. Applying the impulse-momentum equation in the direction tangential a to the plate, we obtain

$$(\rho Q_1 V - \rho Q_2 V) - \rho Q V \cos\theta = 0$$

$$Q_1 - Q_2 - Q\cos\theta = 0 \qquad (i)$$

Also,

$$Q_1 + Q_2 = Q \qquad \text{(Continuity equation)} \quad ...(ii)$$

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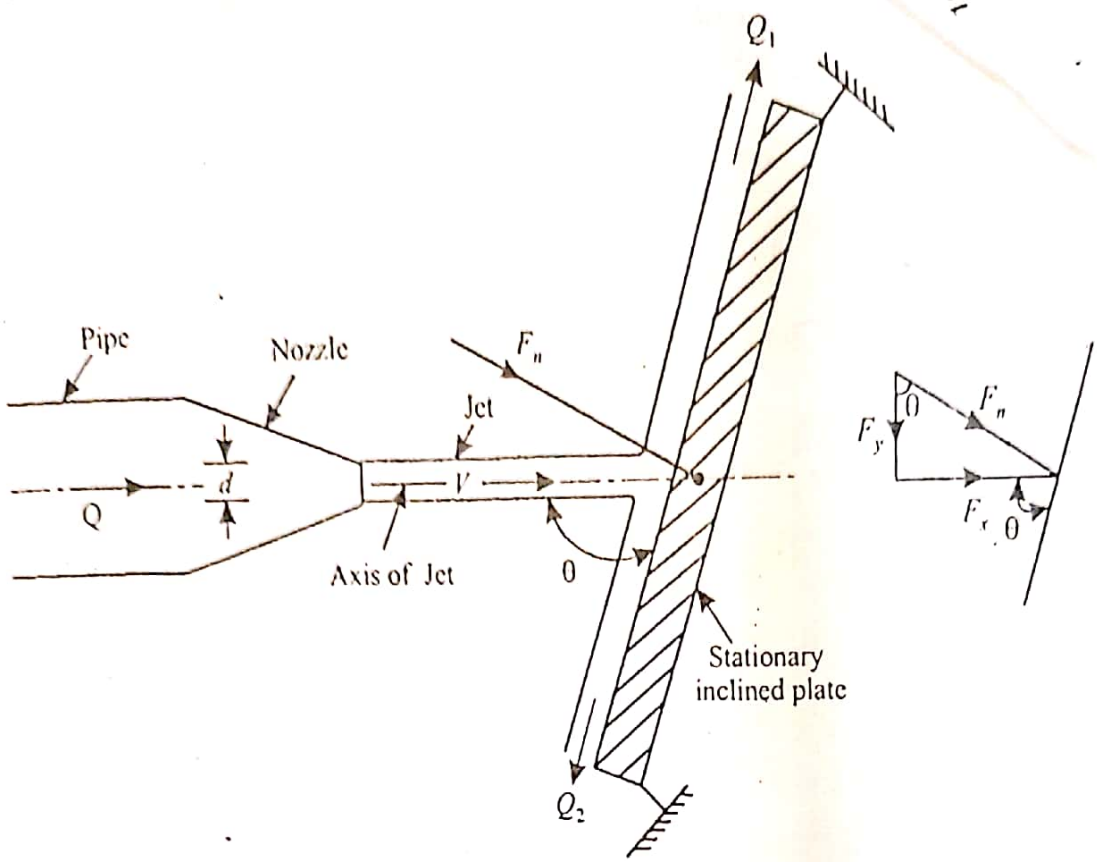


Fig. 17.2. Fluid jet striking a stationary inclined plate.

$$Q_1 = \frac{Q}{2}(1 + \cos\theta), \qquad ...(17\cdot5)$$

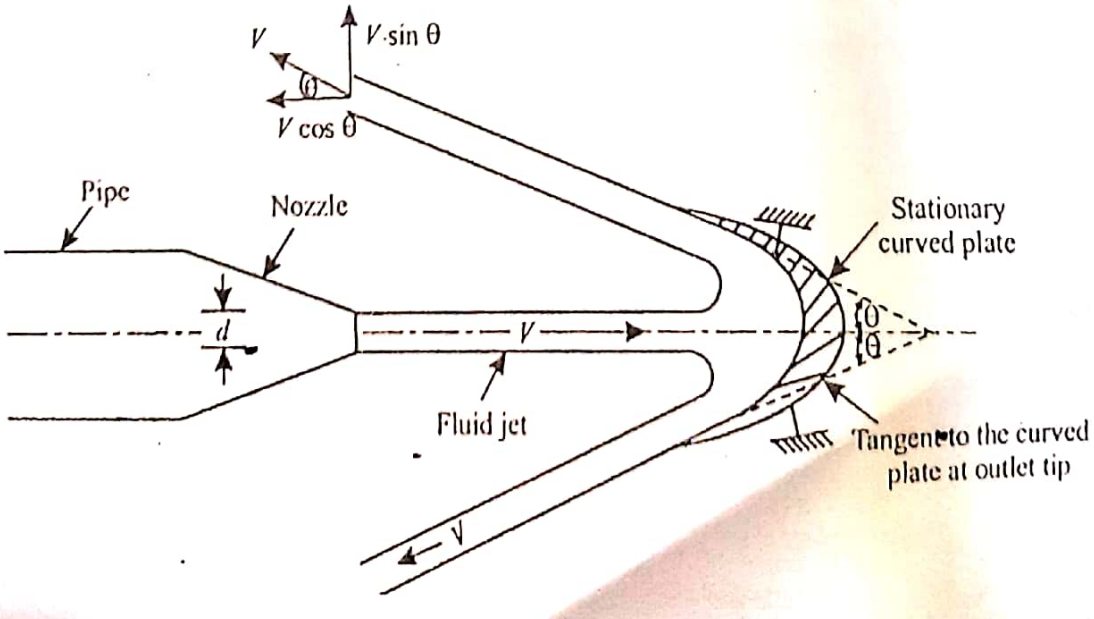
and,
$$Q_2 = \frac{Q}{2}(1 - \cos\theta) \qquad ...(17\cdot6)$$

Ratio of discharges,
$$\frac{Q_1}{Q_2} = \frac{1 + \cos\theta}{1 - \cos\theta} \qquad ...(17\cdot7)$$

17·4. Force exerted on Stationary Curved Plate

Case I. Jet strikes the curved plate at the centre:

Cosider a fluid jet striking a stationary curved plate (*smooth*) at the centre as shown in Fig. 17·3. The jet after striking the plate comes out with the same velocity, in the tangential direction of the curved plate.



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The velocity at the outlet of the plate can be resolved into the following *two components*:

(*i*) Component of velocity in the direction of jet $= -V\cos\theta$

(–ve sign indicates that the velocity at the outlet is in a direction *opposite* to that of the fluid jet)

(*ii*) Component of velocity perpendicular to the jet $= V\sin\theta$

Applying *impulse-momentum equation*, we have:

Force exerted by the jet (in the direction of jet),

$$F_x = \rho a V(V_{1x} - V_{2x})$$

where ρ = Mass density of the fluid,

a = Cross-sectional area of the jet $= \frac{\pi}{4} d^2$ (d = diameter of the jet),

V = Velocity of the jet,

V_{1x} = Initial velocity in the direction of jet $= V$

V_{2x} = Final velocity in the direction of jet $= -V\cos\theta$

$$\therefore \quad F_x = \rho a V[V - (-V\cos\theta) = \rho a V(V + V\cos\theta]$$

or,
$$F_x = \rho a V^2(1 + \cos\theta) \qquad ...(17\cdot8)$$

Similarly,
$$F_x = \rho a V(V_{1y} - V_{2y})$$

where V_{1y} = Initial velocity in the direction of $y = 0$

V_{2y} = Final velocity in the direction of $y = V\sin\theta$

$$\therefore \quad F_y = \rho a V(0 - V\sin\theta) = -\rho a V^2\sin\theta \qquad ...(17\cdot9)$$

–ve sign indicates that force is acting in the *downward* direction.

Note: The angle of defection of the jet $= (180° - \theta)$.

Case II. Jet strikes the curved plate at one end tangentially when the plate is symmetrical:

Fig. 17·4. shows a fluid jet striking a stationary symmetrical curved plate (*smooth*) at one end tangentially, the plate is symmetrical about X-axis. Let V be the velocity of the jet and θ be the angle made the jet with X-axis at inlet tip of the curved plate. The velocity of fluid at outlet tip of the curved plate will be equal to V (since the curved plate is assumed smooth).

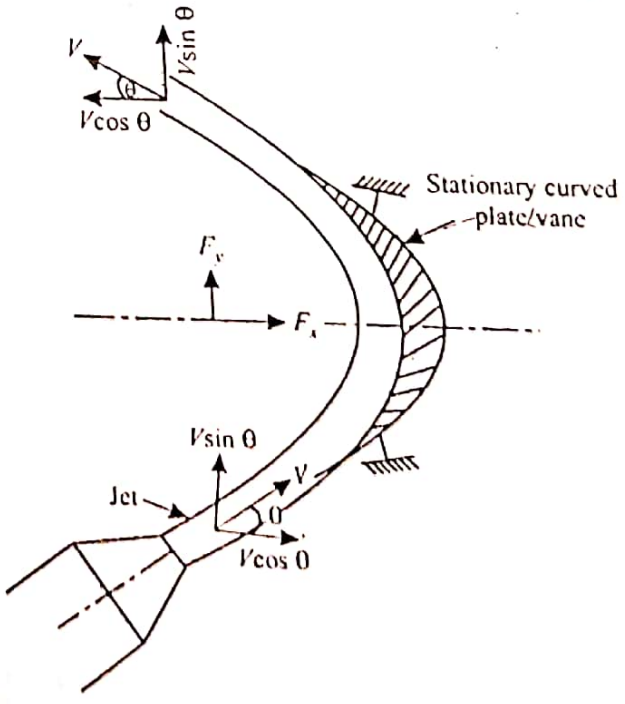


Fig. 17.4. Fluid jet striking stationary curved plate/vane.

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The forces exerted by the fluid jet in X and Y directions are:

$$F_x = \rho a V (V_{1x} - V_{2x})$$
$$= \rho a V [V \cos\theta - (-V \cos\theta)]$$
$$= \rho a V (V \cos\theta + V \cos\theta) = \rho a V \times 2V \cos\theta$$

or, $$F_x = 2\rho a V^2 \cos\theta \quad ...(17\cdot10)$$

$$F_y = \rho a V (V_{1y} - V_{2y})$$
$$= \rho a V [V \sin\theta - V \sin\theta] = 0$$

i.e., $$F_y = 0$$

Case III. Jet strikes the curved plate or vane at one end tangentially when the plate is unsymmetrical:

In this case, as the plate is unsymmetrical about X-axis, therefore, the angles made by the tangents drawn at inlet and outlet tips of the plate with X-axis will be different. Let θ and ϕ be the angles made by the tangents at inlet tip and outlet tip respectively with X-axis.

$\therefore$ Components of velocity at inlet: $V_{1x} = V\cos\theta$; $V_{1y} = V\sin\theta$

Components of velocity at outlet: $V_{2x} = -V\cos\phi$; $V_{2y} = V\sin\phi$

Now, the forces exerted by the fluid jet in X and Y direction are:

$$F_x = \rho a V (V_{1x} - V_{2x})$$
$$F_x = \rho a V [V\cos\theta - (-V\cos\phi)] = \rho a V (V\cos\theta + V\cos\phi)$$

or $$F_x = \rho a V^2 (\cos\theta + \cos\phi) \quad ...(17\cdot11)$$

and, $$F_y = \rho a V (V_{1y} - V_{2y})$$
$$= \rho a V (V\sin\theta - V\sin\phi)$$

or $$F_y = \rho a V^2 (\sin\theta - \sin\phi) \quad ...(17\cdot12)$$

Example 17·1. *A jet of water, 75 mm in diameter, issues with a velocity of 30 m/s and impinges on a stationary flat plate which destroys its forward motion. Find the force exerted by the jet on the plate and work done.*

Solution. Diameter of jet, $d = 75$ mm $= 0{\cdot}075$ m

Velocity of jet, $V = 30$ m/s

The force exerted by the jet on a stationary vertical plate is given by

$$F = \rho a V^2$$

where, ρ = Mass density of water = 1000 kg/m^3

$$a = \text{Area of jet} = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 0.075^2 = 0.004418 \text{ m}^2$$

$\therefore$ $$F = 1000 \times 0.004418 \times 30^2 = 3976{\cdot}2 \text{ N (Ans.)}$$

As the plate is *stationary*, **the work done is zero. (Ans.)**

Example 17·2 *A jet of water srikes with a velocity of 35 m/s a flat plate inclined at 30° with the axis of the jet. If the corss-sectional area of the jet is 25 cm^2, determine:*

(i) The force exerted by the jet on the plate,

(ii) The components of the force in the direction normal to the jet

(iii) The ratio in which the discharge gets divided after striking the plate

Solution. Velocity of the jet, $V = 35$ m/s

Inclination of the plate with the jet axis, $\theta = 30°$

Area of the jet, $a = 25$ cm$^2 = 25 \times 10^{-4}$ m^2

(i) The force exerted by the jet, F:

$$F = \rho a V^2 \sin\theta \quad \text{[Eqn. (17·3)]}$$
$$= 1000 \times (25 \times 10^{-4}) \times 35^2 \times \sin 30° = 1531{\cdot}25 \text{ N (Ans.)}$$

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(*ii*) **The components of the force, *F*:**

$$F_x = F \sin 0 = 1531{\cdot}25 \times \sin 30^\circ = 765{\cdot}625 \text{ N (Ans.)}$$

$$F_y = F \cos 0 = 1531{\cdot}25 \times \cos 30^\circ = 1326{\cdot}1 \text{ N (Ans.)}$$

(*iii*) **The ratio in which the discharge gets divided:**

$$\frac{Q_1}{Q_2} = \frac{1 + \cos 0}{1 - \cos 0} \qquad \text{[Eqn. (17·7)]}$$

or $$\frac{Q_1}{Q_2} = \frac{1 + \cos 30^\circ}{1 - \cos 30^\circ} = \frac{1 + 0{\cdot}866}{1 - 0{\cdot}866} = 13{\cdot}92 \text{ (Ans.)}$$

Example 17·3. *A jet of water of diameter 40 mm moving with a velocity of 30 m/s, strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet water in the direction of the jet if the jet is deflected through an angle 120° at the outlet of the curved plate.*

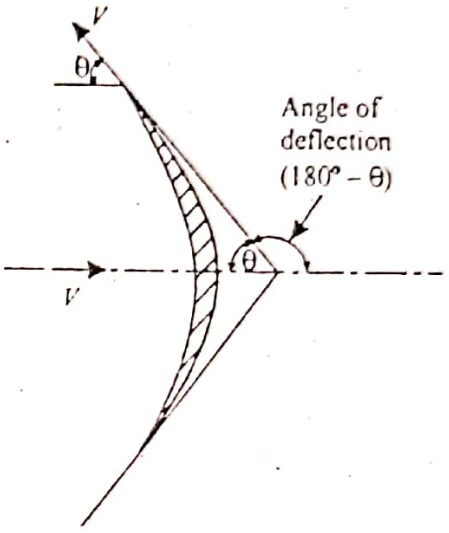


Fig. 17.5

Solution. Diameter of jet, $d = 40 \text{ mm} = 0{\cdot}04 \text{ m}$

∴ Area of jet,

$$a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 0{\cdot}04^2$$

$$= 0{\cdot}001256 \text{ m}^2$$

Velocity of jet, $V = 30 \text{ m/s}$

Angle of deflention $= 120^\circ$, or, $180 - \theta = 120^\circ$

∴ $\theta = 180^\circ - 120^\circ = 60^\circ$

Force exerted by the jet of water in the direction of jet,

$$F_x = \rho a V^2 (1 + \cos \theta) \qquad \text{[Eqn. (17·8)]}$$

$$= 1000 \times 0{\cdot}001256 \times 30^2 (1 + \cos 60^\circ)$$

$$= \mathbf{1695{\cdot}6 \text{ N (Ans.)}}$$

Example 17·4. *A jet of water of 20 mm diameter and moving at 15 m/s, strikes upon the centre of a symmetrical vane. After impingement, the jet gets deflected through 160° by the vane. Presuming vane to be smooth determine:*

(*i*) *The force exerted by jet on the vane, and*

(*ii*) *The ratio of velocity at outlet to that at inlet if actual reaction of the vane is 127 N.*

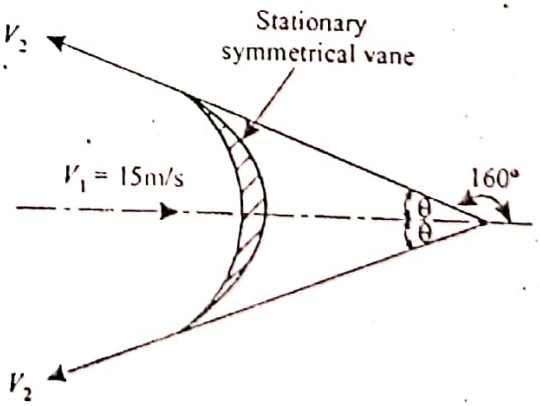


Fig. 17.6

Solution. Diameter of the jet, $d = 20 \text{ mm} = 0{\cdot}02 \text{ m}$

Velocity of jets, $V = 15 \text{ m/s}$

Angle of deflection $= 160^\circ$.

(*i*) The force exerted by the jet on the vane, F:

Reger to Fig 17·6.

$160^\circ = 180 - 0$, or, $0 = 180 - 160 = 20^\circ$

For *smooth vane*, the theoretical force (or thrust) by the jet on the vane is,

$$F = \rho a V^2 (1 + \cos 0) \qquad \text{...(i)}$$

$$= 1000 \times \left(\frac{\pi}{4} \times 0{\cdot}02^2\right) \times 15^2 (1 + \cos 20^\circ)$$

$$= \mathbf{137{\cdot}1 \text{ N (Ans.)}}$$

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(*ii*) $\frac{V_2}{V_1}$:

Actual reaction of the vane = 127 N (*Given*)

If the vane is not smooth, then outgoing velocity at the vane tip is less than the incoing velo

i.e., $\frac{V_2}{V_1} = K$ where K < 1. The eqn. (*i*) gets modified to

$$F = \rho a V^2 (1 + K \cos \theta)$$

$$127 = 1000 \times (\frac{\pi}{4} \times 0{\cdot}02^2) \times 15^2 (1 + K \cos 20^\circ)$$

or, $$1 + K \cos 20^\circ = \frac{127}{1000 \times (\frac{\pi}{4} \times 0.02^2) \times 15^2} = 1{\cdot}796$$

or, $$K = \frac{1{\cdot}796 - 1}{\cos 20^\circ} = 0{\cdot}847 \text{(Ans.)}$$

Example 17·5. *A jet of water from a nozzle is deflected through 60° from its original direction by curved plate which it enters tangentially without shock with a velocity of 30 m/s and leaves with a mean velocity of 25 m/s. If the discharge form the nozzle is 0·8 kg/s, calculate the magnitude and direction of the resultant force on the vane, if the vane is stationary.* [AMIE]

Solution. Velocity at inlet, V_1 = 30 m/s

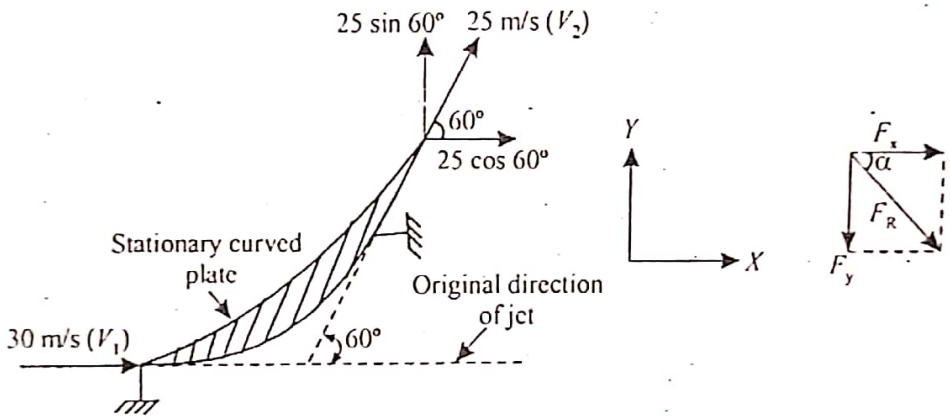


Fig. 17.7

Velocity at outlet, V_2 = 25 m/s

Mass per second = 0.8 kg/s

Force in the direction of jet, F_x = Mass per sec. × $(V_{1x} - V_{2x})$

where, $V_{1x} = V_1 = 30$ m/s

$V_{2x} = V_2 \cos 60^\circ = 25 \cos 60^\circ = 12{\cdot}5$ m/s

$\therefore$ $F_x = 0{\cdot}8 (30 - 12{\cdot}5) = 14$ N

Similarly, force normal to the jet, F_y = mass per sec. × $(V_{1y} - V_{2x})$

where, $V_{1y} = 0$, and

$V_{2y} = V_2 \sin 60^\circ = 25 \sin 60^\circ = 21{\cdot}65$ N

$\therefore$ $F_y = 0{\cdot}8 (0 - 21{\cdot}65) = -17{\cdot}32$ N

(–ve sign indicates that the force F_y *is acting in the vertically downward direction*)

$\therefore$ Resltant force in the curved plate,

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{14^2 + 17{\cdot}32^2} = 22{\cdot}27\text{N (Ans.)}$$

The angle made by the resultant force with X-axis (Refer to Fig. 17·7),

$$\tan \alpha = \frac{F_y}{F_x} = \frac{17{\cdot}32}{14} = 1{\cdot}237$$

$\therefore$ $\alpha = \tan^{-1} 1{\cdot}237 = 51^\circ$ **(Ans.)**

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Example 17·6. *A jet of water of diameter 60 mm moving with a velocity of 40 m/s, strikes a curved fixed plate tangentially at one end at an angle of 30° to horizontal. The jet leaves the plate at an angle of 20° to the horizontal. Find the force exerted by the jet on the plate in the horizontal and vertical directions.*

Solution. Diameter of jet, $d = 60 \text{ mm} = 0{\cdot}06 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 0{\cdot}06^2 = 0{\cdot}002827 \text{ m}^2$

Velocity of the jet, $V = 40 \text{ m/s}$

Angle made by the jet at inlet tip with horizontal, $\theta = 30°$

Angle made by the jet at outlet tip with horizontal, $\phi = 20°$.

Force exerted by the jet:

Force exerted by the jet in X-direction,

$$F_x = \rho a V^2 (\cos\theta + \cos\phi)$$

$$= 1000 \times 0{\cdot}002827 \times 40^2 (\cos 30° + \cos 20°) = 8167{\cdot}6 \text{ N (Ans.)}$$ [Eqn. (17·11)]

Force exerted by the jet in Y-direction,

$$F_y = \rho a V^2 (\sin\theta - \sin\phi)$$

$$= 1000 \times 0{\cdot}002827 \times 40^2 (\sin 30° - \sin 20°) = 714{\cdot}57 \text{ N (Ans.)}$$ [Eqn. (17·12)]

Example 17·7. *A rectangular plate, weighing 60 N is suspended vertically by a hinge on the top horizontal edge. The centre of gravity of the plate is 100 mm from the hinge. A horizontal jet of water 20 mm diameter, whose axis is 150 mm below the hinge impinges normally on the plate with a velocity of 5 m/s. Determine:*

(*i*) *The horizontal force applied at the centre of gravity to maintain the plate in its vertical portion.*

(*ii*) *The corresponding velocity of the jet, if the plate is deflected through 30° and the same force continues to act at the centre of gravity of the plate.* **[Delhi University]**

Solution. Weight of plate, $W = 60 \text{ N}$

Distance of weight W from hinge, $x = 100 \text{ mm} = 0.1 \text{ m}$

Diameter of the jet, $d = 20 \text{ mm} = 0.02 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (0{\cdot}02)^2 = 0{\cdot}000314 \text{ m}^2$

Distance of the axis of the jet from hinge = 150 mm = 0·15 m

Velocity of jet, $V = 15 \text{ m/s}$

Refer to Fig. 17·8 for the plate and its orientation in the vertical and deflected positions.

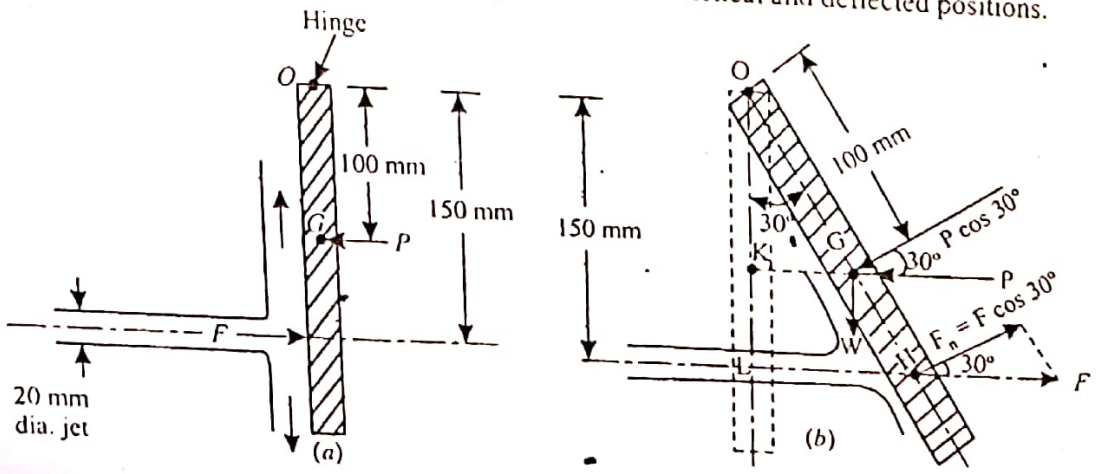


Fig. 17.8

(*i*) Horizonatal force, P:

Normal force exerted by the jet on plate,

$$F = \rho a V^2 = 1000 \times 0{\cdot}000314 \times 5^2 = 7{\cdot}85 \text{ N}$$

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Let P be the force to be applied at the centre of gravity G to keep the plate in its vertical position. Then taking moments about the hinge point O, we get

$$F \times 150 = P \times 100$$

or,
$$P = \frac{F \times 150}{100} = \frac{7{\cdot}85 \times 150}{100} = 11{\cdot}77 \text{(Ans.)}$$

(ii) Velocity of the jet;

Let, V = Velocity of the jet, if the plate is deflected through 30° and the same force continues to act at the centre of gravity of the plate.

Refer to Fig. 17·8. (*b*). Taking moments about the hinge point O, we get

$$W \times GK + P \cos 30° \times OG = F \cos 30° \times OH \quad (i)$$

But, $GK = 0{\cdot}1 \sin 30° = 0{\cdot}05$ m, $OG = 0{\cdot}1$ m, $OH = \frac{OL}{\cos 30} \quad \frac{015}{\cos 30°} = 0{\cdot}173$ m

Substituting the various values, we get

$$60 \times 0{\cdot}05 + 11{\cdot}77 \times 0{\cdot}866 \times 0{\cdot}1 = F \times 0{\cdot}1732$$

$$3 + 1{\cdot}02 = 0{\cdot}15F$$

$$\therefore \quad F = 26{\cdot}8 \text{ N}$$

Now jet force $F = \rho a V^2$, or, $26{\cdot}8 = 1000 \times 0{\cdot}000314 \times V^2$

$$\therefore \quad V = \left(\frac{26{\cdot}8}{1000 \times 0{\cdot}000314}\right)^{1/2} = 9{\cdot}24 \text{ m/s (Ans.)}$$

Example 17·8. *A jet of water of diameter 20 mm strikes a 200 mm × 200 mm square plate of uniform thickness with a velocity of 10 m/s at the centre of the plate which is suspended vertically by a hinge on its top horizontal edge. The weight of the plate is 98 N. The jet strikes normal to the plate.*

(i) What force must be applied at the lower edge of the plate so that plate is kept vertical?

(ii) If the plate is allowed to deflect freely, what will be the inclination of the plate with vertical due to the force exerted by jet water?

Solution. Diameter of jet = 20 mm or 0·02 m

$$\therefore \text{Area, } a = \frac{\pi}{4} \times 0{\cdot}02^2 = 0{\cdot}000314 \text{ m}^2$$

Size of the plate = 200 mm × 200 mm

Weight of the plate, $W = 98$ N

Velocity of jet, $V = 10$ m/s

(i) Force to be applied at lower edge, P:

Let, P = The force applied at the lower edge of the plate so that plate is kept vertical.

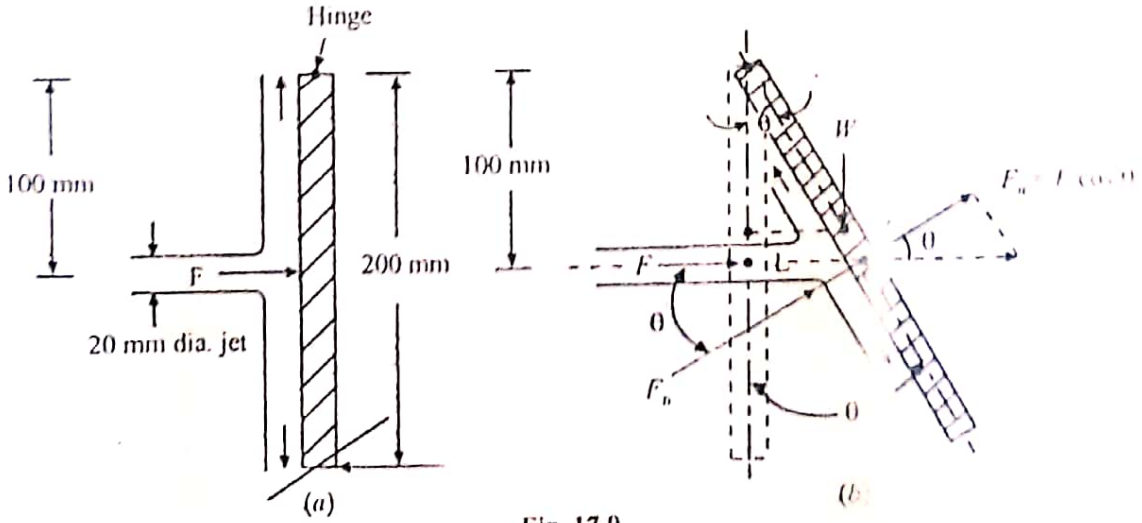


Fig. 17.9

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Fo e exerted by the jet on the plate,

$$F = \rho a V^2 = 1000 \times 0{\cdot}000314 \times 10^2 = 31{\cdot}4 \text{ N}$$

N , taking moments about the hinge point O, we get

$$P \times 200 = F \times 100$$

or

$$P = \frac{F \times 100}{200} = \frac{31{\cdot}4 \times 100}{200} = 15{\cdot}7 \text{ N (Ans.)}$$

(*ii*) **nclination of the plate:**

Le θ = Inclination of the plate with vertical due to force exerted by the jet.

Fo e exerted by water normal to the plate is given by,

$$F_n = F \cos\theta = \rho a V^2 \cos\theta = 31{\cdot}4 \cos\theta$$

[$\because \rho a V^2 = 31{\cdot}4$ N calculated earlier]

Re r to Fig 17·9 (*b*). Taking moments about the hinge point O, we get

$$F_n \times OH = W \times GK \qquad ...(i)$$

wh e,

$$OH = \frac{OL}{\cos\theta} = \frac{0{\cdot}1}{\cos\theta} \text{ m}, \quad GK = 0{\cdot}1 \sin\theta \text{ m}$$

Su tituting the values in eqn. (*i*), we have

$$31{\cdot}4 \cos\theta \times \frac{0{\cdot}1}{\cos\theta} = 98 \times 0{\cdot}1 \sin\theta$$

$$\therefore \quad \sin\theta = \frac{31{\cdot}4}{98} = 0{\cdot}32$$

He e,

$$\theta = \sin^{-1} 0{\cdot}32 = 18{\cdot}66° \text{ (Ans.)}$$

B. ORCE EXERTED BY THE JET ON THE MOVING PLATE

17·5. I rce exerted on Moving Flat Plate held Normal to Jet

Fig 17·10 shows a fluid jet striking a flat vertical plate moving with a uniform velocity away from the jet.

Let

V = Absolute velocity of the jet,

a = Cross-sectional area of the jet, and

u = Velocity of the flat plate held normal to the jet.

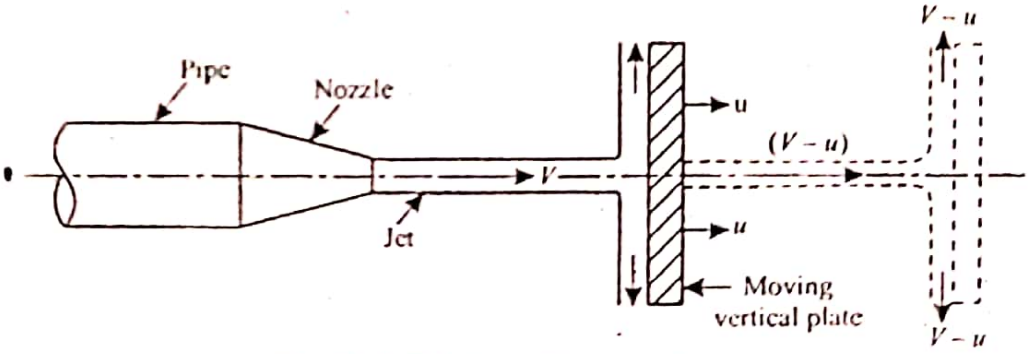


Fig. 17.10. Fluid jet striking a moving plate.

Th lative velocity with which the jet strikes the plate is $(V - u)$.

Ma of water striking the plate per second = $\rho a (V - u)$

∴ I ce exerted by the jet on the plate in the direction of jet,

= Mass of water striking the plate/sec × (initial velocity with which water strikes–final velocity)

$$= \rho a (V - u)[(V - u) - 0] \qquad (\because \text{Final velocity in the diction of jet} = 0)$$

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or, $F_x = \rho a (V - u)^2$ ...(

The work done = Force × the distance through which the body moves in the direction of fore

$\therefore$ *Work done* $= \rho a (V - u)^2 \times u$ ...(17·14)

Note. This case is not practically feasible because the distance between the nozzle and the plate will go on increasing. However, if a series of plates is fitted on the wheel, there is always a plate facing the jet. Thus, the entire fluid issuing from the jet strikes the plates.

17·6. Force exerted on a Moving Plate held inclined to the Direction of Jet

Fig. 17·11 shows a fluid jet striking an inclined plate, which is moving with uniform velocity in the direction of the jet.

Let, V = Absolute velocity of the jet,

u = Velocity of plate in the direction of jet

a = Cross-secrional area of jet, and

θ = Angle between jet and the plate.

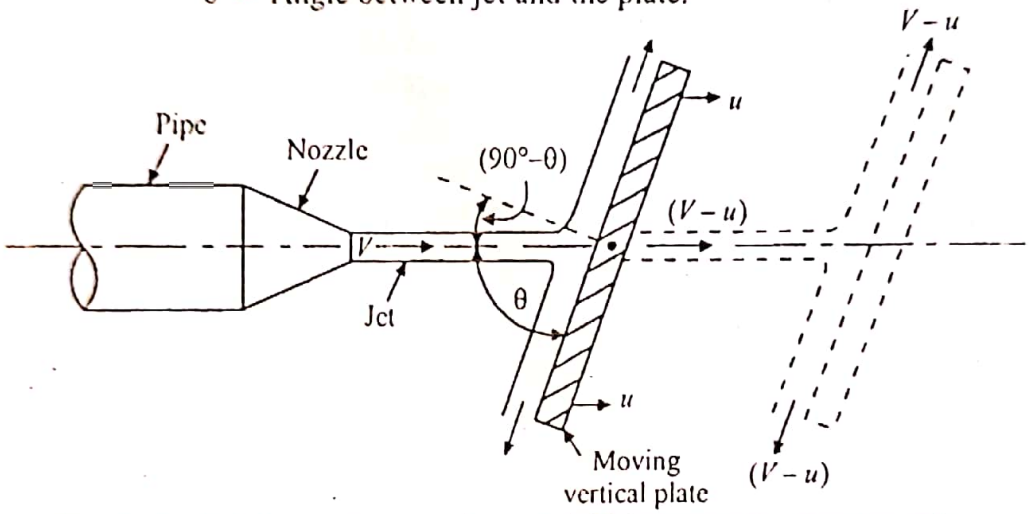


Fig. 17.11. Fluid jet striking a moving plate held inclined to the direction of jet.

Relative velocity with which the jet strikes the plate = $(V - u)$

The mass of fluid striking the plate per second = $\rho a (V - u)$. The force exerted by the jet on the plate in the direction *normal* to the plate is given as:

$$F_n = \rho a (V - u) [(V - u) \sin \theta - 0]$$

or $$F_n = \rho a (V - u)^2 \sin \theta \quad ...(17·15)$$

Component of this force in the direction of jet,

$$F_x = F_n \sin \theta = \rho a (V - u)^2 \sin \theta \times \sin \theta = \rho (V - u)^2 \sin^2 \theta \quad ...(17·16)$$

$\therefore$ *Work done* $= F_x \times u = \rho a (V - u)^2 \sin \theta \times u$ ...(17·17)

Example 17·9. *A nozzle of 60 mm diameter delivers a stream of water at 24 m/s perpendicular to a plate that moves away from the jet at 6 m/s.*

Find: (*i*) *The force on the plate,*

(*ii*) *The work done, and*

(*iii*) *The efficiency of the jet.*

Solution. Diameter of the nozzle/jet, $d = 60$ mm $= 0·06$ m

Velocity of water, $V = 24$ m/s

Velocity of the plate, $u = 6$ m/s

(*i*) **The force in the plate, *F*:**

$$F = \rho a (V - u)^2 \quad \text{[Eqn. (17·13)]}$$

$$= 1000 \times \left(\frac{\pi}{4} \times 0·06^2\right) \times (24 - 6)^2 = \mathbf{916\ N\ (Ans.)}$$

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(*ii*) **The work done:**

Work done $= F \times u = 916 \times 6 = 5496$ Nm/s **(Ans.)**

(*iii*) **The efficiency of jet, η_{jet}:**

Kinetic energy of issuing jet $= \frac{1}{2} mV^2 = \frac{1}{2} (\rho aV) \times V^2$ ($\because$ Mass, $m = \rho aV$)

$= \frac{1}{2} (1000 \times \frac{\pi}{4} \times 0{\cdot}06^2 \times 24) \times 24^2 = 19543{\cdot}2$ Nm/s

$\therefore \quad \eta_{jet} = \frac{\text{Work done}}{\text{Kinetic energy of issuing jet}}$

$= \frac{5496}{19543.2} = 0{\cdot}281$ or **28·1%** **(Ans.)**

Example 17·10. *A 75 mm diameter jet having a velocity of 30 m/s strikes a flat plate, the normal of which is inclined at 45° to the axis of the jet. Find the normal pressure on the plate,*

(*i*) *When the plate is stationary;*

(*ii*) *When the plate is moving with a velocity of 15 m/s in the direction of jet, away from the jet. Also determine the power and efficiency of the jet when the plate is moving.*

[Allahabad University]

Solution. Diameter of the jet, $d = 75$ mm $= 0{\cdot}075$ m

$\therefore$ Area, $a = \frac{\pi}{4} \times 0.075^2 = 0.004418$ m^2

Angle between the jet and the plate, $\theta = 90° - 45° = 45°$

Velocity of the jet, $V = 30$ m/s

(*i*) *When the plate is stationary,* the normal force on the plate is:

$F_n = \rho a V^2 \sin\theta$ [Eqn. (17·2)]

$= 1000 \times 0{\cdot}004418 \times 30^2 \times \sin 45° = 2811{\cdot}6$ N **(Ans.)**

(*ii*) *When the plate is moving with a velocity of 15 m/s and moving away from the jet,* the normal force on the plate is

$F_n = \rho a (V - u)^2 \sin\theta$ (where, $u = 15$ m/s) [Eqn. 17·15]

$= 1000 \times 0{\cdot}004418 (30 - 15)^2 \times \sin 45° = 702{\cdot}9$ N **(Ans.)**

Work done per second by the jet

= Force in the direction of jet × distance moved by the plate in the direction of jet/sec.

$= F_x \times u$

where, $F_x = F_n \sin\theta = 702{\cdot}9 \times \sin 45° = 497$ N

$\therefore$ Work done $= 497 \times 15 = 7455$ Nm/s

$\therefore$ *Power of the jet* $= 7455$ J/s $= 7455$ W $= 7{\cdot}455$ **kW (Ans.)**

Efficiency of the jet $= \frac{\text{Work done on the plate}}{\text{Kinetic energy supplied by the jet}}$

$= \frac{7455}{\frac{1}{2} (\rho aV) \times V^2} = \frac{7455}{\frac{1}{2} \times (1000 \times 0{\cdot}004418 \times 30) \times 30^2}$

$= 0{\cdot}125$ or **12·5%** **(Ans.)**

Example 17.11. *A 75 mm diameter water jet having a velocity of 12 m/s impinges on a plane, smooth plate at an angle of 60° to the normal to the plate. What will be the impact when (i) the plate is stationary and (ii) the plate is moving in the direction of the jet at 6 m/s? Estimate the work done per unit time by the jet on the plate in each case. Take the density of water as 998 kg/m^3.* **(AMIE)**

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Solution. *Given:* $d = 75$ mm $= 0.075$ m; $V = 12$ m/s; $\theta = 90 - 60° = 30°$; $u = 6$ m/s; $\rho = 998$ kg/m^3.

Force on the plate and work done:

(i) When the plate is stationary:

Force on the plate, $F = (\rho a V) V \sin 30°$

$$= \left[998 \times \frac{\pi}{4} \times (0.075)^2 \times 12\right] \times 12 \sin 30° = 17.45 \text{ N (Ans.)}$$

Work done = 0, since the plate is stationary (Ans.)

(ii) When the plate is moving :

Normal force on the plate, $F_n = [\rho a (V - u)] \times (V - u) \sin 30°$

$$= \rho a (V - u)^2 \sin 30° = 998 \times \frac{\pi}{4} \times (0.075)^2 \times (12 - 6)^2 \times 0.5 = 79.36 \text{ N (Ans.)}$$

Work done on the plate $= F_n \sin\theta \times u$

$= 79.36 \sin 30° \times 6 = 238.08$ Nm/s (Ans.)

Example 17·12. *(a) Derive the following expression for force F exerted by a jet of area a which strikes a flat plate at an angle θ to the normal to the plate with velocity V. The plate is moving with velocity u in the direction of normal to the plate surface.*

$$F = \rho a \frac{(V \cos\theta - u)^2}{\cos\theta}$$

(b) A 40 mm diameter jet having a velocity of 20 m/s strikes a flat plate, the normal of which is inclined at 30° to the axis of jet. If the plate itself is moving with a velocity of 8 m/s parallel to and in the direction of normal to its surface, calculate:

(i) Normal force exerted on the plate,

(ii) Work done per second, and

(iii) Efficiency of the jet [Panjab Univer

Solution. *(a)* Fig 17·12 shows the orientation of the jet and the arrangement of the plate.

Component of plate velocity in the direction of jet $= \frac{u}{\cos\theta}$

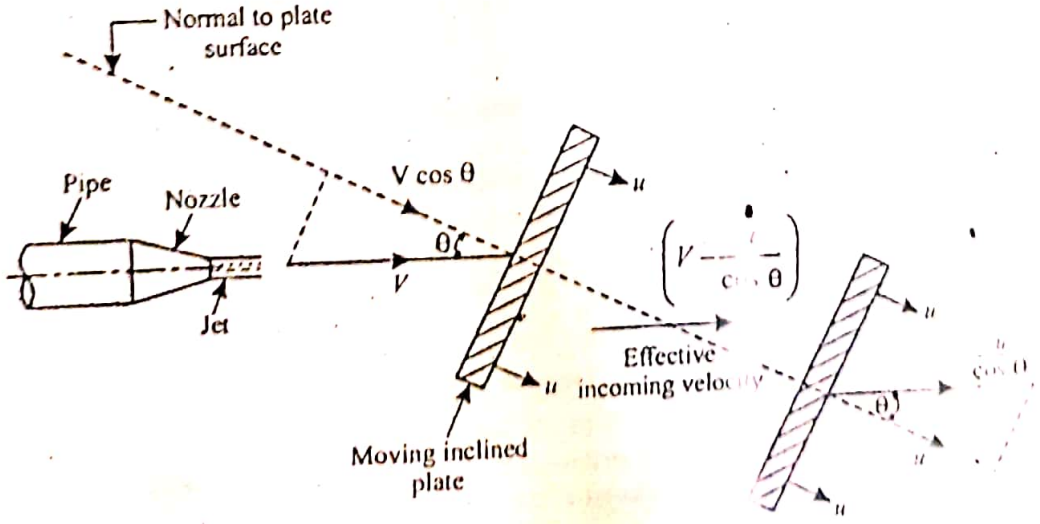


Fig. 17.12. Fluid jet and an inclined plate—plate moves in a direction normal to its surface

$\therefore$ Effective incominge velocity $= V - \frac{u}{\cos\theta}$

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Mass of fluid striking the plate $= \rho a \left(V - \frac{u}{\cos\theta} \right) = \rho a \left(\frac{V\cos\theta - u}{\cos\theta} \right)$

Component of absolute velocity of jet *normal* to plate $= V\cos\theta$

∴ Change of velocity *normal* to plate $= (V\cos\theta - u)$

Force exerted on the plate (F) = Mass × change of velocity

or $$F = \rho a \left(\frac{V\cos\theta - u}{\cos\theta} \right) \times (V\cos\theta - u)$$

or $$F = \rho a \frac{(V\cos\theta - u)^2}{\cos\theta}$$ **...Proved**

(*b*) Diameter of the jet, $d = 40$ mm $= 0{\cdot}04$ m

∴ Area, $a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 0{\cdot}04^2 = 0{\cdot}001256\ \text{m}^2$

Velocity of jet (absolute), $V = 20$ m/s

Velocity of the plate, $u = 8$ m/s

The inclination of the normal to the plate with the axis of jet, $\theta = 30°$

(*i*) **Normal force exerted on the plate, F:**

$$F = \rho a \frac{(V\cos\theta - u)^2}{\cos\theta} = 1000 \times 0{\cdot}001256 \times \frac{(20\cos 30° - 8)^2}{\cos 30°} = \mathbf{125{\cdot}98\ N\ (Ans.)}$$

(*ii*) **Work done per second:**

Work done/sec = Normal force exerted on the plate × distance moved by the plate

$= 125{\cdot}98 \times 8 = \mathbf{1007{\cdot}84\ Nm/s\ (Ans.)}$

(*iii*) **Efficiency of the jet, η_{jet}:**

$$\eta_{jet} = \frac{\text{Work done}}{\text{Kinetic energy of issuing jet}} = \frac{1007{\cdot}84}{\frac{1}{2}mV^2}$$

$$= \frac{1007{\cdot}84}{\frac{1}{2}(\rho aV)V^2} = \frac{1007{\cdot}84}{\frac{1}{2}(1000 \times 0{\cdot}001256 \times 20) \times 20^2} = 0{\cdot}20 \text{ or } \mathbf{20\%\ (Ans.)}$$

17.7. Force exerted on a Curved Plate (or Vane) when the Plate (or Vane) is moving in the Direction of Jet.

A. Single vane:

Fig. 17·13 shows a fluid jet striking at the centre of curved vane moving with a unform velocity in the direction of jet.

Let V = Absolute velocity of the jet,

a = Area of jet, and

u = Velocity of the plate in the direction of the jet.

When the jet strikes the moving vane, the effective velocity is the relative velocity $(V - u)$. The component of this velocity [*i.e.*, $(V-u)$] in the direction of jet

$= -(V-u)\cos\theta$

(– ve sign indicates that the component is in the direction *opposite* to that of the jet.)

Mass of fluid striking the plate $= \rho a (V-u)$

∴ Force exerted by the jet on the vane,

F = Mass striking per sec × (initial velocity with which the jet strikes the vane in the direction of jet – final velocity)

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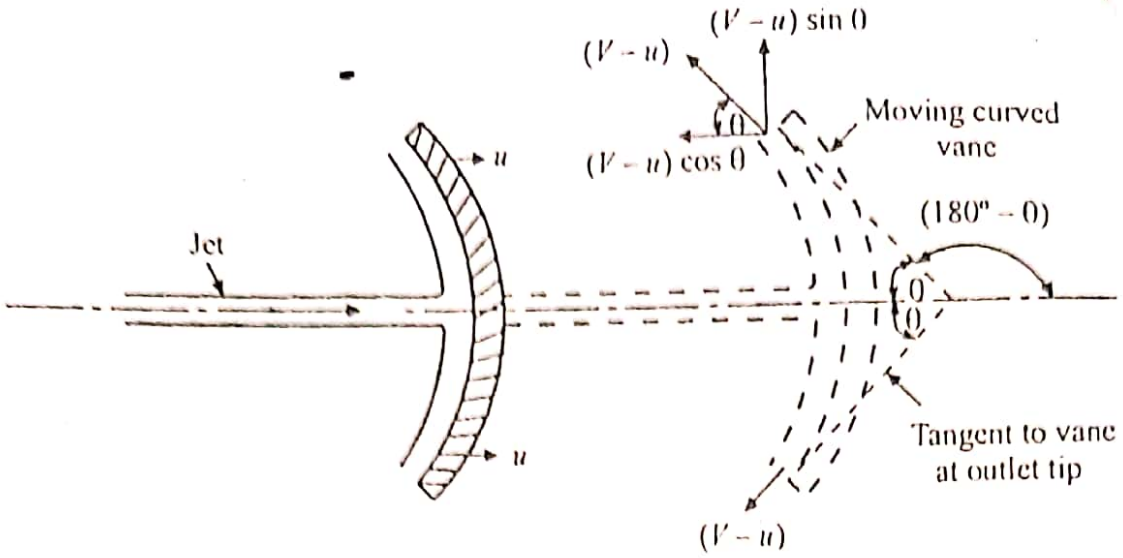


Fig. 17.13. Jet striking a curved moving vane.

$$= \rho a (V - u) [(V - u) - \{-(V - u) \cos \theta\}]$$

$$= \rho a (V - u) [(V - u) + (V - u) \cos \theta]$$

or, $$F_x = \rho a (V - u)^2 (1 + \cos \theta) \quad \text{...[17·}$$

Work done on the vane per second

$$= F_x \times u = \rho a (V - u)^2 (1 + \cos \theta) \times u \quad \text{...(17·}$$

Kinetic energy of issuing jet $= \frac{1}{2} mV^2 = \frac{1}{2} (\rho a V) V^2 = \frac{1}{2} \rho a V^3$

Efficiency, $$\eta = \frac{\text{Work done}}{\text{Kinetic energy supplied by the jet}}$$

$$= \frac{\rho a (V - u)^2 (1 + \cos \theta) \times u}{\frac{1}{2} \rho a V^3}$$

or, $$\eta = \frac{2 (V - u)^2 (1 + \cos \theta) u}{V^3} \quad \text{...(17·}$$

For a given jet velocity, the efficiency will be *maximum* when

$$\frac{d\eta}{du} = 0$$

or, $$\frac{d}{du} \left[\frac{2(V - u)^2 (1 + \cos \theta) u}{V^3} \right] = 0$$

or, $$\frac{2}{V^3} (1 + \cos \theta) \frac{d}{du} [(V^2 - u)^2 \times u] = 0$$

or, $$\frac{2}{V^3} (1 + \cos \theta) \frac{d}{du} [(V^2 + u^2 - 2Vu) u] = 0$$

or, $$\frac{2}{V^3} (1 + \cos \theta) \frac{d}{du} (V^2 u + u^3 - 2Vu^2) = 0$$

or, $$\frac{2}{V^3} (1 + \cos \theta) (V^2 + 3u^2 - 4Vu) = 0$$

For a given system $$\frac{2}{V^3} (1 + \cos \theta) \neq 0$$

$\therefore$ $$V^2 + 3u^2 - 4Vu = 0$$

or, $$(V - u) (V - 3u) = 0$$

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i.e., $u = V$ or $u = \frac{V}{3}$ ...(17·21)

When $u = V$, the work done becomes zero. Thus, for maximum efficiency $u = V/3$, *i.e. vane velocity is 1/3 rd of the jet velocity.*

B. Series of vanes:

In this case, there is always one vane or the other facing the jet (e.g., curved vanes mounted equidistantly around the periphery of a wheel) and the entire fluid is utilized. The mass of fluid striking the vanes per second equals ρaV.

Thrust exerted by the jet on the vane,

$$F_x = \rho aV[(V-u) - \{-(V-u)\cos\theta)\}]$$
$$= \rho aV[(V-u) + (V-u)\cos\theta)]$$

or, $F_x = \rho aV(V-u)(1+\cos\theta)$ ...(17·22)

Work done per second = $F_x \times u$

$= \rho aV[(V-u)(1+\cos\theta) \times u$ ...(17·23)

Kinetic energy supplied by the jet

$$= \frac{1}{2}(\rho aV)V^2 = \frac{1}{2}\rho aV^3$$

$\therefore$ Efficiency, $\eta = \frac{\text{Work done on the wheel}}{\text{Kinetic energy supplied by the jet}}$

$$= \frac{\rho aV\,(V-u))\,(1+\cos\theta) \times u}{\frac{1}{2}\rho aV^3}$$

or, $\eta = \frac{2u(V-u)(1+\cos\theta)}{V^2}$ ...(17·24)

The efficiency is *maximum* if

$$\frac{d\eta}{du} = 0$$

or, $$\frac{d}{du}\left[\frac{2u(V-u)(1+\cos\theta)}{V^2}\right] = 0$$

or, $$\frac{2(1+\cos\theta)}{V^2} \times \frac{d}{du}(uV - u^2) = 0$$

or, $$\frac{2(1+\cos\theta)}{V^2}(V-2u) = 0$$

For a given system, $\frac{2(1+\cos\theta)}{V^2} \neq 0$ and therefore

$$V - 2u = 0 \text{ or } u = \frac{V}{2}$$ ...(17·25)

Hence for the *maximum efficiency of the wheel, the peripheral speed (u) is one-half the velocity of jet.*

Maximum efficiency, $\eta_{max} = \frac{2\left(\frac{V}{2}\right)\left(V - \frac{V}{2}\right)(1+\cos\theta)}{V^2}$

$\left[\text{Substituting } u = \frac{V}{2} \text{ in eqn.(17·24)}\right]$

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or, $$\eta_{max} = \frac{1 + \cos 0}{2}$$

When $0 = 0$, *i.e.* the vanes are *semicircular*, $\eta_{max} = \frac{1+1}{2} = 1$ or 100%. Thus the theoretical maximum efficiency of the wheel is 100%. This concept (of having vanes of semicircular configuration) is utilized in the design of buckets for Pelton wheel. The buckets are, however not exactly semicircular. The angle of deflection is limited to 160° - 170° depending upon a particular design. This *ensures that the jet coming out of a bucket does not interfere with the jet striking the bucket*

Example 17·13 *A jet water of 60 mm diameter strikes a curved vane at its centre with a velocity of 18 m/s. The curved vane is moving with a velocity of 6 m/s in the direction of the jet. The jet is deflected through an angle of 165°. Assuming the plate to be smooth find:*

(i) Thrust on the plate in the direction of jet,

(ii) Power of the jet, and

(iii) Efficiency of the jet.

Solution. Diameter of the jet, $d = 60$ mm $= 0{\cdot}06$ m

$\therefore$ Area, $a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 0{\cdot}06^2 = 0{\cdot}002827$ m^2

Velocity of the jet, $V = 18$ m/s

Velocity of the vane, $u = 6$ m/s

Angle of deflection of the jet $= 165°$

$\therefore$ Angle made by the relative velocity at the outlet of the vane,

$$\theta = 180 - 165 = 15°$$

(i) Thrust on the plate:

Thrust on the plate in the direction of the jet, ...[Eqn. 17·18]

$$F_x = \rho a (V-u)^2 (1 + \cos\theta)$$
$$= 1000 \times 0{\cdot}002827 (18-6)^2 (1 + \cos 15°) = \mathbf{800{\cdot}3\ N}\ \textbf{(Ans.)}$$

(ii) Power of the jet:

Work done by the jet on the vane per second

$$= F_x \times u = 800{\cdot}3 \times 6 = 4801{\cdot}8 \text{ Nm/s}$$

$\therefore$ *Power of the jet* $= 4801{\cdot}8$ J/s $= 4801{\cdot}8$ W $\simeq$ **4·8 kW (Ans.)**

(iii) Efficiency of the jet, η_{jet}:

$$\eta_{jet} = \frac{\text{Work done by the jet/sec.}}{\text{Kinetic energy of the jet/sec.}}$$

$$= \frac{4801{\cdot}8}{\frac{1}{2}(\rho a V) \times V^2} = \frac{4801{\cdot}8}{\frac{1}{2}(1000 \times 0{\cdot}002827 \times 18) \times 18^2} = 0{\cdot}5825 \text{ or } 58{\cdot}25\ \%\ \textbf{(Ans.)}$$

Example 17·14. *(a) A stationary vane having an inlet angle of zero degree and an outlet angle of 25° receives water at velocity of 50 m/s. Determine the components of force acting on it in the direction of the jet velocity and normal ot it. Also find the resultant force in magnitude and direction per kg of flow.*

(b) If the vane stated above is moving with a velocity of 20 m/s in the direction of the jet, calculate the force components in the direction of the vane velocity and across it, also the resultant force in magnitude and direction. Calculate the work done and power developed per kg of flow. **[AMIE]**

Solution. (a) Stationary vane:

Inlet angle of the vane $= 0°$

Outlet angle of the vane $= 25°$

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Velocity of water jet, $V = 50$ m/s

The force in the direction of jet per kg of flow,

$$F_x = 1 \times (V_{1x} - V_{2x})$$

where, $V_{1x} = 50$ m/s

$$V_{2x} = -V\cos 25° = -50\cos 25° = -45{\cdot}315 \text{ m/s}$$

$$\therefore \quad F_x = 1 \times [50 - (-45{\cdot}315)] = 95{\cdot}315 \text{ N}$$

Similarly, the force exerted by the jet in the direction perpendicular to the direction of the jet per kg of flow,

$$F_y = 1 \times (V_{1y} - V_{2y})$$

where, $V_{1y} = 0$, and

$$V_{2y} = V\sin 25° = 50 \sin 25° = 21{\cdot}13 \text{ m/s}$$

$$\therefore \quad F_y = 1\,(0 - 21{\cdot}13) = -21{\cdot}13 \text{ N}$$

(– ve sign indicates that F_y is acting in the *downward direction*.)

∴ Resulting force per N of water (Refer to Fig. 17·14),

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{95{\cdot}315^2 + 21{\cdot}13^2} = \mathbf{97{\cdot}63 \text{ N (Ans.)}}$$

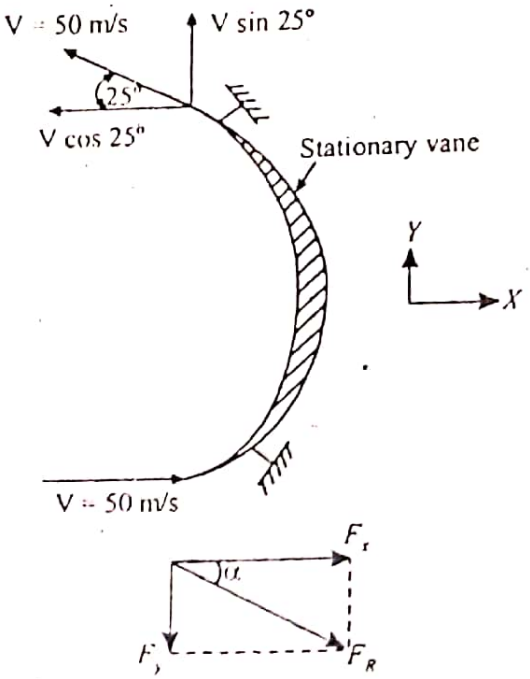


Fig. 17.14. Stationary vane.

The angle made by the resultant with X-axis,

$$\tan \alpha = \frac{F_y}{F_x} = \frac{21{\cdot}13}{95{\cdot}315} = 0{\cdot}2217$$

$$\therefore \quad \alpha = \tan^{-1} 0{\cdot}2217 = \mathbf{12{\cdot}5°\ (Ans.)}$$

(*b*) **Moving vane:**

Velocity of the vane, $u = 20$ m/s

The force exerted by the jet on the moving vane in the direction of jet,

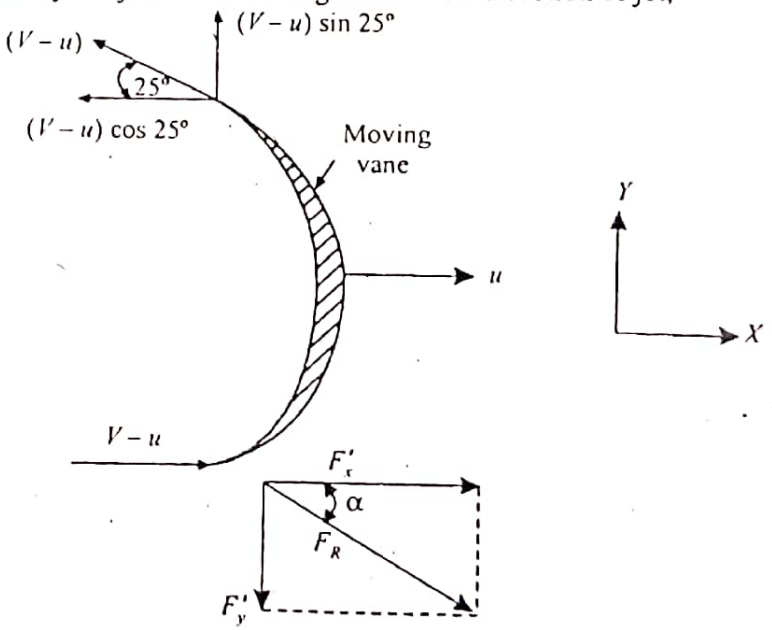


Fig. 17.15. Moving vane.

$$F'_x = 1 \times (V_{1x} - V_{2x})$$

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where, $V_{1x} = V - u = 50 - 20 = 30$ m/s

$V_{2x} = -(V-u)\cos 25° = -(50-20)\cos 25° = -27{\cdot}19$ m/s

$\therefore F'_x = 1 \times [30 - (-27{\cdot}19)] = \mathbf{57{\cdot}19\ N}$ **(Ans.)**

Similarly, $F'_y = 1 \times (V_{1y} - V_{2y})$

where $V_{1y} = 0$,

$V_{2y} = (V-u)\sin 25° = (50-20)\sin 25° = 12{\cdot}67$ m/s

$F'_y = 1 \times (0 - 12{\cdot}67) = \mathbf{-12{\cdot}67\ N}$ **(Ans.)**

$\therefore$ *Resultant force,* $F_R = \sqrt{F'^2_x + F'^2_y} = \sqrt{57{\cdot}19^2 + 12{\cdot}67^2} = \mathbf{58{\cdot}57\ N}$ **(Ans.)**

The angle made by the resultant with X-axis,

$$\tan\alpha = \frac{F'_y}{F'_x} = \frac{12{\cdot}67}{57{\cdot}19} = 0{\cdot}2215$$

or, $\alpha = \tan^{-1} 0{\cdot}2215 = \mathbf{12{\cdot}5°}$ **(Ans.)**

Work done per second $= F'_x \times u = 57{\cdot}19 \times 20 = \mathbf{1143 \cdot 8\ Nm/s}$ **(Ans.)**

$\therefore$ *Power developed* $= 1143{\cdot}8$ J/s or W or $1{\cdot}1438$ kW **(Ans.)**

Example 17·15. *A jet of water 80 mm diameter and having a velocity of 20 m/s impinges at the centre of hemispherical vane. The liner velocity of vane is 10 m/s in the direction of jet. Find the force exerted on the vane. How this force would change if the jet impinges on a series of vanes attached to the circumference of wheel ?*

Solution. Diameter of the jet, $d = 80$ mm $= 0{\cdot}08$ m

$\therefore$ Area, $a = \frac{\pi}{4} \times 0{\cdot}08^2 = 0{\cdot}005026\ m^3$

Velocity of the jet, $V = 20$ m/s

Velocity of the vane, $u = 10$ m/s

When the jet strikes at the centre of a moving vane, then effective incoming velocity $= V - u$

Force exerted on the vane, $F = \rho a (V-u)^2 (1 + \cos\theta)$ ...[Eqn. (17·18)]

For a hemispherical vane, $\theta = 0$

$\therefore$ $F = \rho a (V-u)^2 (1 + \cos 0°) = \rho a (V-u)^2 \times 2 = 2\rho a (V-u)^2$

$= 2 \times 1000 \times 0{\cdot}005026 (20-10)^2 = \mathbf{1005{\cdot}2\ N}$ **(Ans.)**

When the jet impinges on a *series of vanes*, the entire fluid mass issuing from the jet ($m = \rho a V$) hits the vane and the force exerted by the jet on the vane system is

$F = \rho a V (V-u)(1 + \cos\theta)$ ...[Eqn. (17.22)]

$= \rho a V (V-u)(1 + \cos 0°)$

$= 2\rho a V (V-u)$ $(\because \theta = 0°)$

$= 2 \times 1000 \times 0{\cdot}005026 \times 20 \times (20-10) = \mathbf{2010{\cdot}4\ N}$ **(Ans.)**

Example 17·16. *A jet strikes tangentially a smooth curved vane moving in the same direction as the jet, and the jet gets reversed in the direction. Show that the maximum efficiency is slightly less than 60 per cent.*

Solution. Refer to Fig 17·16

Let, V = Velocity of the jet, and

u = Velocity of the vane.

($V > u$ for impact)

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The force exerted by jet in the direction of motion of the vane,

$$F_x = \text{Mass/sec} \times \text{change in velocity}$$
$$= \rho a (V-u)[(V-u) - \{-(V-u)\}]$$

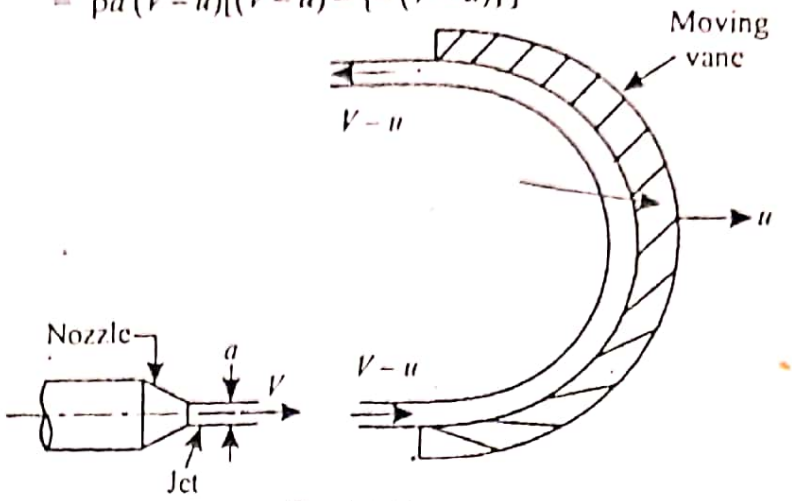


Fig. 17.16

$$= \rho a (V-u)[(V-u) + (V-u)]$$
$$= 2\rho a (V-u)^2$$

Work done $= F_x \times u = 2\rho a (V-u)^2 \times u$

$$\eta = \frac{\text{Work done on the vane}}{\text{Kinetic energy of the jet}}$$

$$= \frac{2\rho a (V-u)^2 \times u}{\frac{1}{2}(\rho a V) V^2} = 4 \times \frac{u (V-u)^2}{V^3} \quad ...(i)$$

For *maximum efficiency*, $\frac{d\eta}{dV} = 0$

or, $$\frac{d}{dV}\left[\frac{4u (V-u)^2}{V^3}\right] = 4u\left[\frac{V^3 \times 2(V-u) - (V-u)^2 \times 3V^2}{V^6}\right] = 0$$

or, $$\frac{2V^3 (V-u) - (V-u)^2 \times 3V^2}{V^6} = 0 \qquad (\because u \neq 0)$$

or, $$2V(V-u) - 3(V-u)^2 = 0$$

or, $$(V-u)[2V - 3(V-u)] = 0$$

or, $$(V-u)(2V - 3V + 3u) = 0$$

or, $$(V-u)(3u - V) = 0, (V-u) \neq 0, \text{ for impact,}$$

$\therefore$ $$3u - V = 0 \text{ or } u = \frac{V}{3}$$

Substituting this value of $u \left(= \frac{V}{3}\right)$ in expression (*i*), we get

$$\eta_{max} = 4 \times \frac{V/3\left(V - \frac{V}{3}\right)^2}{V^3} = \frac{16}{27} = 0.593 \text{ or } 59.3\%$$

which is slightly less than 60%. **(Ans.)**

17·8. Jet striking a moving Curved Vane Tangentially at One Tip and leaving at the Other

A. Single vane:

Fig. 17·17, shows a jet striking a moving curved vane tangentially at one tip and leaving at the other. The effective velocity with which the jet strikes the vane is the relative velocity (V_{r1}). The relative velocity V_{r1} may be obtained by drawing the velocity triangle at the *inlet*. The absolute

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velocity (V_2) at the exit may be obtained from the velocity triangle at the *outlet*. Fi 17·17 shows the velocity triangles.

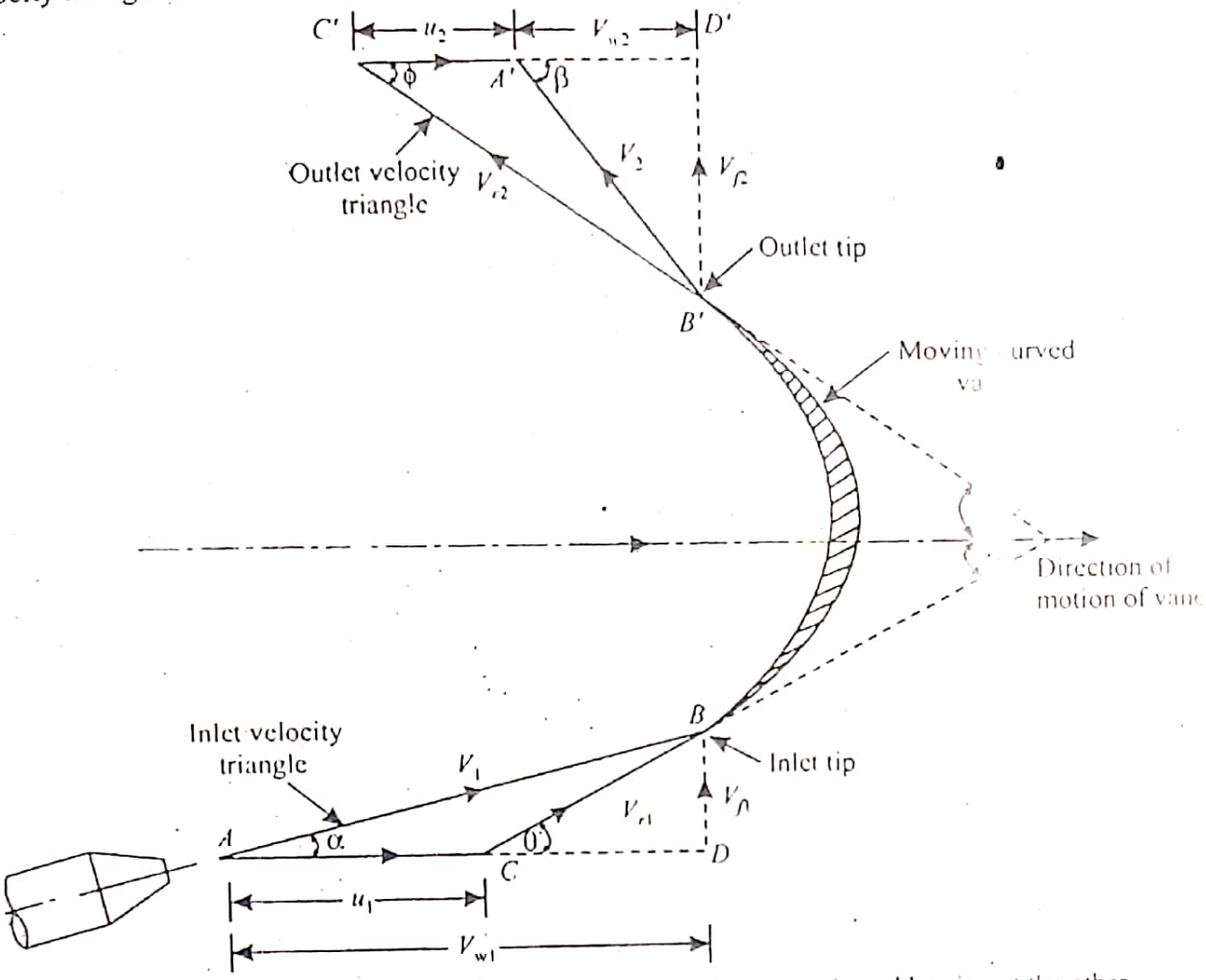


Fig. 17.17. Jet striking a moving curved vane tangentially at one tip and leavin at the other.

Let, V_1, V_2 = Absolute velocities of the jet at the inlet and outlet respec ely,

u_1, u_2 = Peripheral velocities of the vane at the inlet and outlet res ctively,

V_{r1}, V_{r2} = Relative velocities at the inlet and outlet respectively,

V_{f1}, V_{f2} = Velocities of the flow at the inlet and outlet respectively,

V_{w1}, V_{w2} = Velocities of the whirl at the inlet and outlet respectively,

θ, ϕ = Tip angles at the inlet and outlet respectively,

α, β = Angles which the absolute velocities make at the inlet an utlet respectively

It may be *noted* that:

- all angles are measured with the direction of motion of vane,
- the velocity of whirl is the component of the absolute velocity in the d ction of motion
- the velocity of flow is the component of the absolute velocity norm to the direction of motion.

The triangles *ABD* and *B'C'D'* are called *inlet* and *outlet velocity triangles* and e drawn as follow

1. Inlet velocity triangle:

- Take any point *A* and draw a line $AB = V_1$ (in magnitude), making n angle α with the horizontal line *AD*.
- Draw a line $AC = u_1$ and join *C* to *B*, *CB* then represents relative velo ty (V_{r1}) of the jet at inlet. If the loss of energy at inlet due to *impact is zero*, then *CB must be in the tangential direction* to the vane at inlet.

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- From B draw a perpendicular BD meeting the horizontal line AC produced at D. Then BD represents the velocity of flow at Inlet (V_{f1}). AD represents the velocity of whirl at inlet (V_{w1}). $\angle BCD = \theta$ = vane angle at inlet.

2. Outlet velocity triangle:

If the vane surface is assumed *smooth*, the energy loss due to friction will be zero and thus $= V_{r2}$ will be in tangential direction to the vane at outlet:

- Draw $B'C'$ in the tangential direction of the vane at outlet and cut $B'C' = V_{r2}$.
- From C' draw a line $C'A'$ in the direction of vane at outlet and equal to u_2 (the velocity of vane at outlet). Join $B'A'$. Then $B'A'$ represents the absolute velocity of the jet (V_2) at outlet in magnitude and direction.
- From B' draw a prependicular $B'D'$ to meet the line $C'A'$ produced at D'. Then $B'D'$ and $A'D'$ represent the velocity of flow (V_{f2}) and velocity of whirl (V_{w2}) at outlet respectively.
- ϕ = angle of vane at outlet, β = angle made by V_2 with the direction of motion of vane at outlet.

If vane is *smooth* and is having velocity in the direction of motion at inlet and outlet *equal*, then

$$u_1 = u_2 = u \text{ (velocity in the direction of motion), and } V_{r1} = V_{r2}$$

Mass of water striking the vane per second $= \rho a V_{r1}$

(where, a = area of jet water)

∴ Force exerted in the direction of motion,

F_x = Mass of water striking the vane per second × (Initial velocity with which the jet strikes in the *direction of motion* – final velocity)

$$= \rho a V_{r1}[V_{r1}\cos\theta - (-V_{r2}\cos\phi)]$$

But, $V_{r1}\cos\theta = (V_{w1} - u_1)$, and $V_{r2}\cos\phi = (u_2 + V_{w2})$ (Refer to Fig. 17·17)

$$F_x = \rho a V_{r1}[(V_{w1} - u_1) - \{-(u_2 + V_{w2})\}]$$

$$= \rho a V_{r1}[V_{w1} - u_1 + u_2 + V_{w2}]$$

$$F_x = \rho a V_{r1}(V_{w1} + V_{w2}) \quad (\because u_1 = u_2) \quad ...(i)$$

The eqn. (*i*) is *true only when* β *is an actue angle* (See Fig. 17·17), When $\beta = 90°$, $V_{w2} = 0$ the eqn. ( ) redues to

$$F_x = \rho a V_{r1}(V_{w1})$$

If β is an *obtuse angle*, the expression for F_x will become

$$F_x = \rho a V_{r1}(V_{w1} - V_{w2})$$

Thus, in general F_x is written as:

$$F_x = \rho a V_{r1}(V_{w1} \pm V_{w2}) \quad ...(17·27)$$

Work done per second by the jet on the vane

$$= F_x \times u = \rho a V_{r1}(V_{w1} \pm V_{w2}) \times u \quad ...(17·28)$$

∴ Work done per second per unit weight of fluid striking

$$= \frac{\rho a V_{r1}(V_{w1} \pm V_{w2}) \times u}{\text{Weight of fluid striking}}$$

$$= \frac{\rho a V_{r1}(V_{w1} \pm V_{w2}) \times u}{\rho a V_{r1} \times g}$$

$$= \frac{1}{g}(V_{w1} \pm V_{w2}) \times u \quad ...(17·29)$$

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B. Force exerted on a series of radial curved vanes :

Consider a series of radial curved vanes mounted on a wheel as shown in Fig. 17·18. The water impinges upon the vanes and the wheel starts rotating at a constant angular speed.

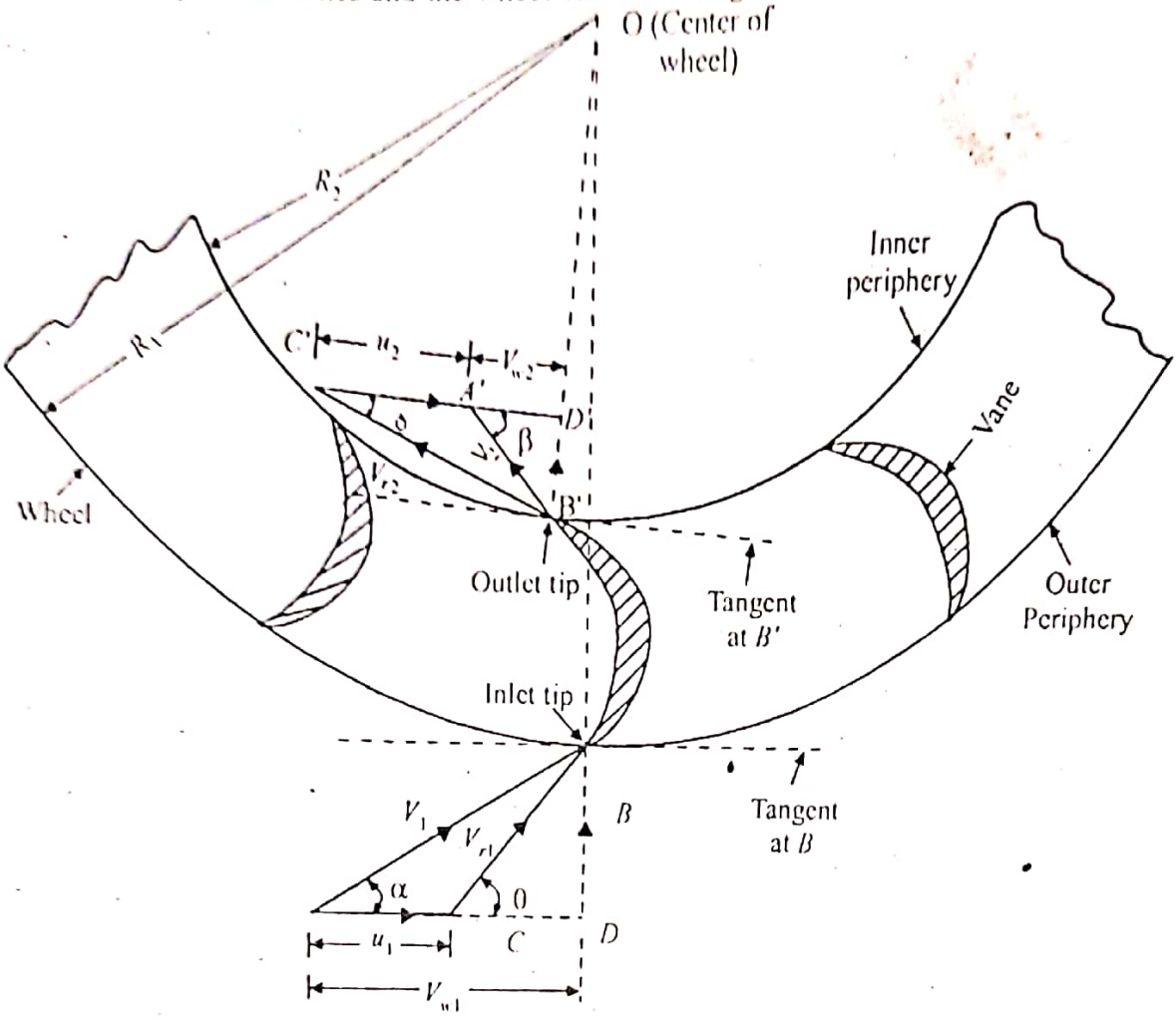


Fig. 17.18. Series of radially curved vanes mounted on a wheel.

Let, ω = Angular speed of the wheel, and

R_1, R_2 = Radii of the wheel at the inlet and outlet of the vane respectively.

As the vanes are situated radially round the wheel the blade velocities at the inlet and outlet tips of the vane would be different, *i.e.*,

$$u_1 = \omega R_1 \quad \text{and} \quad u_2 = \omega R_2$$

The flow system is inward or outward, depending upon whether the jet enters the outer periphery or the inner periphery.

The inlet and outlet triangles are shown in Fig. 17·18.

The mass of water striking per second (for a series of vanes)

= Mass of water issuing from the nozzle per second

$= \rho a V_1$ (where, a = area of jet, V_1 = velocity of jet).

Momentum of water striking the vanes (in tangential direction) per second *at inlet*

$= (\rho a V_1) \times V_{w1}$

[where, V_{w1} ($= V_1 \cdot \cos \alpha$) = component of V_1 in the *tangential direction.*]

Similarly, momentum of water per second at outlet

$= \rho a V_1 \times (-V_{w2}) = -\rho a V_1 \times V_{w2}$

[where, $V_{w2} = (V_2 \cos \beta)$ = component of V_2 in tangential direction]

–ve sign in taken as V_2 at outlet is in opposite direction.

Now, angular momentum per second at *inlet* = momentum at inlet × radius at inlet.

$= (\rho a V_1) \times V_{w1} \times R_1$

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